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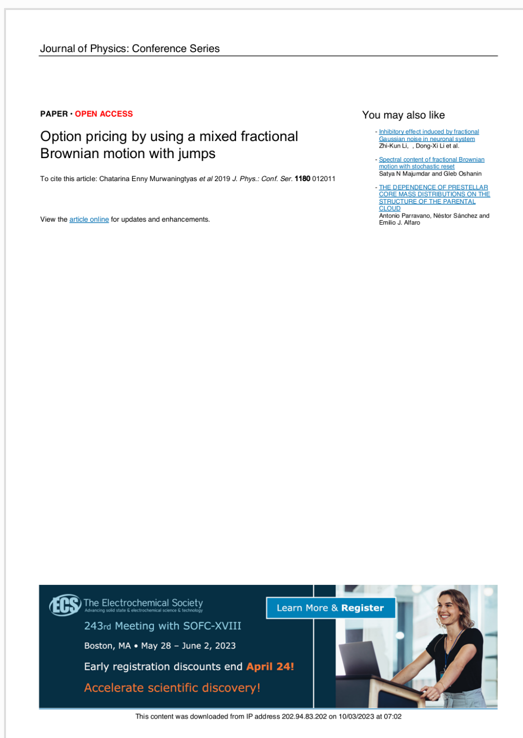
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Option pricing by using a mixed fractional Brownian motion with jumps

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Abstract. Option pricing is conventionally based on a Brownian motion (Bm). The Bm is a semimartingale process with stationary and independent increments. However, there are several stock returns that have a long memory or have high autocorrelation for long lags. A fractional Brownian motion (fBm) is one of the models that can solve this problem, but a model option with fBm is not arbitrage-free. A mixed fractional Brownian motion (mfBm) is a linear combination of a Bm and an independent fBm which can overcome the arbitrage problem. A jump process in time series is another problem found in stock price modeling. This paper deals with the problem of options pricing by using mfBm with jumps. Based on quasi-conditional expectation and Fourier transform method, we obtain a pricing formula for a stock option.

1. Introduction

Some empirical studies show that stock returns exhibit long-range dependence, see for example [1–5]. One of the model that can describe long-range dependence is the fractional Brownian motion (fBm). The fBm with Hurst index $H \in (0, 1)$ is a continuous centered Gaussian process with dependent and stationary increments. For $H = ½$ we recover the classical case of Brownian motion (Bm) . The fBm is a long-range dependence process or a long memory process if $H > ½$.

The fBm is neither a semimartingale nor a Markov process, except for $H = ½$. Therefore, we cannot use the standard theory of Itô integral. One of the stochastic integrals that can be used in option pricing is the Wick-Itô integral [6,7]. Hu and Oksendal [6] obtained a formula for European call option under an fBm using a Wick-Itô integral and then this formula was expanded by Necula [8].

Option pricing using the Wick-Itô integral still has an arbitrage opportunity [9,10]. Cheridito [11] introduced a mixed fractional Brownian motion (mfBm) which can reduce the arbitrage opportunity. The mfBm is a family of Gaussian processes which are linear combinations of a Bm and an independent fBm. An mfBm is equivalent to a Bm for $H \in (¾, 1)$, so this option pricing model is arbitrage free [12]. A formula for option pricing under an mfBm using the Wick-Itô integral was derived in [13].

In this paper we will discuss a combination of Poisson jumps and an mfBm called jump mixed fractional Brownian motion (jmfBm). The jmfBm model is used to capture fluctuations, discontinuities or jumps as well as to take into account long-range dependence properties. The jmfBm is based on the assumption that stock returns are generated by a two-part stochastic process: (1) continuous price movements are generated by an mfBm and (2) infrequent price jumps are generated by a Poisson process. This process are able to describe a distribution of empirical data from stock prices that are long-range dependence, leptokurtic, skewed, and have fatter tails.

A currency options pricing by using jump fractional Brownian motion (jfBm) has been studied in [14]. A currency options pricing by using a jmfBm has been studied in [15]. Our aim is to investigate the price of European option under a jmfBm model.

