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Chatarina Enny Murwaningtyas

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European Option Pricing by Using the Mixed Fractional Brownian Motion

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Option Pricing by Using the Mixed Fractional Brownian Motion

Chatarina Enny Murwaningtyas[1,2, a)], Sri Haryatmi Kartiko[1, b)],
Gunardi[1, c)], and Herry Pribawanto Suryawan [3, d)]

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Abstract. Financial modeling is conventionally based on semi-martingale processes with stationary and independent increments. However, empirical investigations on financial data do not always support these assumptions. Fractional Brownian motion (fBm) is a continuous Gaussian process with dependent increments. Actually, the main problem of applying fBm in finance is that an option with fBm is not arbitrage-free. To handle this problem, it is reasonable to use a mixed fractional Brownian motion (mfBm) in order to capture fluctuations of the financial assets. The mfBm is a family of Gaussian processes that is a linear combination of a Brownian motion and an independent fractional Brownian motion. This paper deals with the problem of pricing European options by using mfBm. Based on quasi-conditional expectation and Fourier transform method, we present a pricing model for a stock option and obtain a pricing formula for European call options.

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European Option Pricing by Using the Mixed Fractional Brownian Motion

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Abstract. Financial modeling is conventionally based on semi-martingale processes with stationary and independent increments. However, empirical investigations on financial data do not always support these assumptions. Fractional Brownian motion is a continuous Gaussian process with dependent increments. Actually, the main problem in applying a fractional Brownian motion in option pricing is not arbitrage-free. To handle this problem, it is reasonable to use a mixed fractional Brownian motion in order to capture fluctuations of the financial assets. The mixed fractional Brownian motion is a family of Gaussian processes that is a linear combination of a Brownian motion and an independent fractional Brownian motion. This paper deals with the problem of European option pricing by using the mixed fractional Brownian motion. Based on quasi-conditional expectations and Fourier transform method, we present a pricing model for a stock option and obtain a pricing formula for European call options.

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INTRODUCTION

An option is a contract that gives a person the right, but not the obligation, to buy or sell an underlying asset at a specific price on or before a certain date. The right to buy is called a call option and the right to sell is called a put option. The option contract can be either an American style or a European style. American options allow the owner to exercise his option between purchase and the strike date, while European options can be exercised only on the strike date. In this paper, we will concentrate on a European call option.

Black and Scholes [1] in 1973 proposed the famous Black-Scholes model in a stock market based on a geometric Brownian motion and gave an option pricing formula. A Brownian motion is a semi-martingale process with stationary and independent increments. The existence of sort memory or long memory in financial stock returns has been an important subject of both theoretical and empirical research. If stock returns display the sort memory or long memory, the series realizations are said to be not independent over time. This is the case of a fractional Brownian motion. A number of studies have tested the sort memory and long memory hypotheses for the stock market returns, for example [2–8].

The Fractional Brownian Motion is a stochastic process introduced by Kolmogorov [9] in 1940. Mandelbrot and Van Ness [10] gave a representation theorem for Kolmogorov's process and introduced the name of fractional Brownian motion in 1968. The fractional Brownian motion has further been developed by Hurst [11,12] and Mandelbrot [13]. Nowadays, the fractional Brownian motion plays an increasingly important role in many fields of studies such as hydrology [12,14], insurance [15,16] and finance [17–19].

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The stochastic integral with respect to a fractional Brownian motion is different from the classical Itô integral because a fractional Brownian motion is not a semi-martingale. Duncan et al [20] introduced the Wick product in the definition of stochastic integration for a fractional Itô's formula. They also introduced Girsanov's theorem under the fractional Brownian motion. Hu and Øksendal [21] and Elliott and van der Hoek [22] show that there is no arbitrage if the Wick product is used in the definition of stochastic integration. Hu and Oksendal [21] derived a formula for the price at time $t = 0$ of a European call option. Necula [23] extended the formula in [21] for $t \in [0, T]$. Moreover, Necula proved some results regarding the quasi-conditional expectation by using Fourier transform.

