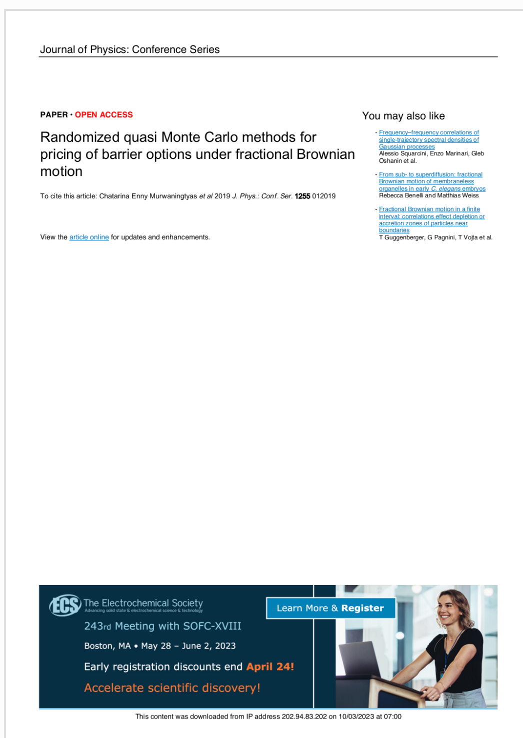
Digital Receipt

This receipt acknowledges that Turnitin received your paper. Below you will find the receipt information regarding your submission.

The first page of your submissions is displayed below.

Submission author:	Enny Murwaningtyas
Assignment title:	Periksa similarity
Submission title:	Randomized quasi Monte Carlo methods for pricing of barri...
File name:	pricing_of_barrier_options_under_fractional_Brownian_motio...
File size:	993.41K
Page count:	7
Word count:	2,875
Character count:	14,201
Submission date:	10-Mar-2023 02:57PM (UTC+0700)
Submission ID:	2033754822



Journal of Physics: Conference Series

PAPER • OPEN ACCESS

Randomized quasi Monte Carlo methods for pricing of barrier options under fractional Brownian motion

To cite this article: Chatarina Enny Murwaningtyas *et al* 2019 *J. Phys.: Conf. Ser.* **1255** 012019

View the article online for updates and enhancements.

You may also like

- Frequency–frequency correlations of single-trajectory spectral densities of Gaussian processes
  Alessio Squarcini, Enzo Marinari, Gleb Oshanin et al.
- From sub- to superdiffusion: fractional Brownian motion of membraneless organelles in early *C. elegans* embryos
  Rebecca Benelli and Matthias Weiss
- Fractional Brownian motion in a finite interval: correlations effect depletion or accretion zones of particles near boundaries
  T Guggenberger, G Pagnini, T Vojta et al.

Randomized quasi Monte Carlo methods for pricing of barrier options under fractional Brownian motion

by Murwaningtyas Enny

Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

Randomized quasi Monte Carlo methods for pricing of barrier options under fractional Brownian motion

IOP Conf. Series: Journal of Physics: Conf. Series **1255** (2019) 012019 doi:10.1088/1742-6596/1255/1/012019

Randomized quasi Monte Carlo methods for pricing of barrier options under fractional Brownian motion

Chatarina Enny Murwaningtyas [1,2], Sri Haryatmi Kartiko [1], Gunardi [1], and Herry Pribawanto Suryawan [3]

[1] Department of Mathematics, Universitas Gadjah Mada, Indonesia,
[2] Department of Mathematics Education, Universitas Sanata Dharma, Indonesia,
[3] Department of Mathematics, Universitas Sanata Dharma, Indonesia.

