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Time-Invariant Fuzzy Max-Plus Linear Systems

Marcellinus Andy Rudhito
Department of Mathematics Education
Universitas Sanata Dharma, Pajajaran Magelang Yogyakarta
e-mail: arudhito@ubohs.co.id

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Abstract

In time-invariant max-plus linear systems (IMLS), the activity times are real numbers. In time-invariant fuzzy max-plus linear systems (IFMLS), as there is uncertainty in the activity times, the activity times are modeled as fuzzy numbers. This article discusses a generalization of IMLS to IFMLS, especially IFMLS with single input single output (SISO), and input-output analysis of IFMLS-SISO. We show that input-output analysis IFMLS-SISO associated with the latest times input problem can be solved by using a solution of a system of fuzzy number max-plus linear equations.

Keywords: Linear Systems, Max-Plus, Fuzzy Number, Time-Invariant, Input-Output.

Sistem Linier Max-Plus Samar yang Invariant terhadap Waktu

Abstrak

Dalam sistem linear max-plus waktu invariant (SLMI), waktu aktifitas berupa bilangan real. Dalam sistem linear max-plus kabur waktu invariant (SLMKI), ada ketidakpastian dalam waktu aktifitas, sehingga waktu aktifitas dimodelkan sebagai bilangan kabur. Artikel ini membahas suatu generalisasi SLMI menjadi SLMKI, khususnya SLMKI dengan satu input dan satu output (SISO), dan analisis input-output SLMKI-SISO. Dapat ditunjukkan bahwa analisis input-output SLMKI-SISO yang berkaitan dengan masalah waktu input paling lambat, dapat diselesaikan dengan menggunakan suatu penyelesaian sistem persamaan linear max-plus kabur.

Kata kunci: Sistem Linear, Max-Plus, Bilangan Kabur, Waktu-Invariant, Input-Output.

1. Introduction

In modeling and analyzing a network sometimes its activity time is not known, because, for example its phase design, data on time activity or distribution which are not fixed. The activity can be estimated based on the experience and opinions from experts and network operators. This network activity times are modeled using fuzzy number, and it is called fuzzy activity times (Chanas and Zielinski, 2001; Solitoni and Hajj, 2007).

Max-plus algebra, namely the set of all real numbers \mathbb{R} with operations max and plus, has been used to model and analyze network problems algebraically, such as problem of project scheduling and queuing system, with a deterministic time activity (Chanas and Zielinski, 2001; Heidergott et al., 2005; Krivulin, 2001; Rudhito, 2003). Rudhito (2004), de Schutter (1996) have discussed a model of simple production system dynamics using max-plus algebra approach. In general, this model is a system of max-plus linear time-invariant.

Fuzzy number max-plus algebra is an extension of max-plus algebra, in which the elements are fuzzy numbers. (Rudhito et al., 2008, 2011a). Moreover Rudhito et al. (2011b) studied matrices over fuzzy

number max-plus algebra and systems of fuzzy number linear equations.

In a manner analogous to Rudhito (2003) and de Schutter (1996), and taking into account the results on fuzzy numbers max-plus algebra, this paper will discuss the time-invariant fuzzy max-plus linear system using fuzzy number max-plus algebra.

2. Results and Discussion

First, we review some definitions and basic concepts of max-plus algebra and matrices over max-plus algebra. For further details see (Chanas and Zielinski, 2001; Rudhito, 2003). Let $\mathbb{R}_+ = \mathbb{R} \cup \{-\infty\}$ be the set of all real numbers and $\mathbb{R}_+ := -\infty$. In \mathbb{R}_+ we define two operations as follows:

for every $a, b \in \mathbb{R}_+$,

$$a \oplus b := \max(a, b) \text{ and } a \otimes b := a + b.$$

We can show that $(\mathbb{R}_+, \oplus, \otimes)$ is a commutative semiring whose neutral element $e = -\infty$ and unity element $\varepsilon = 0$. Moreover $(\mathbb{R}_+, \oplus, \otimes)$ is a semifield, that is $(\mathbb{R}_+, \oplus, \otimes)$ is a commutative semiring, where for every $a \in \mathbb{R}_+$ there exists $-a$ such that $a \oplus (-a) = e$. We call $(\mathbb{R}_+, \oplus, \otimes)$ a max-plus algebra, and write it as \mathbb{R}_{\max} . The max-plus algebra \mathbb{R}_{\max} has no zero divisors, that is for every