The formula for evaluation of a European call option obtained in [21] is arbitrage-free and complete market. However, Bender et al. [24] and Bjork et al. [25] still saw the possibility of arbitrage opportunities in the resulting model in [21]. Cheridito [26] and Bender et al. [27] proposed the mixed fractional Brownian motion to reduce arbitrage opportunities. A mixed fractional Brownian motion is a linear combination of a Brownian motion and an independent fractional Brownian motion.

The purpose of this article is to obtain the option pricing formula for European call options where the underlying price is assumed to follow the mixed fractional Brownian motion model. This paper is organized as follows. Section 2 introduces definitions and properties of a mixed fractional Brownian motion. Section 3 explores some results regarding the quasi-conditional expectation. Section 4 uses theorems in Section 3 in the study of the European option pricing. Section 5 concludes.

MIXED FRACTIONAL BROWNIAN MOTION

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Let $0 < H < 1$. The fractional Brownian motion with Hurst parameter H is the Gaussian process $B^H = \left(B^H(t); t \geq 0\right)$ with mean $\mathbb{E}\left[B^H(t)\right] = 0$ and covariance

$$\mathbb{E}\left[B^H(t)B^H(s)\right] = \frac{1}{2}\left(|t|^{2H} + |s|^{2H} - |t-s|^{2H} \right), \tag{1}$$

for all $s,t \geq 0$. Here $\mathbb{E}[\cdot]$ denotes the expectation with respect to a probability measure $\mathbb{P}^H$. The process has the following properties :

- $B^H(0) = 0$ and $\mathbb{E}\left[\left(B^H(t)\right)^2\right] = t^{2H}$, $t \geq 0$;
- $B^H(t)$ has stationary increments, i.e., $B^H(t+s) - B^H(s)$ has the same distribution with $B^H(t)$ for all $s,t \geq 0$;
- $B^H(t)$ is H-self similar, i.e., $B^H(at) = a^H B^H(t)$ for $s,t \geq 0$;
- $B^H(t)$ has continuous trajectories.

If $H = \frac{1}{2}$, then $B^H(t)$ coincides with the standard Brownian motion $B(t)$. The constant H determine the sign of the covariance of the future and past increments. This covariance is negative when $0 < H < \frac{1}{2}$, zero when $H = \frac{1}{2}$ and positive when $\frac{1}{2} < H < 1$. As a consequence, for $0 < H < \frac{1}{2}$ it has a short memory and for $\frac{1}{2} < H < 1$ it has a long memory.

A fractional Brownian motion is neither a Markov nor a semimartingale unless $H = \frac{1}{2}$. To avoid arbitrage opportunities, the mixed fractional Brownian motion is introduced by Cheridito in [28]. A mixed fractional Brownian motion of parameter H, a, and b is a stochastic process $M^H = \left(M^H(t); t \geq 0\right) = \left(M^{H,a,b}(t); t \geq 0\right)$ defined by

$$M^H(t) = M^{H,a,b}(t) = aB(t) + bB^H(t)$$

where $B(t)$ is a Brownian motion and $B^H(t)$ is an independent fractional Brownian motion of Hurst parameter H.

QUASI-CONDITIONAL EXPECTATIONS

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In this section, we will present some results regarding the quasi-conditional expectation which is needed for the rest of this paper. These results were introduced by Necula [23] and then developed by Sun [19] and Xiao et al [18] for the mixed fractional Brownian motion. The proof of theorems in this section can be seen in [18]. Let

$(\Omega, \mathcal{F}^H, \mathbb{P}^H)$ be the probability space such that $B(t)$ is a Brownian motion with respect to $\mathbb{P}^H$ and $B^H(t)$ is an independent fractional Brownian motion with respect to $\mathbb{P}^H$.

Theorem 1. For every $0 < t < T$ and $\lambda, \varepsilon \in \mathbb{C}$ we have

$$\tilde{\mathbb{E}}\left[\exp\left(\lambda\left(B(T)+\varepsilon B^H(T)\right)\right)\middle|\mathcal{F}^H(t)\right] = \exp\left(\lambda\left(B(t)+\varepsilon B^H(t)\right)+\tfrac{1}{2}\lambda^2\left(T-t\right)+\tfrac{1}{2}\lambda^2\varepsilon^2\left(T^{2H}-t^{2H}\right)\right)$$

where $\mathcal{F}^H(t)$ is a σ-algebra generated by $\left(B^H(s); 0 \le s \le t\right)$ and $\tilde{\mathbb{E}}\left[\cdot\middle|\mathcal{F}^H(t)\right]$ denotes a quasi-conditional expectation with respect to $\mathcal{F}^H(t)$ under a probability measure $\mathbb{P}^H$.