2. Preliminaries

An fBm $B^H = \left(B_t^H\right)_{t\geq 0}$ of Hurst index $H \in (0,1)$ is a centered and continuous Gaussian process with covariance function

$$\mathbb{E}\left[B_t^H B_s^H\right] = \frac{1}{2}\left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right), \tag{1}$$

for all $t, s \geq 0$, see [16]. The sign of the covariance of the fBm is determined by a Hurst index H. This covariance is zero when $H = ½$, negative when $H \in (0, ½)$, and positive when $H \in (½, 1)$. As a consequence, for $H \in (0, ½)$, it exhibits a short-range dependence (short memory) and for $H \in (½, 1)$ it exhibits a long-range dependence.

Definition 1. [12,17] An mfBm is a stochastic process $M^H = \left(M_t^{H,a,b}\right)_{t\geq 0}$, defined on a probability space $\left(\Omega, \mathcal{F}, \mathbb{P}^H\right)$ by

$$M_t^{H,a,b} = aB_t^H + bB_t$$

where B_t^H is an fBm with Hurst index H and B_t is an independent Bm.

Proposition 2. [17] The mfBm $M_t^{H,a,b}$ is a centered Gaussian process which satisfies the following properties:

(i) $M_t^{H,a,b}$ is not a Markovian process for all $H \in (0,1) \setminus \frac{1}{2}$;

(ii) $M_0^{H,a,b} = 0$ $\mathbb{P}^H$-almost surely;

(iii) the covariation function of $M_t^{H,a,b}$ and $M_s^{H,a,b}$ for any $t, s \geq 0$ is given by

$$\operatorname{cov}\left(M_t^{H,a,b}, M_s^{H,a,b}\right) = a^2 \min(t,s) + \tfrac{1}{2}b^2\left(t^{2H} + s^{2H} - |t-s|^{2H}\right);$$

(iv) the increments of $M_t^{H,a,b}$ are stationary and mixed-self-similar, i.e. for any $h > 0$

$$\left(M_{ht}^{H,a,b}\right)_{t\geq 0} \overset{d}{=} \left(M_t^{H,ah^{1/2},bh^H}\right)_{t\geq 0};$$

(v) the increments of $M_t^{H,a,b}$ are negatively correlated if $H \in \left(0, \frac{1}{2}\right)$, uncorrelated if $H \neq \frac{1}{2}$ and positively correlated if $H \in \left(\frac{1}{2}, 1\right)$;

(vi) the increments of $M_t^{H,a,b}$ are short-range dependent if $H \in \left(0, \frac{1}{2}\right)$ and long-range dependent if $H \in \left(\frac{1}{2}, 1\right)$;

(vii) for $t \geq 0$, we have the moment formula

$$\mathbb{E}\left[\left(M_t^{H,a,b}\right)^n\right] = \begin{cases} 0 & n = 2l+1 \\ \dfrac{(2l)!}{2^l l!}\left(a^2 t + b^2 t^{2H}\right)^l & n = 2l \end{cases}$$

Cheridito [12] introduced mfBm, to avoid arbitrage opportunities. The stock price model under mfBm is given by

$$dS_t = \sigma S_t dB_t^H + \sigma S_t dB_t + \mu S_t dt, \qquad S_0 > 0, \qquad t \in [0,T], \tag{2}$$

where B_t is a Bm and B_t^H is an fBm with respect to $\hat{\mathbb{P}}^H$, μ is an expected return and σ is a volatility coefficient. Furthermore, we get the solution from (2) using the Ito formula in [16] as follows

$$S_t = S_0 \exp\left(\mu t + \sigma B_t^H + \sigma B_t - \tfrac{1}{2}\sigma^2 t^{2H} - \tfrac{1}{2}\sigma^2 t\right).$$

3. Results and discussion

Let $(\Omega, \mathcal{F}, \mathbb{P}^H)$ be a probability space. The Black Scholes market is a model consisting of two assets, one risky asset (stock) and one riskless asset (bank account). A bank account satisfies

$$dA_t = rA_t dt, \qquad A_0 = 1, \qquad t \in [0,T], \tag{3}$$

where r is a interest rate. A stock price satisfies

$$dS_t = S_t\left(\mu - \lambda\mu_{J_t}\right)dt + \sigma S_t d\hat{B}_t^H + \sigma S_t d\hat{B}_t + S_t(e^{J_t} - 1)dN_t \qquad S_0 > 0, \qquad t \in [0,T], \tag{4}$$

where S_t denote a stock price with an expected return μ and a volatility σ, N_t is a Poisson process with rate λ, J_t is the jump size percent at time t which is a sequence of independent identically distributed, $(e^{J_t} - 1) \sim N(\mu_{J_t}, \delta_t^2)$, $\hat{B}_t$ is a Bm, $\hat{B}_t^H$ is a fBm with respect to $\hat{\mathbb{P}}^H$ and Hurst index $H > 3/4$.

