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Preface of the Conference Proceeding of the 2nd International Conference on Mathematics, its Applications, and Mathematics Education (ICMAME) 2024

We are pleased to present the proceedings of the 2nd International Conference on Mathematics, its Applications, and Mathematics Education (ICMAME) 2024. This esteemed event was a collaborative effort organized by the Department of Mathematics, Faculty of Science and Technology, and the Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Sanata Dharma, Yogyakarta, Indonesia.

The conference was held on September 21, 2024, at Universitas Sanata Dharma, Indonesia, under the theme *"21st Century Mathematics and Mathematics Education."* This theme reflects the urgent need for innovative approaches to mathematics and mathematics education, aligned with the demands and opportunities of the 21st century. ICMAME 2024 brought together mathematicians, educators, and researchers to discuss emerging trends and share insights in these critical areas.

We were privileged to have distinguished keynote and invited speakers who enriched the conference with their expertise and perspectives. Our heartfelt thanks go to:

- **Assoc. Prof. Dr. Wanty Widjaya** (Deakin University, Australia)
- **Dr. rer. nat. Wolfgang Bock** (Linnaeus University, Sweden)
- **Dr. Martianus Frederic Ezerman** (Nanyang Technological University, Singapore)
- **Asst. Prof. Dr. Parkpoom Phetpradap** (Chiang Mai University, Thailand)
- **Veronica Fitri Rianasari, M.Sc., Ph.D.** (Universitas Sanata Dharma, Indonesia)

We extend our deepest gratitude to the organizing committee for their dedication and hard work, to the authors for their invaluable contributions, and to the reviewers for their diligent evaluations and constructive feedback. Without their combined efforts, this conference would not have been possible.

We hope this collection of proceedings, featuring contributions from authors across various countries (Indonesia, Philippines, Thailand, Japan, etc.), serves as a significant resource for researchers, educators, and practitioners. May it inspire future innovations, foster collaboration, and contribute to the advancement of mathematics and mathematics education for the betterment of our global community.

Thank you for being a part of the 2nd ICMAME 2024. We look forward to continuing this journey together and to exploring new horizons in the fascinating realms of mathematics and mathematics education.

Yogyakarta, 15 January 2025
Eko Budi Santoso
Chair, The 2nd ICMAME 2024

Statement of Peer review

In submitting conference proceedings to ITM Web of Conferences, I certify to the Publisher that I adhere to the **Policy on Publishing Integrity** of the journal in order to safeguard good scientific practice in publishing.

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2. Reviews have been conducted by expert referees, who have been requested to provide unbiased and constructive comments aimed, whenever possible, at improving the work.
3. Proceedings editors have taken all reasonable steps to ensure the quality of the materials they publish and their decision to accept or reject a paper for publication has been based only on the merits of the work and the relevance to the journal.

International Conference on Mathematics, its Applications and Mathematics Education (ICMAME), 21 st September 2024, Yogyakarta, Indonesia

Proceedings editor(s): Hartono, Ph.D., Eko Budi Santoso, Ph.D., Prof.Dr.rer.nat Herry Pribawanto

January 29, 2025

The 2nd International Conference on Mathematics, Its Applications, and Mathematics Education (ICMAME) 2024

ICMAME

21st Century Mathematics and Mathematics Education

Hybrid: Driyarkara Seminar Room (offline) & Zoom (online)

SANATA DHARMA UNIVERSITY, INDONESIA

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ABOUT THE CONFERENCE

The 2nd International Conference on Mathematics, its Applications, and Mathematics Education (ICMAME) 2024 with the theme **"21st Century Mathematics and Mathematics Education"** will be held at Sanata Dharma University, Yogyakarta, Indonesia, on 21 September 2024. Selected papers of this conference will be published in ITM Web of Conferences Proceedings, Journal of The Indonesian Mathematical Society (SCOPUS), Barekeng: Jurnal Ilmu Matematika dan Terapan (SINTA 2), and Jurnal Pendidikan Matematika (SINTA 3).

KEYNOTE SPEAKER

Assoc. Prof. Dr. Wanty Widjaja
Deakin University, Australia

INVITED SPEAKERS

Dr. rer.nat. Wolfgang Bock
Linnaeus University, Sweden

Dr. Martianus Frederic Ezerman
Nanyang Technological University, Singapore

Assoc. Prof. Dr. Parkpoom Phetpradap
Chiang Mai University, Thailand

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Sanata Dharma University, Indonesia

GOALS

This conference is conducted to bring together mathematicians and other scientists working on new trends of mathematics and mathematics education for 21st century. The aim of this conference is to promote research interests in different fields of mathematics as well as in mathematics education.