Using Theorem 1, one can determine the quasi-conditional expectation of a function of the mixed fractional Brownian motion as shown in the following theorem.

Theorem 2. Let f be a function such that $\tilde{\mathbb{E}}\left[f\left(B(T), B^H(T)\right)\right] < \infty.$ Then for every $0 < t < T$ and $\lambda, \varepsilon \in \mathbb{C}$, we have

$$\tilde{\mathbb{E}}\left[f\left(\lambda B(T)+\lambda\varepsilon B^H(T)\right)\middle|\mathcal{F}^H(t)\right] = \int_{\mathbb{R}} \frac{1}{\sqrt{2\pi\lambda^2\left(T-t+\varepsilon^2\left(T^{2H}-t^{2H}\right)\right)}} \exp\left(\frac{-\left(x-\lambda B(t)-\lambda\varepsilon B^H(t)\right)^2}{2\lambda^2\left(T-t+\varepsilon^2\left(T^{2H}-t^{2H}\right)\right)}\right) f(x)dx.$$

If *f* is an indicator function, $f(x) = 1_A(x)$, then we can easily obtain following corollary.

Corollary 3. Let $A \in \mathcal{B}(\mathbb{R}).$ Then

$$\tilde{\mathbb{E}}\left[1_A\left(\lambda B(T)+\lambda\varepsilon B^H(T)\right)\middle|\mathcal{F}^H(t)\right] = \int_{A} \frac{1}{\sqrt{2\pi\lambda^2\left(T-t+\varepsilon^2\left(T^{2H}-t^{2H}\right)\right)}} \exp\left(\frac{-\left(x-\lambda B(t)-\lambda\varepsilon B^H(t)\right)^2}{2\lambda^2\left(T-t+\varepsilon^2\left(T^{2H}-t^{2H}\right)\right)}\right) dx.$$

Let $\vartheta, \theta \in \mathbb{R}$ and for $0 \le t \le T$, consider the process

$$\vartheta B^*(t) + \theta B^{H^*}(t) = \vartheta B(t) - \vartheta^2 t + \theta B^H(t) - \theta^2 t^{2H}. \tag{2}$$

From the fractional Girsanov theorem in [29], there exists a probability measure $\mathbb{P}^{H^*}$ such that $\vartheta B^*(t) + \theta B^{H^*}(t)$ is a new mixed fractional Brownian motion. We will denote $\tilde{\mathbb{E}}^*[\cdot]$ as a quasi-conditional expectation under the probability measure $\mathbb{P}^{H^*}$. Now, we have defined the process

$$\tilde{Z}(t) = \exp\left(-\vartheta B(t) - \tfrac{1}{2}\vartheta^2 t - \theta B^H(t) - \tfrac{1}{2}\theta^2 t^{2H}\right) \tag{3}$$

where $0 \le t \le T$.

Theorem 4. Let f be a function such that $\mathbb{E}\left[f\left(B(T), B^H(T)\right)\right] < \infty.$ Then for every $0 \le t \le T$, we have

$$\tilde{\mathbb{E}}^*\left[f\left(\vartheta B(T)+\theta B^H(T)\right)\middle|\mathcal{F}^H(t)\right] = \frac{1}{\tilde{Z}(t)}\tilde{\mathbb{E}}\left[f\left(\vartheta B(T)+\theta B^H(T)\right)\tilde{Z}(T)\middle|\mathcal{F}^H(t)\right]. \tag{4}$$

Theorem 4 illustrates the relationship between the quasi-conditional expectation $\tilde{\mathbb{E}}$ with respect to $\mathbb{P}^H$ and the quasi-conditional expectation $\tilde{\mathbb{E}}^*$ with respect to $\mathbb{P}^{H^*}$. Based on Theorem 4, a discounted expectation of the function of a mixed fractional Brownian motion is calculated in the following theorem.