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2. Results and Discussion

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for every $a, b \in \mathbb{R}_\varepsilon$,

$$a \oplus b := \max(a, b) \text{ and } a \otimes b := a + b.$$

We can show that $(\mathbb{R}_\varepsilon, \oplus, \otimes)$ is a commutative semiring whose neutral element $\varepsilon = -\infty$ and unity element $e = 0$. Moreover, $(\mathbb{R}_\varepsilon, \oplus, \otimes)$ is a semifield, that is $(\mathbb{R}_\varepsilon, \oplus, \otimes)$ is a commutative semiring, where for every $a \in \mathbb{R}$ there exists $-a$ such that $a \otimes (-a) = 0$. We call $(\mathbb{R}_\varepsilon, \oplus, \otimes)$ a max-plus algebra, and write it as \mathbb{R}_{\max} . The max-plus algebra \mathbb{R}_{\max} has no zero divisors, that is for every

27 $x, y \in \mathbf{R}_\varepsilon$, if $x \otimes y = \varepsilon$, then $x = \varepsilon$ or $y = \varepsilon$. The relation " \preceq_m " as $x \preceq_m y$ iff $x \oplus y = y$. The max-plus algebra \mathbf{R}_{\max} can be partially ordered by introducing a partial order relation. Operations \oplus and \otimes are consistent 23 with respect to the order \preceq_m , that is, for every $a, b, c \in \mathbf{R}_{\max}$, if $a \preceq_m b$, then $a \oplus c \preceq_m b \oplus c$, and $a \otimes c \preceq_m b \otimes c$. We define $x^{\otimes 0} := 0$, $x^{\otimes k} := x \otimes x^{\otimes k-1}$ and $\varepsilon^{\otimes k} := \varepsilon$ for all $k = 1, 2, \dots$.

The operations \oplus and \otimes in \mathbf{R}_{\max} can be extended to matrix operation 16 in $\mathbf{R}_{\max}^{m \times n}$, where $\mathbf{R}_{\max}^{m \times n} := \{A = (A_{ij}) \mid A_{ij} \in \mathbf{R}_{\max}, \text{ for } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n\}$ is the set of all m -by- n matrices over max-plus algebra. Specifically, for $A, B \in \mathbf{R}_{\max}^{n \times n}$ and $\alpha \in \mathbf{R}_{\max}$ we define

$$\begin{aligned} (\alpha \otimes A)_{ij} &= \alpha \otimes A_{ij}, (A \oplus B)_{ij} = A_{ij} \oplus B_{ij} \\ \text{and } (A \otimes B)_{ij} &= \bigoplus_{k=1}^n A_{ik} \otimes B_{kj}. \end{aligned} \quad 12$$

For any matrix $A \in \mathbf{R}_{\max}^{n \times n}$, one can define

$$A^{\otimes 0} = E_n \text{ where } (E)_{ij} := \begin{cases} 0 & \text{where } i = j \\ \varepsilon & \text{where } i \neq j \end{cases} \quad 22$$

$$A^{\otimes k} = A \otimes A^{\otimes k-1} \text{ for } k = 1, 2, \dots$$

The relation " \preceq_m " is defined on $\mathbf{R}_{\max}^{m \times n}$ as A

$\preceq_m B$ iff $A \oplus B = B$. It is a partial order in $\mathbf{R}_{\max}^{m \times n}$. In $(\mathbf{R}_{\max}^{n \times n}, \oplus, \otimes)$, operations \oplus and \otimes are consistent with respect to the order \preceq_m , that is for every $A, B, C \in \mathbf{R}_{\max}^{n \times n}$, if $A \preceq_m B$, then $A \oplus C \preceq_m B \oplus C$ and $A \otimes C \preceq_m B \otimes C$.

We also recall 24 some definitions and some basic concepts of interval max-plus algebra, matrices over interval max-plus algebra. Further details can be found in Rudhitho et al. (2011a) 30 and de Schutter (1996). The (closed) interval x in \mathbf{R}_{\max} is a subset of \mathbf{R}_{\max} of the form $x = [\underline{x}, \bar{x}] = \{x \in \mathbf{R}_{\max} \mid \underline{x} \preceq_m x \preceq_m \bar{x}\}$. The interval x in \mathbf{R}_{\max} 15 called max-plus interval or shortly interval. Define $\mathbf{I}(\mathbf{R})_\varepsilon := \{x = [\underline{x}, \bar{x}] \mid \underline{x}, \bar{x} \in \mathbf{R}, \varepsilon \preceq_m \underline{x} \preceq_m \bar{x}\}$ 9 $\{\varepsilon\}$, where $\varepsilon := [\varepsilon, \varepsilon]$. In $\mathbf{I}(\mathbf{R})_\varepsilon$ we can define two operations \oplus and \otimes where