SCHEDULE:

a. Submission Start : May 1, 2024
b. ~~Deadline Abstract, Batch 1 : July 31, 2024~~
c. ~~Announcement Batch 1 : August 5, 2024~~
d. ~~Deadline Abstract, Batch 2 : August 16, 2024~~
e. ~~Announcement Batch 2 : August 22, 2024~~
f. Deadline Abstract, Batch 3 : August 30, 2024
g. Announcemnet Abstract, Batch 3 : August 31, 2024
f. Full Paper Submission : September 2, 2024
g. Conference Day : September 21, 2024

ARTICLE REGULATIONS :

We welcome researchers to submit their original, unpublished work that hasn't been reviewed elsewhere, and that addresses cutting-edge research in Pure Mathematics, Applied Mathematics, and Mathematics Education. The review process involves

- the submission of a complete paper by the author(s) before the specified deadline, adhering to the provided template. Submission is conducted through an e-conference system.
- Each manuscript undergoes thorough peer review by at least two qualified reviewers. Only manuscripts approved by reviewers with relevant expertise in the field are accepted.
- Evaluation criteria include the paper's contribution to the respective field, originality and suitability of the research method, technical proficiency in research development and execution, as well as paper structure and language quality.
- The peer review process entails manuscript submission, review and selection by editors who may approve, reject, or directly review submissions.
- Plagiarism checks are performed using Turnitin.
- Authors are notified of acceptance, revision requirements, or rejection based on reviewer feedback.
- Authors revise the manuscript according to reviewer suggestions, resubmitting the revised version.
- Acceptance notification is issued by the editor if reviewers are satisfied with the revision.
- Editors conduct a final review and edit to ensure adherence to template guidelines.
- Publication requires conference registration and participation by the author or at least one co-author.

Topics of interest for submission include, but are not limited to:

Mathematics:
- o Analysis and Applied Analysis
- o Analysis and Applied Algebra
- o Graph Theory
- o Discrete Mathematics
- o Statistics and Probability

Applied Mathematics:
- a .Cryptography
- o Dynamical Systems
- o Control Theory
- o Fuzzy Sets and Systems and Fuzzy Logic
- o Mathematical Modeling
- o Optimization and Operational Research
- o Machine Learning and Data Science

Mathematics Education:
- o Realistic Mathematics Education
- o Ethnomathematics
- o Learning Media
- o Curriculum
- o Learning Design
- o Pedagogical Content Knowledge

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Analysis of Pedagogical Content Knowledge (PCK) of Mathematics Education Students in Practice Learning Mathematics and Science Using STEAM Approach

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Abstract. This study aims to analyze the Pedagogical Content Knowledge (PCK) of Mathematics Education students who took the Micro Teaching course in the 2023/2024 academic year in learning practices for Mathematics and Science materials in junior high schools using the STEAM approach. The type of research used in this study is the Cobb and Gravemeijer model design research. The subjects of this research were 19 students of the Mathematics Education study program who took the Micro Teaching course in class C. Data were collected through observations of the learning practices carried out by the students. The results of the study showed that (1) the PCK of students was mostly in the moderate category, (2) the abilities that had been mastered well by students were creating and presenting contextual problems that connected mathematics and science, and (3) the abilities that students did not yet have were the ability to guide students in solving problems, overcoming student difficulties, and making conclusions.

1 Introduction

Education is an important thing in the process of human development. Based on Law No. 2 of 1985 concerning the Education System, education has the goal of improving the life of the nation and developing human beings as a whole. In the education system, there needs to be educators to be able to carry out the education process, in this case, teachers. To be able to channel knowledge to students, teachers must have the ability in educational competence. The competencies that must be fulfilled by teachers are pedagogical competence, personality competence, professional competence and social competence (Law no. 14 of 2005). In addition, according to Aminah and Wahyuni (2018), in-depth knowledge of how to teach (pedagogical knowledge) and the ability to understand the material to be taught (content knowledge) should be possessed by a teacher. In agreement with this, Solihin, Iqbal, and Miun (2021) argue that teachers must be able to develop knowledge about pedagogical knowledge and content knowledge in order to be able to understand the material to be taught and use effective teaching methods for students. Based on the opinions of these experts, it

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can be said that pedagogical knowledge and content knowledge are important for teachers to have so that teachers can carry out good learning to build students' knowledge.