Theorem 5. The price at every $t \in [0, T]$ of a bounded $\mathcal{F}^H(t)$-measurable claim $V \in L^2(\mathbb{P}^H)$ is given by

$$V(t) = e^{-r(T-t)}\tilde{\mathbb{E}}\left[V(T)\middle|\mathcal{F}^H(t)\right] \tag{5}$$

where *r* represents a constant riskless interest rate.

EUROPEAN OPTION PRICING

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The purpose of this section is to derive the pricing formula for a European call option. Now let us consider a mixed fractional Black-Scholes market with two investment possibilities:

1. a money market account:
$$dA(t) = rA(t)dt, \qquad A(0) = 1, \qquad 0 \le t \le T, \tag{6}$$
where r represents the constant riskless interest rate.
2. a stock whose price satisfies the following:
$$dS(t) = \mu S(t)dt + \sigma S(t)d\hat{B}(t) + \sigma S(t)d\hat{B}^H(t), \qquad S(0) > 0, \qquad 0 \le t \le T, \tag{7}$$
where μ is an appreciation rate, σ is a volatility coefficient, $\hat{B}(t)$ is a Brownian motion with respect to $\hat{\mathbb{P}}^H$ and $\hat{B}^H(t)$ is a fractional Brownian motion with respect to $\hat{\mathbb{P}}^H$.

By using a change of variable $B(t) + B^H(t) = \frac{\mu - r}{\sigma} + \hat{B}(t) + \hat{B}^H(t)$, then under a risk-neutral measure, we have that
$$dS(t) = rS(t)dt + \sigma S(t)dB(t) + \sigma S(t)dB^H(t), \qquad S(0) > 0, \qquad 0 \le t \le T. \tag{8}$$
Using the Ito formula in [29], we obtain the solution of (8)~~(8)~~

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$$S(t) = S(0)\exp\left(rt + \sigma\left(B(t) + B^H(t)\right) - \tfrac{1}{2}\sigma^2\left(t + t^{2H}\right)\right). \tag{9}$$
Let $C(t, S(t))$ is the price of a European option at time t with a strike price K and maturity T. We present the pricing formula for a European call option in the following theorem.

Theorem 6. The price at every $t \in [0, T]$ of a European call option with a strike price K and maturity T is given by
$$C(t, S(t)) = S(t)N(d_1) - Ke^{-r(T-t)}N(d_2) \tag{10}$$
where
$$d_1 = \frac{\ln\left(\frac{S(t)}{K}\right) + r(T-t) + \frac{1}{2}\sigma^2\left(T - t + T^{2H} - t^{2H}\right)}{\sqrt{\sigma^2\left(T - t + T^{2H} - t^{2H}\right)}}, \tag{11}$$
$$d_2 = \frac{\ln\left(\frac{S(t)}{K}\right) + r\left(T-t\right) - \frac{1}{2}\sigma^2\left(T - t + T^{2H} - t^{2H}\right)}{\sqrt{\sigma^2\left(T - t + T^{2H} - t^{2H}\right)}}, \tag{12}$$
and $N(\cdot)$ is a cumulative probability function of a standard normal distribution.