According to the fractional Girsanov theorem [16], it is known that there is a risk-neutral measure $\mathbb{P}^H$, so that if $\sigma\hat{B}_t + \sigma\hat{B}_t^H = \sigma B_t + \sigma B_t^H - \mu + r$ is obtained

$$dS_t = S_t\left(r - \lambda\mu_{J_t}\right)dt + \sigma S_t dB_t^H + \sigma S_t dB_t + S_t(e^{J_t} - 1)dN_t \qquad S_0 > 0, \qquad t \in [0,T]. \tag{5}$$

Furthermore, we obtain a solution of (5) by using an Itô formula in [16], as

$$S_t = S_0 \prod_{i=1}^{N_t} e^{J_{t_i}} \exp\left(\left(r - \lambda\mu_{J_t}\right)t + \sigma B_t^H + \sigma B_t - \frac{1}{2}\sigma^2 t^{2H} - \frac{1}{2}\sigma^2 t\right). \tag{6}$$

Theorem 3. *Suppose stock price* S_t *is modeled with a jmfBm* (6)*, a price at time* $t \in [0,T]$ *of a European call option with a strike price K and an expiry date T is given by*

$$C(t, S_t) = \sum_{n=0}^{\infty} \frac{e^{\lambda(T-t)}\lambda^n (T-t)^n}{n!} \varepsilon_n \times \left[S_t \prod_{i=1}^{n} e^{J_{t_i}} e^{-\lambda\mu_{J_t}(T-t)} \Phi(d_1) - K e^{-r(T-t)} \Phi(d_2) \right], \tag{7}$$

where

$$d_1 = \frac{\frac{1}{2}\sigma^2\left(T^{2H} - t^{2H}\right) + \frac{1}{2}\sigma^2(T-t) + \ln\left(\frac{S_t}{K}\prod_{i=1}^{n} e^{J_{t_i}}\right) - \left(r - \lambda\mu_{J_t}\right)(T-t)}{\sqrt{\sigma^2\left(T^{2H} - t^{2H}\right) + \sigma^2(T-t)}}, \tag{8}$$

$$d_2 = \frac{-\frac{1}{2}\sigma^2\left(T^{2H} - t^{2H}\right) - \frac{1}{2}\sigma^2(T-t) + \ln\left(\frac{S_t}{K}\prod_{i=1}^{n} e^{J_{t_i}}\right) - \left(r - \lambda\mu_{J_t}\right)(T-t)}{\sqrt{\sigma^2\left(T^{2H} - t^{2H}\right) + \sigma^2(T-t)}}, \tag{9}$$

ε_n *denote a expectation operator over the distribution of* $\prod_{i=1}^{n} e^{J_{t_i}}$ *and* $\Phi(\cdot)$ *is a cumulative normal distribution function.*

Proof: Equation (6) can be written as

$$S_T = S_t \prod_{i=1}^{N_{T-t}} e^{J_{t_i}} \exp\left(\left(r - \lambda\mu_{J_t}\right)(T-t) + \sigma\left(B_T^H - B_t^H\right) + \sigma\left(B_T - B_t\right) - \frac{1}{2}\sigma^2\left(T^{2H} - t^{2H}\right) - \frac{1}{2}\sigma^2(T-t)\right). \tag{10}$$

Let

$$S_T^n = S_t \prod_{i=1}^{n} e^{J_{t_i}} \exp\Big(\left(r-\lambda\mu_{J_t}\right)(T-t) + \sigma\left(B_T^H - B_t^H\right) + \sigma\left(B_T - B_t\right) - \frac{1}{2}\sigma^2\left(T^{2H}-t^{2H}\right) - \frac{1}{2}\sigma^2 (T-t) \Big). \tag{11}$$

Using the independence of N_{T-t} and J_{t_i} and the theory of Poisson distribution with intensity $\lambda(T-t)$ we have

$$S_T = \sum_{n=0}^{\infty} P\left(N_{T-t}=n\right) S_T^n = \sum_{n=0}^{\infty} e^{-\lambda(T-t)} \frac{\lambda^n (T-t)^n}{n!} S_T^n \tag{12}$$