The combination of the ability to understand learning materials (content knowledge) with the ability to teach materials (pedagogical knowledge) is called pedagogical content knowledge (PCK) (Shulman, 1986). According to Loughran (2012), PCK is the ability possessed by teachers in creating learning situations that allow students to understand material or content based on scientific facts. Sari (2021) argues that PCK is a combination of understanding of teaching materials (content knowledge) and teaching skills (pedagogical knowledge) that are harmoniously integrated. Shulman (1986) said that teacher PCK is a crucial thing that concerns learning and greatly influences the improvement of student understanding. In line with this, Wulandari and Iriana (2018) argue that in order for meaningful learning to be created for students, PCK skills are very important for teachers to have.

According to Grossman (1990), there are three components in PCK, namely: (1) Knowledge of subject matter knowledge which includes a deep understanding of the learning material to be taught; (2) Knowledge of general pedagogy (general pedagogical knowledge) which includes general pedagogical knowledge, such as understanding of teaching principles, teaching strategies, classroom management, interaction with students and learning assessment; and (3) Knowledge of context, including understanding of the learning context including student characteristics, school environment and other factors that influence learning. The intersection of the three aspects, namely: knowledge of subject matter, knowledge of general pedagogy and knowledge of context will form the components of PCK.

The components of the intersection of the three things are divided into four, namely the purpose of teaching a material, knowledge of student understanding, knowledge of the curriculum and knowledge of learning strategies. The purpose of teaching a material is the teacher's understanding of the purpose of teaching a learning material. This also includes the reasons for teaching the material and the relevance of the material to different levels of students. Knowledge of student understanding is knowledge of how students understand the subject matter including misconceptions experienced by students and how to overcome errors that occur in student understanding. Curriculum knowledge is the teacher's knowledge to understand the content and structure of the curriculum that is in accordance with the subject matter and its relationship to other topics in the curriculum. Knowledge of learning strategies is the teacher's knowledge of teaching strategies that are in accordance with the subject matter such as how to deliver the material effectively and determine methods to explain the material so that it is easier for students to understand (Grossman, 1990).

From the results of observations of videos of learning practices carried out by Mathematics Education students who took Micro Teaching class D in the 2022/2023 academic year when they practiced teaching Mathematics and Science to junior high school students using the STEAM approach and the Problem-Based Learning model, the following problems were obtained: the problems given by the practicum are still in the form of instructions to do something, not questions that must be answered independently by students, the practicum has not ensured whether the students they teach understand the problems they have to solve or not, the practicum does not help students when they are unable to solve the problem, as a result, students do not solve the problems given properly, the practicum does not provide guidance questions when students have difficulty solving the problem, but provides instructions on how students can solve the problem, the practicum does not explore students' answers and connect them with mathematical and science concepts related to the problems that have been solved by students, the practicum does not involve students when drawing conclusions.

In reality, although this PCK ability is very important for teachers and prospective teachers to have, there are still teachers or prospective teachers who do not fully have this ability. In a study conducted by Irfan et al. (2018) at a university in Aceh, the results showed that the PCK ability of prospective teacher students in the Mathematics Education study program at the university was in the low to medium category with a score of 1 to 2. Furthermore, in a study conducted by Dyastika (2023) at a university in Yogyakarta, the results showed that the PCK ability of undergraduate students in teaching high school mathematics material with the flipped classroom learning model was 33.3% of students in the high category, 55.3% of students in the medium category, and 12.4% of students in the low category. In a study conducted by Anindita (2018) at a high school in Yogyakarta, the results showed that teachers had not been able to meet several indicators of PCK ability measurement, including understanding students' thinking, providing guidance that was in accordance with students' thinking processes, understanding the curriculum, and attracting and maintaining students' attention.