* enny@usd.ac.id

Abstract. Randomized quasi-Monte Carlo (RQMC) method is presented to compute the problem of a barrier option pricing. It is assumed that stock prices are modeled with a fractional Brownian motion (FBM). The FBM is a Gaussian process with dependent and stationary increments except H = ½. The FBM can model stock prices with short or long memory. We propose a trajectory generation technique based on fast Fourier transforms to simulate stock prices modeled by FBM. A stock price trajectory is utilized to predict pricing of barrier options. Barrier options are options whose payoff function depend on the stock prices during the option's lifetime. Using the results of the stock price trajectory and RQMC method can be determined the price of a barrier option under FBM. We conclude that RQMC is an efficient technique for calculating the price of barrier options rather than a standard Monte Carlo (MC).

1. Introduction

Monte Carlo (MC) method is one way that can overcome financial problems, especially in determining option prices. MC method is a method that uses random numbers to determine an expected value of a random variable. These random numbers are pseudo-random numbers (PRN) that have a certain probability distribution. However, this method has a slow speed of convergence and rather time-consuming, because a root of the average decay error is $O(N^{-1/2})$, where N is a number of samples.

Quasi-Monte Carlo (QMC) method is an efficient alternative to the standard MC method, which is able to achieve faster convergence and higher accuracy [1,2]. The QMC method is based on using a low-discrepancy sequence (LDS), is also called a quasi-random number for sampling point. LDS is designed so that the integration domain resembles a uniform distribution but the process of determining random numbers is deterministic. However, PRN is a random number that has a uniform distribution and fulfills statistical properties.

With deterministic QMC method, it is difficult to estimate integration errors in practice. Randomized quasi-Monte Carlo (RQMC) method is used to replace QMC method. RQMC method combines LDS and PRN. In this paper, Halton and Sobol sequence, which are LDS with a different deterministic model, will be applied in RQMC method for calculating a barrier option pricing.

Analytical formulas for calculating barrier options are not available in many cases such as barrier options under the FBM model and options with many assets. The MC method is one that plays an

IOP Conf. Series: Journal of Physics: Conf. Series **1255** (2019) 012019 doi:10.1088/1742-6596/1255/1/012019

important role in this situation. The MC algorithm is an algorithm based on simulating several stock price trajectories under a risk-neutral probability measure. A price of barrier options with the MC method has been discussed at [3–6]. Meanwhile, the price of barrier options under the FBM model using RQMC method has never been discussed. The purpose of this paper is to determine the price of a barrier option using RQMC method.

2. Fractional Brownian motion

An FBM $B^H = \left(B_t^H\right)_{t\ge 0}$ is a Gaussian process with a zero mean and a covariance function is defined

$$\mathrm{E}\left[B_t^H B_u^H\right] = \frac{1}{2}\left(|t|^{2H} - |t-u|^{2H} + |u|^{2H}\right), \tag{1}$$

where Hurst index $H \in (0, 1)$ and $t, u \ge 0$, see [7]. More precisely, by using (1), we obtain that covariance between $X_t = B_t^H - B_{t-1}^H$ and $X_{t+u} = B_{t+u}^H - B_{t+u-1}^H$ is

$$\rho_H(u) = \tfrac{1}{2}\left((u-1)^{2H} - 2u^{2H} + (u+1)^{2H}\right). \tag{2}$$

A FBM coincides with a standard BM if $H = \frac{1}{2}$. The Hurst index H determines the sign of the covariance of the future and past increments. This covariance is negative when $H \in \left(0, \frac{1}{2}\right)$, positive when $H \in \left(\frac{1}{2}, 1\right)$, and zero when $H = \frac{1}{2}$. As a consequence, it has short-range dependence (short memory) for $H \in \left(0, \frac{1}{2}\right)$ and it has long-range dependence (long memory) for $H \in \left(\frac{1}{2}, 1\right)$.