Motivated from Theorem 3.5 from [18] and using (11) and (12), the call option with a strike price K and an expiry date T is determined by

$$\begin{aligned} C(t,S_t) &= \tilde{\mathbb{E}}\left[e^{-r(T-t)} \max\{S_T - K, 0\} \middle| \mathcal{F}_t^H \right] \\ &= e^{-r(T-t)} \sum_{n=0}^{\infty} e^{-\lambda(T-t)} \frac{\lambda^n (T-t)^n}{n!} \tilde{\mathbb{E}}\left[\max\{S_T^n - K, 0\} \middle| \mathcal{F}_t^H \right] \\ &= e^{-r(T-t)} \sum_{n=0}^{\infty} e^{-\lambda(T-t)} \frac{\lambda^n (T-t)^n}{n!} \left(\tilde{\mathbb{E}}\left[S_T^n 1_{\{S_T^n > K\}} \middle| \mathcal{F}_t^H \right] - K \tilde{\mathbb{E}}\left[1_{\{S_T^n > K\}} \middle| \mathcal{F}_t^H \right] \right) \\ &= e^{-r(T-t)} \sum_{n=0}^{\infty} e^{-\lambda(T-t)} \frac{\lambda^n (T-t)^n}{n!} \tilde{\mathbb{E}}\left[S_T^n 1_{\{S_T^n > K\}} \middle| \mathcal{F}_t^H \right] \\ &\quad - K e^{-r(T-t)} \sum_{n=0}^{\infty} e^{-\lambda(T-t)} \frac{\lambda^n (T-t)^n}{n!} \tilde{\mathbb{E}}\left[1_{\{S_T^n > K\}} \middle| \mathcal{F}_t^H \right]. \end{aligned} \tag{13}$$

In the meantime, if $S_T > K$, option holders would exercise an option. Solving (10) at time $t=0$ on this boundary, we have

$$\sigma B_T^H + \sigma B_T > \frac{1}{2}\sigma^2 T^{2H} + \frac{1}{2}\sigma^2 T + \ln\left(\frac{K}{S_0 \prod_{i=1}^{N_T} e^{J_{t_i}}} \right) - \left(r - \lambda\mu_{J_t}\right)T.$$

Let

$$d_2^* = \frac{1}{2}\sigma^2 T^{2H} + \frac{1}{2}\sigma^2 T + \ln\left(\frac{K}{S_0 \prod_{i=1}^{N_T} e^{J_{t_i}}} \right) - \left(r - \lambda\mu_{J_t}\right)T. \tag{14}$$

Using (14) and applying Corollary 3.3 from [18] on the second of the right-hand side of (13), we have

$$\begin{aligned} \tilde{\mathbb{E}}\left[1_{\{S_T^n > K\}} \middle| \mathcal{F}_t^H \right] &= \tilde{\mathbb{E}}\left[1_{\{x > d_2^*\}} \left(\sigma B_T^H + \sigma B_T\right) \middle| \mathcal{F}_t^H \right] \\ &= \int_{d_2^*}^{\infty} \frac{1}{\sqrt{2\pi\left(\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2(T-t)\right)}} \exp\left(-\frac{\left(y - \sigma B_t^H - \sigma B_t\right)^2}{2\left(\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2(T-t)\right)} \right) dy \\ &= \int_{-\infty}^{\frac{\sigma B_t^H + \sigma B_t - d_2^*}{\sqrt{\left(\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2(T-t)\right)}}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2} \right) dz \\ &= \Phi(d_2), \end{aligned} \tag{15}$$

where $d_2 = \dfrac{\sigma B_t^H + \sigma B_t - d_2^*}{\sqrt{\left(\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2(T-t)\right)}}$. Furthermore, (6) can be written as

$$\sigma B_t^H + \sigma B_t = \frac{1}{2}\sigma^2 t^{2H} + \frac{1}{2}\sigma^2 t + \ln\left(\frac{S_t}{S_0 \prod_{i=1}^{N_t} e^{J_{t_i}}}\right) - \left(r - \lambda \mu_{J_t}\right)t. \tag{16}$$