The problem-based learning model is a learning model that asks students at the beginning of learning to solve problems, so that students are able to develop new knowledge based on the knowledge that students already have (Syamsidah and Hamidah, 2017). According to Duch (1995), the problem-based learning model is a learning model that asks students to solve problems as a means of developing critical thinking skills, problem-solving skills and gaining new knowledge. Furthermore, according to Arends (2008) the Problem-Based Learning model is a learning model that presents situations by giving students authentic problems in order to encourage students to investigate and obtain problem solving so that they are able to build new knowledge. The advantages of the Problem-Based Learning model, according to Kurniasih and Berlin (2015), are that students are able to develop critical and creative thinking, students are able to improve problem-solving skills, increase motivation to train independence and others. STEAM stands for Science, Technology, Engineering, Art and Mathematics. The STEAM approach is an approach that combines science, technology, engineering, art and mathematics in learning (Nurhikmayati, 2019). According to Katz-Buinicontro (2018) the STEAM approach is an approach that integrates art into the curriculum and learning process in the fields of science, technology, engineering and mathematics. The advantages of the STEAM learning approach are that students can develop active, creative and innovative problem solving, students can express ideas with technology, and students are able to implement the knowledge they have acquired in their lives. The steps for learning with the Problem Based Learning model and the STEAM approach are as follows: problem orientation, organizing students to learn, guiding student investigations, developing and presenting results and analyzing and evaluating the problem solving process (Arends, 2008).

The formulation of the problem that will be attempted to be solved in this study is as follows: how is the Pedagogical Content Knowledge (PCK) of undergraduate students of Mathematics education who take micro-teaching courses in the 2023/2024 academic year in the practice of teaching Mathematics and Science using the STEAM approach for junior high school students?

2 Method Study

The type of research used in this study is design research. Design research is research that aims to design a structured learning system, prepare plans, and conduct educational evaluations through interventions to provide solutions to complex problems that occur in educational practice, and expand understanding of interventions through the design and development process (Plomp, 2013). The stages of design research that will be used are preparation, in-class research and retrospective analysis (Gravemeijer & Coob, 2006). This design research will be used to design learning trajectory hypotheses for students who will practice Mathematics and Science learning for junior high school students using the PBM model and the STEAM approach. Furthermore, the PCK profile of each practitioner will be described qualitatively. This research was conducted from March to April 2024 at Sanata Dharma University in the 2023/2024 academic year.

The subjects in this study were 19 students of Mathematics Education undergraduate program at Sanata Dharma University who took Micro Teaching class C lectures in the 2023/2024 academic year. The instrument used in this study was a teaching practice observation sheet. Instrument validation in this study used expert validation techniques. The instrument will be validated by the supervising lecturer. In compiling the observation sheet that will be used to observe learning practices, the researcher derived and developed the things observed in learning practices based on the PCK indicators proposed by Grossman (1990). The observation sheet that was compiled will be used to record the results of the researcher's observations of the learning practices carried out by each practicum. The observation sheet used by the researcher consists of (1) three indicators used to observe the practicum activities in the opening section, (2) 16 indicators used to observe the practicum activities in this activity section, and (3) two indicators used to observe the practicum activities in the closing section. Details of each indicator can be seen in table 1.

To conduct data analysis on the observation data obtained by researchers in this study, researchers will use the qualitative data analysis stages proposed by Miles and Huberman (1984) which consist of three stages, namely: data reduction stage, data presentation stage and drawing conclusions.

Table 1. Detail of PCK indicators

PCK Aspects	Learning steps	Observed indicators
Learning model knowledge, learning content knowledge and subject knowledge of students	Opening	1. Conduct learning opening activities, pray, check students' attendance and organize the class to be ready for learning.
		2. Provide apperception: done by linking students' experiences with the material to be learned.
		3. Convey learning objectives and benefits to motivate students.
Learning content knowledge	Problem orientation	1. Presents contextual problems that are meaningful and can connect math to science.
Learning model knowledge		2. Motivate students to participate in finding solutions to problems.
Learning content knowledge, subject knowledge of students		3. Ask about students' understanding of the problem and ensure students have understood the problem.
Learning model knowledge	Organizing students to learn	1. Explaining the conditions in solving the problem a. The investigation of the problem will be conducted independently or in groups. b. Estimated time to solve the problem
		2. Ask about students' understanding of the provisions in solving the problem.
Learning model knowledge, learning content knowledge and subject knowledge of students	Guiding student inquiry	1. Ask about problem-solving ideas.
		2. Ask about the difficulties experienced when solving the problem.
		3. Provide feedback to students in the form of guiding questions and comments to help students build student concepts.
		4. Using feedback to make students aware of mistakes