Proof: Motivated from Theorem 5, the European call option with a strike price K and maturity T is theoretically equivalent to
$$\begin{aligned} C(t, S(t)) &= \tilde{\mathbb{E}}\left[e^{-r(T-t)}\max\{S(T) - K, 0\}\middle|\mathcal{F}^H(t)\right] \\ &= \tilde{\mathbb{E}}\left[e^{-r(T-t)}S(T)1_{\{S(T)>K\}} - Ke^{-r(T-t)}1_{\{S(T)>K\}}\middle|\mathcal{F}^H(t)\right] \\ &= \tilde{\mathbb{E}}\left[e^{-r(T-t)}S(T)1_{\{S(T)>K\}}\middle|\mathcal{F}^H(t)\right] - \tilde{\mathbb{E}}\left[Ke^{-r(T-t)}1_{\{S(T)>K\}}\middle|\mathcal{F}^H(t)\right] \\ &= e^{-r(T-t)}\tilde{\mathbb{E}}\left[S(T)1_{\{S(T)>K\}}\middle|\mathcal{F}^H(t)\right] - Ke^{-r(T-t)}\mathbb{E}\left[1_{\{S(T)>K\}}\middle|\mathcal{F}^H(t)\right]. \end{aligned} \tag{13}$$
Option holders would exercise the option only when $S(T) > K$. Solving for the boundary and using (9), we have
$$\sigma\left(B(T) + B^H(T)\right) > \ln\left(\frac{K}{S(0)}\right) - rT + \tfrac{1}{2}\sigma^2\left(T + T^{2H}\right) \tag{14}$$
and set
$$d_2^* = \ln\left(\frac{K}{S(0)}\right) - rT + \tfrac{1}{2}\sigma^2\left(T + T^{2H}\right). \tag{15}$$
Using Corollary 3, we have

$$\tilde{\mathbb{E}}\left[1_{\{S(T)>K\}}\middle|\mathcal{F}^H(t)\right]=\tilde{\mathbb{E}}\left[1_{\{x>d_2^*\}}\left(\sigma\left(B(T)+B^H(T)\right)\right)\middle|\mathcal{F}^H(t)\right]$$

$$=\int_{d_2^*}^{\infty}\frac{1}{\sqrt{2\pi\sigma^2\left(T-t+T^{2H}-t^{2H}\right)}}\exp\left(-\frac{\left(x-\sigma\left(B(t)+B^H(t)\right)\right)^2}{2\sigma^2\left(T-t+T^{2H}-t^{2H}\right)}\right)dx$$

$$=\int_{\frac{d_2^*-\sigma\left(B(t)+B^H(t)\right)}{\sqrt{\sigma^2\left(T-t+T^{2H}-t^{2H}\right)}}}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)dz$$

$$=\int_{-\infty}^{\frac{\sigma\left(B(t)+B^H(t)\right)-d_2^*}{\sqrt{\sigma^2\left(T-t+T^{2H}-t^{2H}\right)}}}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)dz$$

$$=N(d_2) \tag{16}$$

where $d_2=\frac{\sigma\left(B(t)+B^H(t)\right)-d_2^*}{\sqrt{\sigma^2\left(T-t+T^{2H}-t^{2H}\right)}}$. Furthermore, (9) can be write

$$\sigma\left(B(t)+B^H(t)\right)=\ln\left(\frac{S(t)}{S(0)}\right)-rt+\tfrac{1}{2}\sigma^2\left(t+t^{2H}\right). \tag{17}$$

Hence, we have

$$d_2=\frac{\sigma B(t)+\sigma B^H(t)-d_2^*}{\sqrt{\sigma^2\left(T-t+T^{2H}-t^{2H}\right)}}$$

$$=\frac{\ln\left(\frac{S(t)}{S(0)}\right)-rt+\frac{1}{2}\sigma^2\left(t+t^{2H}\right)-\left(\ln\left(\frac{K}{S(0)}\right)-rT+\frac{1}{2}\sigma^2\left(T+T^{2H}\right)\right)}{\sqrt{\sigma^2\left(T-t+T^{2H}-t^{2H}\right)}}$$

$$=\frac{\ln\left(\frac{S(t)}{K}\right)+r\left(T-t\right)-\frac{1}{2}\sigma^2\left(T-t+T^{2H}-t^{2H}\right)}{\sqrt{\sigma^2\left(T-t+T^{2H}-t^{2H}\right)}}. \tag{18}$$

Let us consider the process

$$\sigma\left(B^*(t)+B^{H*}(t)\right)=\sigma\left(B(t)+B^H(t)\right)-\sigma^2\left(t+t^{2H}\right), \tag{19}$$

for $t\in[0,T]$. The fractional Girsanov theorem assures us that there is the probability measure $\mathbb{P}^{H*}$ such that $\sigma\left(B^*(t)+B^{H*}(t)\right)$ is the mixed fractional Brownian motion under $\mathbb{P}^{H*}$. We will denote

$$\tilde{Z}(t)=\exp\left(\sigma\left(B(t)+B^H(t)\right)-\tfrac{1}{2}\sigma^2\left(t+t^{2H}\right)\right) \tag{20}$$