$$x \oplus y = [\underline{x} \oplus \underline{y}, \bar{x} \oplus \bar{y}]$$

and

$$x \otimes y = [\underline{x} \otimes \underline{y}, \bar{x} \otimes \bar{y}] \quad 28$$

for every $x, y \in \mathbf{I}(\mathbf{R})_\varepsilon$. The algebraic structure $(\mathbf{I}(\mathbf{R})_\varepsilon, \oplus, \otimes)$ is a commutative semiring with neutral

element $\varepsilon = [\varepsilon, \varepsilon]$ and unity element $0 = [0, 0]$. This commutative semiring $(\mathbf{I}(\mathbf{R})_\varepsilon, \oplus, \otimes)$ is called the interval max-plus algebra and 10 written as $\mathbf{I}(\mathbf{R})_{\max}$. Relation " \preceq_{Im} " defined on $\mathbf{I}(\mathbf{R})_{\max}$ as $x \preceq_{\text{Im}} y \Leftrightarrow x \oplus y = y$ 34, and it is a partial order on $\mathbf{I}(\mathbf{R})_{\max}$. Notice that $x \oplus y = y \Leftrightarrow \underline{x} \preceq_m \underline{y}$ and $\bar{x} \preceq_m \bar{y}$.

19 Define $\mathbf{I}(\mathbf{R})_{\max}^{m \times n} := \{A = (A_{ij}) \mid A_{ij} \in \mathbf{I}(\mathbf{R})_{\max}, \text{ for } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n\}$. The elements of $\mathbf{I}(\mathbf{R})_{\max}^{m \times n}$ are called matrices over interval max-plus algebra or shortly interval matrices. The operations \oplus and \otimes in $\mathbf{I}(\mathbf{R})_{\max}$ can be extended to matrix operations on $\mathbf{I}(\mathbf{R})_{\max}^{m \times n}$. Specifically, for $A, B \in \mathbf{I}(\mathbf{R})_{\max}^{n \times n}$ and $\alpha \in \mathbf{I}(\mathbf{R})_{\max}$ we define $(\alpha \otimes A)_{ij} = \alpha \otimes A_{ij}$, $(A \oplus B)_{ij} = A_{ij} \oplus B_{ij}$ and $(A \otimes B)_{ij} = \bigoplus_{k=1}^n A_{ik} \otimes B_{kj}$.

Relation " \preceq_{Im} " defined on $\mathbf{I}(\mathbf{R})_{\max}^{m \times n}$ with $A \preceq_{\text{Im}} B \Leftrightarrow A \oplus B = B$ is a partial order. Notice that $A \oplus B = B \Leftrightarrow A_{ij} \oplus B_{ij} = B_{ij} \Leftrightarrow A_{ij} \preceq_{\text{Im}} B_{ij} \Leftrightarrow \underline{A}_{ij} \preceq_m \underline{B}_{ij}$ and $\bar{A}_{ij} \preceq_m \bar{B}_{ij}$ for every i and j .

In further discussion, we assume that the reader knows about some basic concepts in fuzzy set and fuzzy number. For further details see Lee (2005) and Zimmermann (1991). Firstly, we will review a theorem in the fuzzy set that we will use later on.

Theorem 2.1 (Decomposition 6 Theorem, Zimmermann, 1991) If A^α is a α -cut of fuzzy set \tilde{A} in X and \tilde{A}^α is a fuzzy set in X with membership function $\mu_{\tilde{A}^\alpha}(x) = \alpha \chi_{A^\alpha}(x)$, where χ_{A^α} is the characteristic function of set A^α , then $\tilde{A} = \bigcup_{\alpha \in [0,1]} \tilde{A}^\alpha$.

Proof: (Zimmermann, 1991).