The stock price model under the FBM is given by

$$dS_t = \mu S_t dt + \sigma S_t d\hat{B}_t^H, \qquad t \in [0,T], S_0 > 0, \tag{3}$$

where μ and σ are constants, $\hat{B}_t^H$ is the FBM with respect to $\hat{\mathrm{P}}^H$. The fractional Black-Scholes model consists of one riskless asset (bank account) and one risk asset (stock). The stock price satisfies a stochastic differential equation (3). By using a change of variable $\sigma\hat{B}_t^H = \sigma B_t^H - \mu + r$ and using the Girsanov theorem in [7], we have

$$dS_t = rS_t dt + \sigma S_t dB_t^H, \qquad t \in [0,T], S_0 > 0. \tag{4}$$

Furthermore, we obtain a solution of (4) as

$$S_t = S_0 \exp\left(rt + \sigma B_t^H - \frac{1}{2}\sigma^2 t^{2H} \right), \tag{5}$$

by using a Itô formula in [7].

3. Pricing of barrier options

Path-dependent options are options whose value depends on a behavior of stock prices during its lifetime. Unlike vanilla options, the price of path-dependent options depends not only on stock price at maturity but also on stock prices trajectory during the contract. There are many types of path-dependent options, such as Asian options, lookback options, and barrier options. Each option has unique characteristics. Path-dependent options offer premium prices that are cheaper than the standard vanilla option.

A barrier option is a option that can be activated or deactivated if the stock price reaches a certain price level (L). Barrier options in general consist of two types, namely knock in and knock out options. A knock out option is an option whose contract is canceled if the stock price crosses the barrier value. A knock in option, on the other hand, is activated if the stock price crosses the barrier value. The relationship between a barrier value L and a current stock price S_0 indicates whether the option is an up or down option. We have an up option if $L > S_0$ and we have a down option if $L < S_0$. Combining the payoffs of call and put options with these features, we can define an array of barrier options.

IOP Conf. Series: Journal of Physics: Conf. Series **1255** (2019) 012019 doi:10.1088/1742-6596/1255/1/012019

An option is said to be an up-and-out option if the stock price crosses a barrier value and the value is greater than the current stock price. A down-and-out option is an option which the stock price crosses a barrier value and the value is below the current stock price. The payoff of an up-and-out call option with barrier value L, expiration time T and strike price K, is given by

$$f(S_T)=(S_T-K)^+\mathbf{1}_{\sup_{t\in(0,T)}\{S_t\le L\}} \tag{6}$$

and the payoff of a down-and-out call option is given by

$$f(S_T)=(S_T-K)^+\mathbf{1}_{\sup_{t\in(0,T)}\{S_t\ge L\}} \tag{7}$$

The payoff function of a put option is defined similarly with $(K-S_T)^+$ in place of $(S_T-K)^+$.

A closed form formula for an option barrier pricing under a fractional Brownian motion model has not been found. This is because a fractional Brownian motion no longer has markovian and martingale properties so the reflective principle used to derive formulas for barrier options is no longer valid. So it is difficult to get an analytical solution from the barrier option pricing.

4. Randomized quasi Monte Carlo

Option pricing using the MC method can be determined in the following three stages

- Simulate sample trajectories from stock prices during a time interval [0, T] as many as m times,
- Calculate a discounted expected value of a payoff function of a barrier option for each trajectory that generated in the first stage,
- Average the value that calculated in the second stage.

In the vanilla option, there is actually no need to make a stock price trajectory, only the stock price at maturity is of concern. Barrier options are options that depend on the trajectory of a stock prices. The barrier option pricing is determined by whether the stock price passes a certain barrier value during the option period. Because of this path-dependent, all stock price simulations are needed during the option period. To simulate a sample trajectory, we must choose a stochastic differential equation that illustrates price dynamics. Stochastic equations for a stock price under FBM are written in (5).

QMC simulation is based on the same procedure as MC simulation but uses LDS instead of PRN. Similar to PRN, LDS is algorithmically generated by a computer, except that the LDS is determined deterministically in a smart way to be more uniformly distributed than PRN. In contrast to an MC sample, LDS do not have the independent and identically distributed (i.i.d.) property.

Therefore, we cannot directly use LDS in the QMC method. However, randomized LDS samples can be constructed by changing LDS into the following form

$$\tilde{U}_i=(U_i+W_i) \mod 1, \tag{8}$$

where W_i is a PRN and U_i is an LDS. The vector $\tilde{U}_i$ is uniformly distributed in the unit hypercube and sequence $\tilde{U}$ have the independent and identically distributed property. Thus, the estimators based on $\tilde{U}_i$ are unbiased.