Using Theorem 4 and (20) we have that

$$\tilde{\mathbb{E}}\left[S(T)1_{\{S(T)>K\}}\middle|\mathcal{F}^H(t)\right]=\tilde{\mathbb{E}}\left[S(0)e^{\left(rT+\sigma\left(B(T)+B^H(T)\right)-\frac{1}{2}\sigma^2\left(T+T^{2H}\right)\right)}1_{\{S(T)>K\}}\middle|\mathcal{F}^H(t)\right]$$

$$\tilde{\mathbb{E}}\left[S(T)1_{\{S(T)>K\}}\middle|\mathcal{F}^H(t)\right] = S(0)e^{rT}\tilde{\mathbb{E}}\left[\tilde{Z}(T)1_{\{S(T)>K\}}\middle|\mathcal{F}^H(t)\right]$$
$$= \tilde{\mathbb{E}}\left[S(0)e^{rT}e^{\left(\sigma\left(B(T)+B^H(T)\right)-\frac{1}{2}\sigma^2\left(T+T^{2H}\right)\right)}1_{\{S(T)>K\}}\middle|\mathcal{F}^H(t)\right]$$
$$= S(0)e^{rT}\tilde{\mathbb{E}}\left[\tilde{Z}(T)1_{\left\{x>d_2^*\right\}}\left(\sigma\left(B(T)+B^H(T)\right)\right)\middle|\mathcal{F}^H(t)\right]$$
$$= S(0)e^{rT}\tilde{Z}(t)\tilde{\mathbb{E}}^*\left[1_{\left\{x>d_2^*\right\}}\left(\sigma\left(B(T)+B^H(T)\right)\right)\middle|\mathcal{F}^H(t)\right]$$
$$= S(0)e^{rT}\tilde{Z}(t)\tilde{\mathbb{E}}^*\left[1_{\{S(T)>K\}}\middle|\mathcal{F}^H(t)\right] \qquad (21)$$

Substituting (19) into (9) is obtained

$$S(t) = S(0)\exp\left(rt+\sigma\left(B(t)+B^H(t)\right)-\tfrac{1}{2}\sigma^2\left(t+t^{2H}\right)\right)$$
$$= S(0)\exp\left(rt+\left(\sigma\left(B^*(t)+B^{H*}(t)\right)+\sigma^2\left(t+t^{2H}\right)\right)-\tfrac{1}{2}\sigma^2\left(t+t^{2H}\right)\right)$$
$$= S(0)\exp\left(rt+\sigma\left(B^*(t)+B^{H*}(t)\right)+\tfrac{1}{2}\sigma^2\left(t+t^{2H}\right)\right). \qquad (22)$$

Solving for the boundary $S(T)>K$ and using (22) in time *T*, we have

$$\sigma\left(B^*(T)+B^{H*}(T)\right) > \ln\left(\frac{K}{S(0)}\right)-rT-\tfrac{1}{2}\sigma^2\left(T+T^{2H}\right).$$

If we denote

$$d_1^* = \ln\left(\frac{K}{S(0)}\right)-rT-\tfrac{1}{2}\sigma^2\left(T+T^{2H}\right), \qquad (23)$$

we get

$$\tilde{\mathbb{E}}^*\left[1_{\{S(T)>K\}}\middle|\mathcal{F}^H(t)\right] = \tilde{\mathbb{E}}^*\left[1_{\left\{x>d_1^*\right\}}\left(\sigma\left(B^*(T)+B^{H*}(T)\right)\right)\middle|\mathcal{F}^H(t)\right]$$
$$= \int_{d_1^*}^{\infty}\frac{1}{\sqrt{2\pi\sigma^2\left(T-t+T^{2H}-t^{2H}\right)}}\exp\left(-\frac{\left(x-\sigma\left(B^*(t)+B^{H*}(t)\right)\right)^2}{2\sigma^2\left(T-t+T^{2H}-t^{2H}\right)}\right)dx$$
$$= \int_{\frac{d_1^*-\sigma\left(B^*(t)+B^{H*}(t)\right)}{\sqrt{\sigma^2\left(T-t+T^{2H}-t^{2H}\right)}}}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)dz$$
$$= \int_{-\infty}^{\frac{\sigma\left(B^*(t)+B^{H*}(t)\right)-d_1^*}{\sqrt{\sigma^2\left(T-t+T^{2H}-t^{2H}\right)}}}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)dz$$
$$= N(d_1) \qquad (24)$$