3 Definition 2.2 (Rudhitho et al., 2011b). Let $\mathbf{F}(\mathbf{R})_\varepsilon := \mathbf{F}(\mathbf{R}) \cup \{\tilde{\varepsilon}\}$, where $\mathbf{F}(\mathbf{R})$ is the set of all fuzzy numbers and $\tilde{\varepsilon} := \{-\infty\}$, with the α -cut of $\tilde{\varepsilon}$ is $\varepsilon^\alpha = [-\infty, -\infty]$ for every $\alpha \in [0, 1]$. We define two operations 31 \oplus and \otimes as follows: for every $\tilde{a}, \tilde{b} \in \mathbf{F}(\mathbf{R})_\varepsilon$ with $\tilde{a}^\alpha = [\underline{a}^\alpha, \bar{a}^\alpha] \in \mathbf{I}(\mathbf{R})_{\max}$ and $\tilde{b}^\alpha = [\underline{b}^\alpha, \bar{b}^\alpha] \in \mathbf{I}(\mathbf{R})_{\max}$.

i) Maximum of \tilde{a} and \tilde{b} , written $\tilde{a} \oplus \tilde{b}$, is a fuzzy number whose α -cut is the interval $[\underline{a}^\alpha \oplus \underline{b}^\alpha, \bar{a}^\alpha \oplus \bar{b}^\alpha]$ for every $\alpha \in [0, 1]$

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- ii) Addition of \tilde{a} and \tilde{b} , written $\tilde{a} \oplus \tilde{b}$, is a fuzzy number whose α -cut is interval $[\underline{a}^\alpha \oplus \underline{b}^\alpha, \overline{a}^\alpha \oplus \overline{b}^\alpha]$ for every $\alpha \in [0, 1]$.

We can show that α -cut in this definition satisfies the conditions of α -cut of a fuzzy number. The commutative semiring $\mathbf{F}(\mathbf{R})_{\max} := ((\mathbf{F}(\mathbf{R}))_{\tilde{\varepsilon}}, \oplus, \otimes)$ is called fuzzy number max-plus algebra, and is written as $\mathbf{F}(\mathbf{R})_{\max}$. Relation " \preceq_{Fm} " defined on $\mathbf{F}(\mathbf{R})_{\max}$ as $\tilde{x} \preceq_{\text{Fm}} \tilde{y} \Leftrightarrow \tilde{x} \oplus \tilde{y} = \tilde{y}$ is a partial order in $\mathbf{F}(\mathbf{R})_{\max}$. Notice that $\tilde{x} \oplus \tilde{y} = \tilde{y} \Leftrightarrow x^\alpha \oplus y^\alpha = y^\alpha \Leftrightarrow x^\alpha \preceq_{\text{Im}} y^\alpha \Leftrightarrow \underline{x}^\alpha \preceq_m \underline{y}^\alpha$ and $\overline{x}^\alpha \preceq_m \overline{y}^\alpha$ for every $\alpha \in [0, 1]$.

Definition 2.3 (Rudhito et al., 2011a) Define $\mathbf{F}(\mathbf{R})_{\max}^{m \times n} := \{ \tilde{A} = (\tilde{A}_{ij}) \mid \tilde{A}_{ij} \in \mathbf{F}(\mathbf{R})_{\max}, \text{ for } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n \}$. The elements of $\mathbf{F}(\mathbf{R})_{\max}^{m \times n}$ are called matrices over fuzzy number max-plus algebra.

These matrices are also called *fuzzy number matrix*. The operations \oplus and \otimes in $\mathbf{F}(\mathbf{R})_{\max}$ can be extended to the operations of fuzzy number matrices in $\mathbf{F}(\mathbf{R})_{\max}^{m \times n}$. Specifically, for the matrices $\tilde{A}, \tilde{B} \in \mathbf{F}(\mathbf{R})_{\max}^{m \times n}$, we define

$$(\tilde{A} \oplus \tilde{B})_{ij} = \tilde{A}_{ij} \oplus \tilde{B}_{ij}$$

and

$$(\tilde{A} \otimes \tilde{B})_{ij} = \bigoplus_{k=1}^n \tilde{A}_{ik} \otimes \tilde{B}_{kj}.$$

For every $\tilde{A} \in \mathbf{F}(\mathbf{R})_{\max}^{m \times n}$ and for some number $\alpha \in [0, 1]$ define a α -cut matrix of \tilde{A} as the interval matrix $A^\alpha = (A_{ij}^\alpha)$, with A_{ij}^α is the α -cut of \tilde{A}_{ij} for every i and j .