		5. Reinforcing important points from students' answers
Learning content knowledge	Develop and present student work	1. Provide reaffirmation on problem solving after the presentation.
Learning model knowledge		2. Asked about students' willingness to present their answers.
		3. Ask other students with different questions, opinions or answers.
Learning model knowledge, learning content knowledge and subject knowledge of students		4. Provide feedback to students to discuss the problem solving process.
Learning model knowledge, learning content knowledge and subject knowledge of students	Analyze and evaluate the problem-solving process	1. Provide feedback to students in the form of guiding questions or comments to help students make conclusions from solving.
		2. Provide reaffirmation of the conclusions obtained by students.
Learning model knowledge	Cove	1. Reflect on the learning with the students.
		2. Closing the lesson.

3 Results and Discussion

3.1 Hypothetical Learning Trajectory (HLT)

Hypothetical Learning Trajectory (HLT) used to teach junior high school mathematics material is arranged based on learning steps with the PBM model and the STEAM approach. The learning steps taken are as follows: opening learning activities, core activities, and closing learning activities. In the opening learning activities, the activities carried out are as follows: opening learning with prayer, preparing the class to be ready to learn, conveying the objectives and benefits of learning, providing an apperception of the material, reminding about the learning models that have been studied previously. The core activities that will be carried out are as follows: conducting learning simulations with the PBM model and the STEAM approach. The researcher acts as a teacher and students act as students. The first step in the core activity is orientation to the problem, namely the researcher presents a problem about making a ship made of aluminum foil that can carry as many coins as possible. The second step is to organize students to learn, namely: the researcher conveys the provisions for solving problems carried out in groups. The researcher divides the class into three groups consisting of six to seven students. In this step, the researcher also distributes LKPD which will be used to answer the problems that have been given. The third step is to guide student investigations, namely: the researcher guides students in groups to be able to solve problems by giving several guiding questions to students and providing feedback that can support students in finding solutions to the problems given. The fourth step is to present the results of student work, namely: the researcher asks students to present the results of group work. In this fourth step, there are two groups that present the ships they have made and answer questions on the LKPD related to volume and buoyancy. The fifth step is to analyze and evaluate the problem-solving process, namely: the researcher invites students to draw conclusions by asking several questions that guide students to be able to find conclusions from the learning that has been done. The conclusions obtained by students are as follows: ships can float because of the buoyancy and are influenced by the volume of the ship. In the closing part of the learning, the researcher invites students to reflect and closes the learning with a closing greeting.

3.2 Pedagogical Content Knowledge

Reviewed through the learning practices carried out by subject 1, the PCK profile for subject 1 is as follows: (1) Subject knowledge about learning models: subject 1 has provided apperception, given problems, divided groups, and given questions so that discussions can occur between students. However, subject 1 has not been able to provide learning benefits, provide guidance questions and see errors in student work; (2) Subject 1's learning content knowledge is as follows: subject 1 has been able to recall previous knowledge, create contextual problems that connect mathematics with science, and provide confirmation of student answers, but has not been able to connect prerequisite material with the material to be taught and invite students to conclude the comparative concept obtained at this meeting; (3) Subject knowledge about students: subject 1 has been able to help students understand the problems given, and guide students in the problem-solving process. However, the subject has not been able to accompany all groups to be able to complete the task, because there are still groups that are unable to complete their tasks.

The PCK profile for subjects 2 and 3 when viewed from the learning practices carried out are as follows: (1) Subject knowledge about learning models: subjects 2 and 3 have been able to open learning, provide apperception, inform the provisions for solving problems, provide guidance with guiding questions, but subjects 2 and 3 have not been able to provide learning benefits and have not asked about students' understanding of the provisions for solving problems. (2) Subject content knowledge of learning 2 and 3 is to remind previous knowledge, present problems that link mathematics with science and confirm students' answers, but subjects 2 and 3 have not been able to explore students' answers. (3) Subject knowledge about students: subjects 2 and 3 have been able to understand students' characteristics and have been able to provide direction to students in solving problems, although in subject 2 there is a group that has not been able to complete their work, while for subject 3 has not provided further discussion space for students who have different answers.