Hence, using (14) and (16) on d_2, we have

$$d_2 = \frac{-\frac{1}{2}\sigma^2\left(T^{2H} - t^{2H}\right) - \frac{1}{2}\sigma^2\left(T-t\right) + \ln\left(\frac{S_t}{K}\prod_{i=1}^{N_{T-t}} e^{J_{t_i}}\right) - \left(r - \lambda\mu_{J_t}\right)(T-t)}{\sqrt{\sigma^2\left(T^{2H} - t^{2H}\right) + \sigma^2\left(T-t\right)}}.$$

$$= \frac{-\frac{1}{2}\sigma^2\left(T^{2H} - t^{2H}\right) - \frac{1}{2}\sigma^2\left(T-t\right) + \ln\left(\frac{S_t}{K}\prod_{i=1}^{n} e^{J_{t_i}}\right) - \left(r - \lambda\mu_{J_t}\right)(T-t)}{\sqrt{\sigma^2\left(T^{2H} - t^{2H}\right) + \sigma^2\left(T-t\right)}}.$$

Let us consider a process

$$\sigma B_t^{H^*} + \sigma B_t^* = \sigma B_t^H + \sigma B - \sigma^2 t^{2H} - \sigma^2 t, \tag{17}$$

for $t \in [0,T]$. The fractional Girsanov theorem assures us that there is a probability measure $\mathbb{P}^{H^*}$ such that $\sigma B_t^{H^*} + \sigma B_t^*$ is a new jmfBm under $\mathbb{P}^{H^*}$. We will denote

$$Z_t = \exp\left(\sigma B_t^H - \tfrac{1}{2}\sigma^2 t^{2H} + \sigma B_t - \tfrac{1}{2}\sigma^2 t\right) \tag{18}$$

By using Theorem 3.4 from [18], (10) and (18) on the first of the right-hand side of (13), we have

$$\begin{aligned}
\tilde{\mathbb{E}}\left[S_T^n 1_{\{S_T^n > K\}} \middle| \mathcal{F}_t^H\right] &= \tilde{\mathbb{E}}\left[S_0 \prod_{i=1}^{N_T} e^{J_{t_i}} e^{\left(r-\lambda\mu_{J_t}\right)T + \sigma B_T^H + \sigma B_T - \frac{1}{2}\sigma^2 T^{2H} - \frac{1}{2}\sigma^2 T} 1_{\{S_T^n > K\}} \middle| \mathcal{F}_t^H\right] \\
&= \tilde{\mathbb{E}}\left[S_0 \prod_{i=1}^{N_T} e^{J_{t_i}} e^{\left(r-\lambda\mu_{J_t}\right)T} e^{\sigma B_T^H + \sigma B_T - \frac{1}{2}\sigma^2 T^{2H} - \frac{1}{2}\sigma^2 T} 1_{\{S_T^n > K\}} \middle| \mathcal{F}_t^H\right] \\
&= S_0 \prod_{i=1}^{N_T} e^{J_{t_i}} e^{\left(r-\lambda\mu_{J_t}\right)T} \tilde{\mathbb{E}}\left[Z_T 1_{\{S_T^n > K\}} \middle| \mathcal{F}_t^H\right] \\
&= S_0 \prod_{i=1}^{N_T} e^{J_{t_i}} e^{\left(r-\lambda\mu_{J_t}\right)T} \tilde{\mathbb{E}}\left[Z_T 1_{\{y > d_2^*\}}\left(\sigma B_T^H + \sigma B_T\right) \middle| \mathcal{F}_t^H\right] \\
&= S_0 \prod_{i=1}^{N_T} e^{J_{t_i}} e^{\left(r-\lambda\mu_{J_t}\right)T} Z_t \tilde{\mathbb{E}}^*\left[1_{\{y > d_2^*\}}\left(\sigma B_T^H + \sigma B_T\right) \middle| \mathcal{F}_t^H\right] \\
&= S_0 \prod_{i=1}^{N_T} e^{J_{t_i}} e^{\left(r-\lambda\mu_{J_t}\right)T} Z_t \tilde{\mathbb{E}}^*\left[1_{\{S_T^n > K\}} \middle| \mathcal{F}_t^H\right]
\end{aligned} \tag{19}$$

By substituting (17) into (6), we obtain

$$S_t = S_0 \prod_{i=1}^{N_t} e^{J_{t_i}} \exp\left(\left(r - \lambda\mu_{J_t}\right)t + \sigma B_t^{H^*} + \sigma B_t^* + \frac{1}{2}\sigma^2 t^{2H} + \frac{1}{2}\sigma^2 t\right). \tag{20}$$