Matrices $\underline{A}^\alpha = (A_{ij}^\alpha) \in \mathbf{R}_{\max}^{m \times n}$ and $\overline{A}^\alpha = (\overline{A}_{ij}^\alpha) \in \mathbf{R}_{\max}^{m \times n}$ are called *lower bound* and *upper bound of matrix* A^α , respectively. We conclude that the matrices $\tilde{A}, \tilde{B} \in \mathbf{F}(\mathbf{R})_{\max}^{m \times n}$ are equal iff $A^\alpha = B^\alpha$, that is $A_{ij}^\alpha = B_{ij}^\alpha$ for every $\alpha \in [0, 1]$ and for every i and j . For every fuzzy number matrix $\tilde{A}, A^\alpha = [\underline{A}^\alpha, \overline{A}^\alpha]$. Let

$\tilde{\lambda} \in \mathbf{F}(\mathbf{R})_{\max}$, $\tilde{A}, \tilde{B} \in \mathbf{F}(\mathbf{R})_{\max}^{m \times n}$. We can show that $(\tilde{\lambda} \otimes \tilde{A})^\alpha = [\underline{\lambda}^\alpha \otimes \underline{A}^\alpha, \overline{\lambda}^\alpha \otimes \overline{A}^\alpha]$ and $(\tilde{A} \oplus \tilde{B})^\alpha = [\underline{A}^\alpha \oplus \underline{B}^\alpha, \overline{A}^\alpha \oplus \overline{B}^\alpha]$ for every $\alpha \in [0, 1]$. Let $\tilde{A} \in \mathbf{F}(\mathbf{R})_{\max}^{m \times p}$, $\tilde{B} \in \mathbf{F}(\mathbf{R})_{\max}^{p \times n}$. We can show that $(\tilde{A} \otimes$

$\tilde{B})^\alpha = [\underline{A}^\alpha \otimes \underline{B}^\alpha, \overline{A}^\alpha \otimes \overline{B}^\alpha]$ for every $\alpha \in [0, 1]$.

Relation " \preceq_{Fm} " defined on $\mathbf{F}(\mathbf{R})_{\max}^{m \times n}$ with $\tilde{A} \preceq_{\text{Fm}} \tilde{B} \Leftrightarrow \tilde{A} \oplus \tilde{B} = \tilde{B}$ is a partial order. Notice that $\tilde{A} \oplus \tilde{B} = \tilde{B} \Leftrightarrow \tilde{A}_{ij} \oplus \tilde{B}_{ij} = \tilde{B}_{ij} \Leftrightarrow \underline{A}_{ij}^\alpha \oplus \underline{B}_{ij}^\alpha = \underline{B}_{ij}^\alpha \Leftrightarrow \underline{A}_{ij}^\alpha \preceq_{\text{Im}} \underline{B}_{ij}^\alpha \Leftrightarrow \underline{A}_{ij}^\alpha \preceq_m \underline{B}_{ij}^\alpha$ and $\overline{A}_{ij}^\alpha \preceq_m \overline{B}_{ij}^\alpha$ for every α

$\in [0, 1]$ and for every i and j . Define $\mathbf{F}(\mathbf{R})_{\max}^n := \{ \tilde{x} = [\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n]^T \mid \tilde{x}_i \in \mathbf{F}(\mathbf{R})_{\max}, i = 1, 2, \dots, n \}$. The elements in $\mathbf{F}(\mathbf{R})_{\max}^n$ are called fuzzy number vectors over $\mathbf{F}(\mathbf{R})_{\max}$ or shortly fuzzy number vectors.

Definition 2.4 (Rudhito et al., 2011b) Given $\tilde{A} \in \mathbf{F}(\mathbf{R})_{\max}^{n \times n}$ and $\tilde{b} \in \mathbf{F}(\mathbf{R})_{\max}^n$. A fuzzy number vector $\tilde{x}^* \in \mathbf{F}(\mathbf{R})_{\max}^n$ is called fuzzy number solution of system $\tilde{A} \otimes \tilde{x} = \tilde{b}$ if \tilde{x}^* satisfies the system. A fuzzy number vector $\tilde{x}' \in \mathbf{F}(\mathbf{R})_{\max}^n$ is called fuzzy number subsolution of system $\tilde{A} \otimes \tilde{x} = \tilde{b}$ if $\tilde{A} \otimes \tilde{x}' \preceq_{\text{Fm}} \tilde{b}$.

Definition 2.5 (Rudhito et al., 2011b) Given $\tilde{A} \in \mathbf{F}(\mathbf{R})_{\max}^{n \times n}$ and $\tilde{b} \in \mathbf{F}(\mathbf{R})_{\max}^n$. A fuzzy number vector $\hat{\tilde{x}} \in \mathbf{F}(\mathbf{R})_{\max}^n$ is called greatest fuzzy number subsolution of system $\tilde{A} \otimes \tilde{x} = \tilde{b}$ if $\tilde{x}' \preceq_{\text{Fm}} \hat{\tilde{x}}$ for every fuzzy number subsolution \tilde{x}' of the system $\tilde{A} \otimes \tilde{x} = \tilde{b}$.