where $d_1 = \dfrac{\sigma\left(B^*(t)+B^{H*}(t)\right)-d_1^*}{\sqrt{\sigma^2\left(T-t+T^{2H}-t^{2H}\right)}}$. Furthermore, (22)(22) can be written as

$$\sigma\left(B^*(t)+B^{H*}(t)\right) = \ln\left(\frac{S(t)}{S(0)}\right)-rt-\tfrac{1}{2}\sigma^2\left(t+t^{2H}\right). \qquad (25)$$

Using (23)(23) and (25)(25), we get

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$$d_1 = \frac{\ln\left(\frac{S(t)}{S(0)}\right) - rt - \frac{1}{2}\sigma^2\left(t + t^{2H}\right) - \left(\ln\left(\frac{K}{S(0)}\right) - rT - \frac{1}{2}\sigma^2\left(T + T^{2H}\right)\right)}{\sqrt{\sigma^2\left(T - t + T^{2H} - t^{2H}\right)}}$$

$$= \frac{\ln\left(\frac{S(t)}{K}\right) + r(T-t) + \frac{1}{2}\sigma^2\left(T - t + T^{2H} - t^{2H}\right)}{\sqrt{\sigma^2\left(T - t + T^{2H} - t^{2H}\right)}}.$$

Substitution of (24) into (21) yields

$$\begin{aligned}\tilde{\mathbb{E}}\left[S(T)1_{\{S(T)>K\}}\middle|\mathcal{F}^H(t)\right] &= S(0)e^{rT}\tilde{Z}(t)\tilde{\mathbb{E}}^*\left[1_{\{S(T)>K\}}\middle|\mathcal{F}^H(t)\right]\\ &= S(0)e^{rT}\tilde{Z}(t)N(d_1)\\ &= S(0)e^{rT}\exp\left(\sigma\left(B(t)+B^H(t)\right) - \tfrac{1}{2}\sigma^2\left(t+t^{2H}\right)\right)N(d_1)\\ &= e^{rT}e^{-rt}S(0)\exp\left(\sigma\left(B(t)+B^H(t)\right) + rt - \tfrac{1}{2}\sigma^2\left(t+t^{2H}\right)\right)N(d_1)\\ &= e^{r(T-t)}S(t)N(d_1).\end{aligned} \tag{26}$$

Finally, from (16), (26)~~(26)~~ and (13) we obtain

$$\begin{aligned}C((t,S(t)) &= e^{-r(T-t)}\mathbb{E}\left[S(T)1_{\{S(T)>K\}}\middle|\mathcal{F}^H(t)\right] - Ke^{-r(T-t)}\mathbb{E}\left[1_{\{S(T)>K\}}\middle|\mathcal{F}^H(t)\right]\\ &= e^{-r(T-t)}e^{r(T-t)}S(t)N(d_1) - Ke^{-r(T-t)}N(d_2)\\ &= S(t)N(d_1) - Ke^{-r(T-t)}N(d_2).\end{aligned}$$

Thus, the proof of the theorem is complete. ■

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Commented [WU6]: Give the comprehensive discussion for the theorems resulted.

CONCLUSION

In this paper, to capture the long memory property and to exclude the arbitrage opportunity in fractional Brownian motion, the underlying price is assumed to follow the mixed fractional Brownian motion. By using the theory of the quasi-conditional expectation, we present a closed-form option pricing formula for European call options. This formula can be used by investors to predict the option price for a stock that has long memory properties.

Commented [WU7]: This section consist of answering of the objectives and may contain the future research.

ACKNOWLEDGMENTS

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