Solving (20) at time T for the boundary $S_T > K$, we have

$$\sigma B_T^{H^*} + \sigma B_T^* > -\frac{1}{2}\sigma^2 T^{2H} - \frac{1}{2}\sigma^2 T + \ln\left(\frac{K}{S_0 \prod_{i=1}^{N_T} e^{J_{t_i}}}\right) - \left(r - \lambda\mu_{J_t}\right)T.$$

If we denote

$$d_1^* = -\frac{1}{2}\sigma^2 T^{2H} - \frac{1}{2}\sigma^2 T + \ln\left(\frac{K}{S_0 \prod_{i=1}^{N_T} e^{J_{t_i}}}\right) - \left(r - \lambda\mu_{J_t}\right)T, \tag{21}$$

we get

$$\begin{aligned}
\tilde{\mathbb{E}}^*\left[1_{\{S_T^n>K\}}\middle|\mathcal{F}_t^H\right] &= \tilde{\mathbb{E}}^*\left[1_{\{y>d_1^*\}}\left(\sigma B_T^{H*}+\sigma B_T^*\right)\middle|\mathcal{F}_t^H\right] \\
&= \int_{d_1^*}^{\infty}\frac{1}{\sqrt{2\pi\left(\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2\left(T-t\right)\right)}}\exp\left(-\frac{\left(y-\sigma B_t^{H*}-\sigma B_t^*\right)^2}{2\left(\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2\left(T-t\right)\right)}\right)dy \\
&= \int_{-\infty}^{\frac{\sigma B_t^{H*}+\sigma B_t^*-d_1^*}{\sqrt{\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2(T-t)}}}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)dz \\
&= \Phi(d_1),
\end{aligned} \tag{22}$$

where $d_1=\dfrac{\sigma B_t^{H*}+\sigma B_t^*-d_1^*}{\sqrt{\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2\left(T-t\right)}}$. Subsequently, (20) can be written as

$$\sigma B_t^{H*}+\sigma B_t^* = -\tfrac{1}{2}\sigma^2 t^{2H}-\tfrac{1}{2}\sigma^2 t+\ln\left(\frac{S_t}{S_0\prod_{i=1}^{N_t}e^{J_{t_i}}}\right)-\left(r-\lambda\mu_{J_t}\right)t. \tag{23}$$

Substituting (21) and (23) on d_1, we get

$$\begin{aligned}
d_1 &= \frac{\tfrac{1}{2}\sigma^2\left(T^{2H}-t^{2H}\right)+\tfrac{1}{2}\sigma^2\left(T-t\right)+\ln\left(\frac{S_t}{K}\prod_{i=1}^{N_{T-t}}e^{J_{t_i}}\right)-\left(r-\lambda\mu_{J_t}\right)(T-t)}{\sqrt{\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2\left(T-t\right)}} \\
&= \frac{\tfrac{1}{2}\sigma^2\left(T^{2H}-t^{2H}\right)+\tfrac{1}{2}\sigma^2\left(T-t\right)+\ln\left(\frac{S_t}{K}\prod_{i=1}^{n}e^{J_{t_i}}\right)-\left(r-\lambda\mu_{J_t}\right)(T-t)}{\sqrt{\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2\left(T-t\right)}}.
\end{aligned}$$

Substitution of (22) into (19) yields

$$\begin{aligned}
\tilde{\mathbb{E}}\left[S_T^n 1_{\{S_T^n>K\}}\middle|\mathcal{F}_t^H\right] &= S_0\prod_{i=1}^{N_T}e^{J_{t_i}}e^{\left(r-\lambda\mu_{J_t}\right)T}Z_t\,\Phi(d_1) \\
&= S_0\prod_{i=1}^{N_T}e^{J_{t_i}}e^{\left(r-\lambda\mu_{J_t}\right)T}\exp\left(\sigma B_t^H+\sigma B_t-\tfrac{1}{2}\sigma^2 t^{2H}-\tfrac{1}{2}\sigma^2 t\right)\Phi(d_1) \\
&= \left(S_0\prod_{i=1}^{N_t}e^{J_{t_i}}\exp\left(\sigma B_t^H+\sigma B_t-\tfrac{1}{2}\sigma^2 t^{2H}-\tfrac{1}{2}\sigma^2 t+\left(r-\lambda\mu_{J_t}\right)t\right)\right)\prod_{i=1}^{N_{T-t}}e^{J_{t_i}}e^{\left(r-\lambda\mu_{J_t}\right)(T-t)}\Phi(d_1) \\
&= S_t\prod_{i=1}^{N_{T-t}}e^{J_{t_i}}e^{\left(r-\lambda\mu_{J_t}\right)(T-t)}\Phi(d_1) \\
&= S_t\prod_{i=1}^{n}e^{J_{t_i}}e^{\left(r-\lambda\mu_{J_t}\right)(T-t)}\Phi(d_1).
\end{aligned} \tag{24}$$