Definition 2.6 (Rudhito et al., 2011) Given $\tilde{A} \in \mathbf{F}(\mathbf{R})_{\max}^{n \times n}$ with the entries of each column are not all equal to $\tilde{\varepsilon}$ and $\tilde{b} \in \mathbf{F}(\mathbf{R})_{\max}^n$. Defined a fuzzy number vector $\hat{\tilde{x}}$ whose components are $\hat{\tilde{x}}_i$, that is a fuzzy number with the α -cut is $\hat{\tilde{x}}_i^\alpha = [\underline{\hat{x}}_i^\alpha, \overline{\hat{x}}_i^\alpha]$. The bounds of $\hat{\tilde{x}}_i^\alpha$ are defined recursively as follows. Let

$\hat{\underline{x}}_i^\alpha = \min\{-(\underline{A}^\alpha)^T \otimes (-\underline{b}^\alpha)\}_i, -((\underline{A}^\alpha)^T \otimes -\underline{b}^\alpha)\}_i\}$ and $\hat{\overline{x}}_i^\alpha = -((\overline{A}^\alpha)^T \otimes -\overline{b}^\alpha)\}_i$, follows

$$\hat{\tilde{x}}_i^\alpha = \begin{cases} \min_{\beta \in [0, 1]} \{\hat{\tilde{x}}_i^\beta\}, & \text{if } \min_{\beta \in [0, 1]} \{\hat{\tilde{x}}_i^\beta\} \preceq_m \hat{\tilde{x}}_i^\alpha \\ \hat{\tilde{x}}_i^\beta, \text{ if } \hat{\tilde{x}}_i^\beta \preceq_m \hat{\tilde{x}}_i^\alpha, \forall \alpha, \beta \in [0, 1], \alpha > \beta, \text{ if } \hat{\tilde{x}}_i^\alpha \prec_m \min_{\beta \in [0, 1]} \{\hat{\tilde{x}}_i^\beta\} \\ \hat{\tilde{x}}_i^\alpha, \text{ if } \hat{\tilde{x}}_i^\beta \succ_m \hat{\tilde{x}}_i^\alpha \end{cases}$$

$$\hat{\tilde{x}}_i^\alpha = \begin{cases} \hat{\tilde{x}}_i^\beta, \text{ if } \hat{\tilde{x}}_i^\beta \preceq_m \hat{\tilde{x}}_i^\alpha, \forall \alpha, \beta \in [0, 1] \text{ with } \alpha < \beta \\ \hat{\tilde{x}}_i^\alpha, \text{ if } \hat{\tilde{x}}_i^\beta \succ_m \hat{\tilde{x}}_i^\alpha \end{cases}$$

We can show that the α -cut family of the components of the fuzzy number vector $\tilde{\mathbf{x}}$ as in Definition 2.6 is really an α -cut family of a fuzzy number. The following theorem gives an existence of the greatest fuzzy number subsolution of the system $\tilde{A} \otimes \tilde{\mathbf{x}} = \tilde{\mathbf{b}}$.

Theorem 2.7 (Ruditho *et al.*, 2011b) Given $\tilde{A} \in \mathbf{F}(\mathbf{R})_{\max}^{n \times n}$ with the entries of each column are not all equal to $\tilde{\varepsilon}$ and $\tilde{\mathbf{b}} \in \mathbf{F}(\mathbf{R})_{\max}^n$. Fuzzy number vector $\tilde{\mathbf{x}}$ which components are defined in Definition 2.7 is the greatest fuzzy number subsolution of system $\tilde{A} \otimes \tilde{\mathbf{x}} = \tilde{\mathbf{b}}$.

Proof : See Ruditho *et al.* (2011b).

Next we will discuss time invariant fuzzy max-plus linear systems (IFMLS) with single input single output (SISO).

Definition 2.8 (IFMLS-SISO $(\tilde{A}, \tilde{B}, \tilde{C}, \hat{x}_0)$). A time-invariant fuzzy max-plus linear system with single input single output is a discrete even system which can be written as follows

$$\tilde{y}(k+1) = \tilde{A} \otimes \tilde{\mathbf{x}}(k) \oplus \tilde{B} \otimes \tilde{u}(k+1)$$

$$\tilde{y}(k) = \tilde{C} \otimes \tilde{\mathbf{x}}(k)$$

for $k = 1, 2, 3, \dots$, with initial condition $\tilde{\mathbf{x}}(0) = \tilde{\mathbf{x}}_0$,

$\tilde{A} \in \mathbf{F}(\mathbf{R})_{\max}^{n \times n}$, $\tilde{B} \in \mathbf{F}(\mathbf{R})_{\max}^n$ and $\tilde{C} \in \mathbf{F}(\mathbf{R})_{\max}^{1 \times n}$.