The best known quasi-random number generations [8] are Halton sequences, Faure sequences, Sobol sequences, and the lattice method. This paper only discusses two quasi-random number generations, namely Sobol sequences and Halton sequences. Sobol sequences are examples of LDS. Ilya M Sobol, a Russian mathematician, first introduced Sobol sequences [9] in 1967. Sobol points can be produced using algorithms introduced by Bratley and Fox [10]. Halton Sequences are sequences that produce points in space using numerical methods such as appear to be random. The Halton sequence was first introduced in 1964 [11] and developed by Kocis and Whiten [12].

5. Numerical Results

In this section, we first simulate sample trajectories from stock prices at time intervals [0, T]. The stock price is modeled using equation (5). The algorithm for building trajectories of stock prices using quasi random numbers is seen in Algorithm 1. Using the algorithm can be generated trajectories of

IOP Conf. Series: Journal of Physics: Conf. Series **1255** (2019) 012019 doi:10.1088/1742-6596/1255/1/012019

stock prices like Figure 1. Figure 1 also says that when $H = 1/2$, the FBM will be the same as the standard BM.

input : Set an expire date T, an initial stock price S_0, an interest rest r, a stock volatility σ, a Hurst index H and a large number n of equally spaced subintervals in $[0, T)$
output: S_t with $t = t_1, t_2, \ldots, t_n \in [0, T)$

Set using pseudo random number, Halton sequences or Sobol sequences;
for $j \leftarrow 1$ **to** n **do**
  Generate two pseudo-random number W_{1j} and W_{2j};
  if *pseudo random number* **then**
    $\tilde{U}_{1j} \longleftarrow W_{1j}$ and $\tilde{U}_{2j} \longleftarrow W_{2j}$;
  else
    Generate two two Halton or Sobol sequences U_{1j} and U_{1j};
    $\tilde{U}_{1j} \longleftarrow (U_{1j} + W_{1j}) \mod 1$ and $\tilde{U}_{2j} \longleftarrow (U_{2j} + W_{2j}) \mod 1$;
  end
  RandomComplex $\longleftarrow \tilde{U}_{1j} + \tilde{U}_{2j}\, i$;
end
$\rho(1) = 1$;
for $k \leftarrow 1$ **to** n **do**
  $\rho_{k+1} \longleftarrow \frac{1}{2}\left((k-1)^{2H} - 2k^{2H} + (k+1)^{2H}\right)$;
end
ρ new $\leftarrow [\rho; \rho(end - 1 : -1 : 2)]$
$\lambda \leftarrow \frac{\text{Real(FFT}(\rho \text{ new}))}{2*n}$
$X \leftarrow \text{FFT}\left(\sqrt{\lambda}\right) * \text{RandomComplex}$
$W \leftarrow \text{CUMSUM(Real}(X(1 : n+1))))$
for $j \leftarrow 1$ **to** n **do**
  $S_{t_j} \leftarrow S_0 \exp\left(r\frac{jT}{n} - \frac{1}{2}\sigma^2\left(\frac{jT}{n}\right)^{2H} + \sigma\left(\frac{T}{n}\right)^H W_j\right)$
end