Finally, from (13), (15), (12), (24) and [19] it is calculated that the price of European call option can be expressed as

$$C(t,S_t)=\sum_{n=0}^{\infty}\frac{e^{\lambda(T-t)}\lambda^n(T-t)^n}{n!}\varepsilon_n\times\left[S_t\prod_{i=1}^{n}e^{J_{t_i}}e^{-\lambda\mu_{J_t}(T-t)}\Phi(d_1)-Ke^{-r(T-t)}\Phi(d_2)\right].\ \blacksquare$$

Using put-call parity, we calculate the price of European put option in the corollary below.

Corollary 4. *Suppose stock price S_t is modeled with a jmfBm (6), a price at time $t \in [0,T]$ of a European put option with a strike price K and an expiry date T is given by*

$$P(t,S_t)=\sum_{n=0}^{\infty}\frac{e^{\lambda(T-t)}\lambda^n(T-t)^n}{n!}\varepsilon_n\times\left[Ke^{-r(T-t)}\Phi(-d_2)-S_t\prod_{i=1}^{n}e^{J_{t_i}}e^{-\lambda\mu_{J_t}(T-t)}\Phi(-d_1)\right], \tag{25}$$

where

$$d_1=\frac{\frac{1}{2}\sigma^2\left(T^{2H}-t^{2H}\right)+\frac{1}{2}\sigma^2\left(T-t\right)+\ln\left(\frac{S_t}{K}\prod_{i=1}^{n}e^{J_{t_i}}\right)-\left(r-\lambda\mu_{J_t}\right)(T-t)}{\sqrt{\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2\left(T-t\right)}},$$

$$d_2=\frac{-\frac{1}{2}\sigma^2\left(T^{2H}-t^{2H}\right)-\frac{1}{2}\sigma^2\left(T-t\right)+\ln\left(\frac{S_t}{K}\prod_{i=1}^{n}e^{J_{t_i}}\right)-\left(r-\lambda\mu_{J_t}\right)(T-t)}{\sqrt{\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2\left(T-t\right)}},$$

ε_n denote a expectation operator over the distribution of $\prod_{i=1}^{n}e^{J_{t_i}}$ and $\Phi(\cdot)$ is a cumulative normal distribution function.

The next part is how to implement the jfmBm model and to present the effects of jump parameters. We compare option prices under several stock price models, among the following models: an option price model under a Bm [20], an option price model under an fBm [8], an option price model under an mfBm [13] and an option price model under a jmfBM. This test will not be based on empirical data, but they will be calculated based on the formula that has been produced with the selected parameter.

Parameters for computing call options are presented in Table 1. The first row presents parameters for calculating a call option under a Bm model. The second row displays parameters for calculating a call option under an fBm model. The third row presents parameters for calculating a call option under a mfBm model. The fourth and fifth row provide the parameters for calculating the prices by the jfmBm which has low and high jump parameters.

Table 1. The valuation of chosen parameters used in these models.

Model type	r	σ	K	H	J	λ	μ_J	δ
Bm	0.04	0.1	100	-	-	-	-	-
fBm	0.04	0.1	100	0.8	-	-	-	-
mfBm	0.04	0.1	100	0.8	-	-	-	-
jmfBm [a]	0.04	0.1	100	0.8	0.008	1.25	0.0005	0.001
jmfBm [b]	0.04	0.1	100	0.8	0.008	6.25	-0.0005	0.001

By comparing the Bm, fBm, mfBm, and jmfBm[a] columns in Table 2 for $T = 0.25$ cases, it produces an option value that is almost close. This is especially a very low jump parameter and the effect of long memory properties has not been seen for options with a short time maturity. As maturity increases, the magnitude of option prices calculated by the four methods increases in the case of high and low jump parameters. The jmfBm[b] column has an option price that is relatively larger than the other three methods for the case $T = 3$.