Fuzzy number vector $\tilde{\mathbf{x}}(k) \in \mathbf{F}(\mathbf{R})_{\max}^n$ represents the state, $\tilde{u}(k) \in \mathbf{F}(\mathbf{R})_{\max}$ is the input fuzzy scalar and $\tilde{y}(k) \in \mathbf{F}(\mathbf{R})_{\max}$ is the output fuzzy scalar of system for the k -th time.

If the initial condition and the input sequences are given for these systems, we can determine recursively the output sequence of the system. The general form of the input-output for the system is given in the following theorem.

Theorem 2.9 (Input-Output IFMLS-SISO $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{\mathbf{x}}_0)$) Given a positive integer p . If $\tilde{\mathbf{u}} = [\tilde{u}(1), \tilde{u}(2), \dots, \tilde{u}(p)]^T$ and $\tilde{\mathbf{y}} = [\tilde{y}(1), \tilde{y}(2), \dots, \tilde{y}(p)]^T$ are fuzzy vector of input sequences and the fuzzy vector of output sequences, respectively, for FMLS-SISO $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{\mathbf{x}}_0)$, then

$$\tilde{\mathbf{y}} = \tilde{K} \otimes \tilde{\mathbf{x}}_0 \oplus \tilde{H} \otimes \tilde{\mathbf{u}}$$

with

$$\tilde{K} = \begin{bmatrix} \tilde{C} \otimes \tilde{A} \\ \tilde{C} \otimes \tilde{A}^{\otimes 2} \\ \vdots \\ \tilde{C} \otimes \tilde{A}^{\otimes p} \end{bmatrix} \text{ and}$$

$$\tilde{H} = \begin{bmatrix} \tilde{C} \otimes \tilde{B} & \tilde{\varepsilon} & \dots & \tilde{\varepsilon} \\ \tilde{C} \otimes \tilde{A} \otimes \tilde{B} & \tilde{C} \otimes \tilde{B} & \dots & \tilde{\varepsilon} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{C} \otimes \tilde{A}^{\otimes p-1} \otimes \tilde{B} & \tilde{C} \otimes \tilde{A}^{\otimes p-2} \otimes \tilde{B} & \dots & \tilde{C} \otimes \tilde{B} \end{bmatrix}$$

Proof: Using mathematical induction we can show that

$$\tilde{\mathbf{x}}(k) = (\tilde{A}^{\otimes k} \otimes \tilde{\mathbf{x}}_0) \oplus \left(\bigoplus_{i=1}^k (\tilde{A}^{\otimes k-i} \otimes \tilde{B} \otimes \tilde{u}(i)) \right)$$

$\tilde{u}(i)$ for $k = 1, 2, 3, \dots$.

$$\text{Hence } \tilde{\mathbf{y}}(k) = (\tilde{C} \otimes \tilde{A}^{\otimes k} \otimes \tilde{\mathbf{x}}_0) \oplus$$

$$\left(\bigoplus_{i=1}^k \tilde{C} \otimes \tilde{A}^{\otimes k-i} \otimes \tilde{B} \otimes \tilde{u}(i) \right), \text{ and then}$$

$$\begin{bmatrix} \tilde{y}(1) \\ \tilde{y}(2) \\ \vdots \\ \tilde{y}(p) \end{bmatrix} = \begin{bmatrix} \tilde{C} \otimes \tilde{A} \\ \tilde{C} \otimes \tilde{A}^{\otimes 2} \\ \vdots \\ \tilde{C} \otimes \tilde{A}^{\otimes p} \end{bmatrix} \otimes \tilde{\mathbf{x}}_0 \oplus \begin{bmatrix} \tilde{C} \otimes \tilde{B} & \tilde{\varepsilon} & \dots & \tilde{\varepsilon} \\ \tilde{C} \otimes \tilde{A} \otimes \tilde{B} & \tilde{C} \otimes \tilde{B} & \dots & \tilde{\varepsilon} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{C} \otimes \tilde{A}^{\otimes p-1} \otimes \tilde{B} & \tilde{C} \otimes \tilde{A}^{\otimes p-2} \otimes \tilde{B} & \dots & \tilde{C} \otimes \tilde{B} \end{bmatrix} \otimes \begin{bmatrix} \tilde{u}(1) \\ \tilde{u}(2) \\ \vdots \\ \tilde{u}(p) \end{bmatrix}.$$