Algorithm 1. Stock price trajectories under FBM

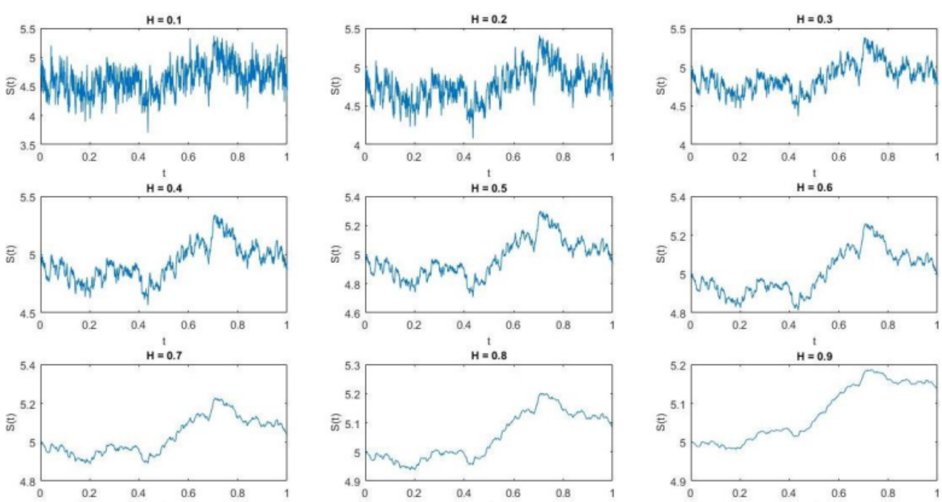


Figure 1. Stock price trajectories under FBM by using Sobol sequences

IOP Conf. Series: Journal of Physics: Conf. Series **1255** (2019) 012019 doi:10.1088/1742-6596/1255/1/012019

If $H < ½$ the trajectory of the stock price fluctuates greatly, and if $H > ½$ the trajectory of the stock price is more likely to be smooth.

Algorithm 2 is an algorithm used to calculate the pricing of a barrier option under an FBM model using MC and RQMC. The pricing of a barrier option can be determined using the trajectory of stock prices generated in Algorithm 1. The pricing of a barrier option generated in Algorithm 2 is an up-and-out call option. Other barrier options can be calculated by changing lines 4-6 in Algorithm 2 according to the payoff function of the option. All algorithms in this paper are written and executed in the Matlab program.

input : Set an expire date T, a strike price K, an initial stock price S_0, a interest rest r, a stock volatility σ, a Hurst index H, a large number n of equally spaced subintervals in $[0,T)$ and sample size m

output: Price of an up-and-out call option

1 Set using pseudo random number, Halton sequences or Sobol sequences
2 **for** $j \leftarrow 1$ **to** m **do**
3   Set $S_T \leftarrow$ stock price trajectory using Algoritma 1
4   $S_{Max} \leftarrow \max\{S_t | S_t \in S_T \text{ and } t \in (0,T)\}$
5   **if** $S_{Max} < L$ **then**
6     $V_j \leftarrow \exp(-rt)\ \max\{S_T(\text{end}) - K, 0\}$
7   **else**
8     $V_j \leftarrow 0$
9   **end**
10 **end**
11 $C_m \leftarrow \sum_{j=1}^{m} V_j \Big/ m$

Algorithm 2. Price of an up-and-out call option using randomized quasi Monte Carlo

We present an example to show the effectiveness of Algorithm 2. We use current stock price $S_0 = 500$, strike price $K = 500$, expiration date $T = 1$, stock volatility $\sigma = 0.05$, index Hurst $H = 0.8$, and interest rate $r = 0.05$. We implement the MC and RQMC simulation algorithms and compare the results obtained from all methods. The RQMCS method is the RQMC method while the LDS used is the Sobol sequence. Whereas, the RQMCH method is the RQMC method by using the Halton sequence.

Table 1. Price of an up-and-out call option with $M = 10^2, 10^3, 10^4, 10^6$

M	N	Price of an up-and-out call option			Error of an option price		
		MC	RQMCS	RQMCH	MC	RQMCS	RQMCH
100	1000	17,24898	14,07713	13,99044	1,52361	0,94219	1,01528
1000	1000	14,49144	14,97568	15,00420	0,47331	0,32219	0,32139
10000	1000	15,09796	15,23683	15,15125	0,15128	0,10101	0,10141
100000	1000	15,27597	15,29528	15,23070	0,04809	0,03201	0,03199

Table 1 is the pricing of an up-and-out call option with the MC and RQMC methods using the Sobol and Halton sequences based on a large number of subintervals, $N = 1000$, and sample sizes, $M = 100$, 1000, 10000 and 100000. Using the results in Table 1, we can conclude that the RGMC method is more efficient than the MC method. Whereas in Table 2 it compares three methods with large numbers of subintervals, $N = 100$, 1000, 10000 and 100000, and sample sizes, $M = 1000$. In this table also concludes the same thing, the RQMC method is more efficient.