Figure 1 and 2 display the values of European call options versus time of maturity and strike prices. The default parameter is $S = 100$, $r = 0.04$, $\sigma = 0.1$, $H = 0.8$, $J = 0.08$, $\lambda = 6.25$, $\delta = 0.01$ and $K = [80,120]$ and $T = [0,5]$. Figure 1 using $\mu_J = -0.005$ and Figure 2 using $\mu_J = 0.005$. Figure 1 illustrates the option price under jmfBm is relatively greater than the option price under mfBm if $\mu_J = -0.005$. This does not apply if $\mu_J = 0.005$ is seen in Figure 2. It can be concluded that an option price below jmfBm depends on the value of jump parameters.

Table 2. Price of call options by different models.

S	$T = 0.25$ (low time to maturity)					$T = 3$ (high time to maturity)				
	C_{Bm}	C_{fBm}	C_{mfBm}	$C_{\text{jmfBm}}{}^{a}$	$C_{\text{jmfBm}}{}^{b}$	C_{Bm}	C_{fBm}	C_{mfBm}	$C_{\text{jmfBm}}{}^{a}$	$C_{\text{jmfBm}}{}^{b}$
80	0.0000	0.0000	0.0002	0.0004	0.0004	2.4740	4.4639	6.1894	6.3756	6.7925
85	0.0015	0.0000	0.0095	0.0143	0.0150	4.3266	6.6063	8.4982	8.7342	9.2589
90	0.0512	0.0017	0.1339	0.1813	0.1881	6.8419	9.2329	11.2015	11.4895	12.1258
95	0.5567	0.1612	0.8449	1.0504	1.0782	9.9833	12.3107	14.2705	14.6113	15.3598
100	2.5216	1.8666	2.9080	3.3809	3.4412	13.6621	15.7893	17.6684	18.0617	18.9205
105	6.2954	6.0452	6.5219	7.2301	7.3155	17.7660	19.6099	21.3549	21.7994	22.7649
110	11.0282	10.9957	11.0941	11.9318	12.0277	22.1837	23.7123	25.2891	25.7831	26.8506
115	15.9971	15.9950	16.0078	16.9103	17.0089	26.8195	28.0404	29.4323	29.9736	31.1378
120	20.9951	20.9950	20.9962	21.9441	22.0431	31.5994	32.5446	33.7488	34.3353	35.5909

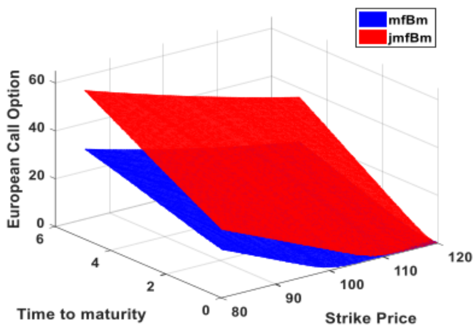

FIGURE 1. Price of call option by mfBm and jmfBm Model with $\mu_J = -0.005$

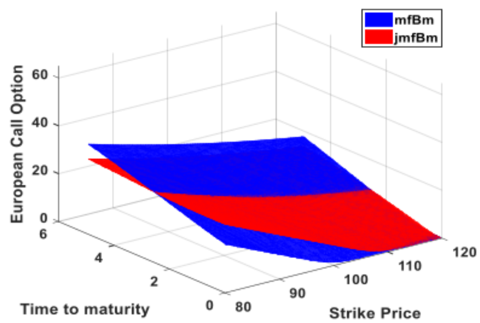

FIGURE 2. Price of call option by mfBm and jmfBm Model with $\mu_J = 0.005$

4. Conclusion

In this paper, stock returns are modeled with a jmfBm to capture long-range dependence and jumps process and also to exclude arbitrage opportunities in the fBm model. We obtain a formula for calculating a European option price under a jmfBm by using a theory of quasi-conditional expectation and Fourier transformation method. This formula may be used by investors to predict option prices for stocks that have long-range dependence and jumps. Moreover, this formula holds for an arbitrage-free market.

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