Let p be a positive integer, $\tilde{\mathbf{u}} = [\tilde{u}(1), \tilde{u}(2), \dots, \tilde{u}(p)]^T$ and $\tilde{\mathbf{y}} = [\tilde{y}(1), \tilde{y}(2), \dots, \tilde{y}(p)]^T$ are fuzzy vector of input sequences and fuzzy vector of output sequences, respectively. We will determine the vector of the latest times input (the greatest fuzzy input vector $\tilde{\mathbf{u}}$) such that $\tilde{K} \otimes \tilde{\mathbf{x}}_0 \oplus \tilde{H} \otimes \tilde{\mathbf{u}} \preceq_{\text{Fm}} \tilde{\mathbf{y}}$, with \tilde{K} and \tilde{H} are given by Theorem 2.9.

Theorem 2.10 Given a IFMLS-SISO $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{\mathbf{x}}_0)$ with $\tilde{C} \otimes \tilde{B} \neq \tilde{\varepsilon}$, where $(\tilde{\varepsilon})_{ij} := \tilde{\varepsilon}$ for every i and j . If $\tilde{K} \otimes \tilde{\mathbf{x}}_0 \preceq_{\text{Fm}} \tilde{\mathbf{y}}$, then solution of the latest times input problem for the system is given by $\tilde{\mathbf{u}}$ whose components are \tilde{u}_i , i.e. it is a fuzzy number with the α -cut is $\tilde{u}_i^\alpha = [\underline{u}_i^\alpha, \overline{u}_i^\alpha]$.

The bounds of \tilde{u}_i^α are defined recursively as follows. Let

$$\underline{u}_i^\alpha = \min \{ -((\underline{H}^\alpha)^T \otimes (-\underline{\mathbf{y}}^\alpha))_i, -((\overline{H}^\alpha)^T \otimes (-\overline{\mathbf{y}}^\alpha))_i \} \text{ and}$$

$$\begin{aligned} \overline{u}_i^\alpha &= -((H^\alpha)^T \otimes -y^\alpha)_i, \\ \overline{u}_i^\alpha &= \\ \begin{cases} \min_{\beta \in [0,1]} \{\overline{u}_i^\beta\}, & \text{if } \min_{\beta \in [0,1]} \{\overline{u}_i^\beta\} \preceq_m \overline{u}_i^\alpha \\ \overline{u}_i^\beta, & \text{if } \overline{u}_i^\beta \preceq_m \overline{u}_i^\alpha, \forall \alpha, \beta \in [0,1] \alpha > \beta, \text{ if } \overline{u}_i^\alpha \prec_m \min_{\beta \in [0,1]} \{\overline{u}_i^\beta\} \\ \overline{u}_i^\alpha, & \text{if } \overline{u}_i^\beta \succ_m \overline{u}_i^\alpha \end{cases}, \\ \overline{u}_i^\alpha &= \begin{cases} \overline{u}_i^\beta, & \text{if } \overline{u}_i^\beta \preceq_m \overline{u}_i^\alpha, \forall \alpha, \beta \in [0,1] \text{ with } \alpha < \beta. \\ \overline{u}_i^\alpha, & \text{if } \overline{u}_i^\beta \succ_m \overline{u}_i^\alpha \end{cases} \end{aligned}$$

Proof: Since $\tilde{K} \otimes \tilde{x}_0 \preceq_{Fm} \tilde{y}$, then $\tilde{K} \otimes \tilde{x}_0 \oplus \tilde{H} \otimes \tilde{u} = \tilde{y} \Leftrightarrow \tilde{H} \otimes \tilde{u} = \tilde{y}$. Hence, the latest times input problem for FMLSI-SISO $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{x}_0)$ become the problem of determining the greatest fuzzy input vector \tilde{u} such that $\tilde{H} \otimes \tilde{u} \preceq_{Fm} \tilde{y}$, that is to find the greatest fuzzy number subsolution of system $\tilde{H} \otimes \tilde{u} = \tilde{y}$. Since $\tilde{C} \otimes \tilde{B} \neq \tilde{\varepsilon}$, then the entries of each column \tilde{H} are not all equal to $\tilde{\varepsilon}$. According to the Theorem 2.7, the greatest fuzzy number subsolution of system $\tilde{H} \otimes \tilde{u} = \tilde{y}$ is given by \tilde{u} whose components are \tilde{u}_i as given by Theorem 2.10 above. ■

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