IOP Conf. Series: Journal of Physics: Conf. Series **1255** (2019) 012019 doi:10.1088/1742-6596/1255/1/012019

Table 2. Price of an up-and-out call option with $N = 10^2, 10^3, 10^4, 10^6$

M	N	Price of an up-and-out call option			Error of an option price		
		MC	RQMCS	RQMCH	MC	RQMCS	RQMCH
1000	100	14,38474	14,97417	15,97086	0,47787	0,32375	0,31689
1000	1000	15,20676	14,74686	15,41580	0,48841	0,32536	0,32023
1000	10000	15,53449	14,98866	14,44457	0,48684	0,31704	0,31358
1000	100000	15,13752	15,30209	15,23531	0,47789	0,32155	0,32103

6. Conclusion

One of the methods to determine the pricing of a barrier option under an FBM model is to use the RQMC methods. The stock price trajectory under the FBM model has been proposed in Algorithm 1. Using Algorithm 1 can be determined the barrier option price under the FBM model with the MC and RQMC methods described in Algorithm 2. We compare the accuracy of MC and RQMC method in the pricing of barrier option under the FBM model. The RQMC method more efficient than the MC method which is shown by smaller errors.

Acknowledgments
The authors would like to thank DRPM Kemenristekdikti, the Republic of Indonesia, due to the funding of PDD 2018 for this work.

References

[1] Jäckel P 2002 *Monte Carlo Methods in Finance* (Wiley)
[2] Glasserman P 2003 *Monte Carlo methods in financial engineering* (Springer Science & Business Media)
[3] Moon K S 2008 Efficient Monte Carlo algorithm for pricing barrier options *Commun. Korean Math. Soc.* **23** 285–94
[4] Shevchenko P V. and Del Moral P 2014 Valuation of Barrier Options using Sequential Monte Carlo 1–30
[5] Alzubaidi H 2016 Efficient Monte Carlo algorithm using antithetic variate and brownian bridge techniques for pricing the barrier options with rebate payments *J. Math. Stat.* **12**
[6] Nouri K and Abbasi B 2017 Implementation of the modified Monte Carlo simulation for evaluate the barrier option prices *J. Taibah Univ. Sci.* **11** 233–40
[7] Biagini F, Hu Y, Øksendal B and Zhang T 2008 *Stochastic Calculus for Fractional Brownian Motion and Applications* (Springer)
[8] Kroese D P, Taimre T and Botev Z I 2011 *Handbook of Monte Carlo Methods* (Wiley Series in Probability and Statistics)
[9] Sobol' I M 1967 On the distribution of points in a cube and the approximate evaluation of integrals *Zhurnal Vychislitel'noi Mat. i Mat. Fiz.* **7** 784–802
[10] Bratley P and Fox B L 1988 ALGORITHM 659: implementing Sobol's quasirandom sequence generator *ACM Trans. Math. Softw.* **14** 88–100
[11] Halton J H 1964 Algorithm 247: Radical-inverse quasi-random point sequence *Commun. ACM* **7** 701–2
[12] Kocis L and Whiten W J 1997 Computational Investigations of Low- Discrepancy Sequences *ACM Trans. Math. Softw.* **23** 266–94

Randomized quasi Monte Carlo methods for pricing of barrier options under fractional Brownian motion

ORIGINALITY REPORT

24%	14%	19%	9%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	Submitted to Universitas Muhammadiyah Surakarta Student Paper	4%
2	Submitted to Universitas Airlangga Student Paper	2%
3	Iddo Eliazar, Tal Kachman. "Anomalous diffusion: fractional Brownian motion vs fractional Ito motion", Journal of Physics A: Mathematical and Theoretical, 2022 Publication	2%
4	Submitted to University of Glasgow Student Paper	2%
5	Hasan Alzubaidi. "Efficient exponential timestepping algorithm using control variate technique for simulating a functional of exit time of one-dimensional Brownian diffusion with applications in finance", Arabian Journal of Mathematics, 2020 Publication	1%

6	Internet Source	1 %
7	www.hrpub.org Internet Source	1 %
8	Wensheng Wang. "Asymptotic analysis for hedging errors in models with respect to geometric fractional Brownian motion", Stochastics, 2018 Publication	1 %
9	"Martingale Methods in Financial Modelling", Springer Nature, 2005 Publication	1 %
10	E N Bogdanov, A V Egorov, I A Polyanin, A V Lysyannikov, Yu F Kaizer, R B Zelykevich, A V Kuznetsov, T Y Matkerimov. "Improvement of the plane-parallel movement of the attachment of the feller-bunchers machine LP-19", Journal of Physics: Conference Series, 2020 Publication	1 %
11	Keith Cuthbertson, Dirk Nitzsche, Niall O'Sullivan. "Derivatives", Wiley, 2019 Publication	1 %
12	www.repo.uni-hannover.de Internet Source	1 %
13	nlistsp.inflibnet.ac.in Internet Source	1 %

14	www.arxiv-vanity.com Internet Source	<1 %
15	Lecture Notes in Electrical Engineering, 2016. Publication	<1 %
16	Pierre Rostan, Alexandra Rostan, François-Éric Racicot. "Pricing discrete double barrier options with a numerical method", Journal of Asset Management, 2015 Publication	<1 %
17	Submitted to University of Birmingham Student Paper	<1 %
18	Submitted to University of Surrey Student Paper	<1 %
19	link.springer.com Internet Source	<1 %
20	ANHUI GU. "RANDOM ATTRACTORS OF STOCHASTIC LATTICE DYNAMICAL SYSTEMS DRIVEN BY FRACTIONAL BROWNIAN MOTIONS", International Journal of Bifurcation and Chaos, 2013 Publication	<1 %
21	Nigel J. Cutland, Alet Roux. "Derivative Pricing in Discrete Time", Springer Science and Business Media LLC, 2013 Publication	<1 %

22	Tao Cui, F. Franchetti. "A Quasi-Monte Carlo approach for radial distribution system probabilistic load flow", 2013 IEEE PES Innovative Smart Grid Technologies Conference (ISGT), 2013 Publication	<1 %
23	ebin.pub Internet Source	<1 %
24	ethesys.lis.nsysu.edu.tw Internet Source	<1 %
25	qme.sggw.pl Internet Source	<1 %
26	Mathematical Engineering, 2015. Publication	<1 %
27	centaur.reading.ac.uk Internet Source	<1 %
28	Alessio Squarcini, Enzo Marinari, Gleb Oshanin, Luca Peliti, Lamberto Rondoni. "Frequency-frequency correlations of single-trajectory spectral densities of Gaussian processes", New Journal of Physics, 2022 Publication	<1 %
29	Hasan Alzubaidi. "Efficient Monte Carlo Algorithm Using Antithetic Variate and Brownian Bridge Techniques for Pricing the	<1 %

Barrier Options with Rebate Payments", Journal of Mathematics and Statistics, 2016
Publication

30	J Beran. "On unified model selection for stationary and nonstationary short- and long-memory autoregressive processes", Biometrika, 12/01/1998 Publication	<1 %
31	Paul Glasserman. "Monte Carlo Methods in Financial Engineering", Springer Science and Business Media LLC, 2003 Publication	<1 %
32	Yuliya S. Mishura. "Chapter 1 Wiener Integration with Respect to Fractional Brownian Motion", Springer Science and Business Media LLC, 2008 Publication	<1 %