The PCK profile for subject 4 when viewed from the learning practices carried out is as follows: (1) Subject knowledge about learning models: subject 4 has been able to open learning, provide apperception, convey learning objectives and benefits, inform problem-solving provisions and conduct reflection, but subject 4 has not been able to provide guidance to students and explore student answers. (2) Subject 4's learning content knowledge has been able to provide contextual problems that connect mathematics and science, help overcome student errors, emphasize important points in student answers and provide conclusions, but subject 4 has not been able to connect apperception with the material taught and has not explored student answers. (3) Subject knowledge about students: subject 4 has been able to understand student characteristics, conduct reflection but has not been optimal in understanding ideas and finding student difficulties.

The PCK profile for subject 5 when viewed from the learning practices carried out is as follows: (1) Subject 5's knowledge about learning models: subject has been able to open learning, convey learning objectives and benefits, explain problem-solving provisions, emphasize problem solving, provide conclusions and conduct reflection, but subject 5 has not been able to provide apperception and guiding questions. (2) Subject 5's learning content knowledge has been able to present contextual problems that connect science and mathematics, convey learning objectives and benefits, explore students' answers, but subject 5 has not been able to provide apperception and has not confirmed the solution to the problem. (3) Subject knowledge about students: subject 5 has been able to understand student characteristics and provide feedback during student discussions, but has not been able to ask about difficulties and ideas that students have.

The PCK profile for subject 6 when viewed from the learning practices carried out is as follows: (1) Subject knowledge about learning models: subject 6 has been able to open learning, provide apperception, inform the provisions for solving problems and conduct reflection, but subject 6 has not been able to provide guidance to students and explore students' answers. (2) Subject 4's learning content knowledge has been able to provide contextual problems that connect mathematics and science, connect apperception with material, help when students experience difficulties and provide conclusions, but subject 4 explores students' answers and provides guiding questions. (3) Subject knowledge about students: subject 6 has been able to understand student characteristics, conduct reflection but has not been optimal in understanding and providing questions to guide students.

The PCK profile for subject 7 when viewed from the learning practices carried out is as follows: (1) Subject knowledge about learning models: the subject has been able to open learning, provide apperception, invite students to conclude, provide feedback to students and reflection, but has not been able to convey the benefits of learning, convey the provisions for completion clearly and provide guidance with guiding questions. (2) Subject 7's learning content knowledge has been able to present problems that link mathematics with science, relate experiences to activities, provide feedback, provide affirmations and conclude, but has not been able to provide apperception that is connected to the material and does not provide guiding questions. (3) Subject knowledge about students: subject 7 has been able to understand student characteristics and provide students with opportunities for further discussion, but has not been able to provide the guiding questions needed by students.

The PCK profile for subjects 8, 9, 10, 15, and 19 when viewed from the learning practices carried out are as follows: (1) Subject knowledge about learning models: subjects have been able to open learning, provide apperception, explain problem-solving provisions, lead to conclusions and carry out reflection, but have not been able to provide guidance questions during discussions. (2) Subject content knowledge of subjects 8, 9, 10, 15 and 19 have been able to provide apperception, present problems related to science and mathematics, provide guidance questions and confirm conclusions, but have not been able to explore students' answers. (3) Subject knowledge about students: subjects 8, 9, 10, 15 and 19 have been able to understand student characteristics in understanding the problems given and have been able to understand the difficulties experienced by students, but have not been able to provide confirmation in solving problems.

The PCK profile for subject 11 when viewed from the learning practices carried out is as follows: (1) Subject knowledge about learning models: the subject has been able to open learning, is able to invite students to participate in solving problems, provides feedback in the form of explanations or guiding questions, provides conclusions and reflections, but has not been able to provide guiding questions for conclusions. (2) Subject 11's learning content knowledge has been able to present contextual problems that connect science and mathematics, explores student answers, but has not been able to provide apperception material. (3) Subject knowledge about students: subject 11 has been able to understand student characteristics in terms of understanding material and organizing group discussions, but has not given students the opportunity to draw conclusions.

The PCK profile for subject 12 when viewed from the learning practices carried out is as follows: (1) Subject knowledge about learning models: the subject has been able to open learning, is able to invite students to participate in solving problems, provides feedback in the form of explanations or guiding questions, provides conclusions and reflections, but has not been able to provide guiding questions for conclusions. (2) Subject 12 learning content knowledge has been able to present contextual problems that connect science and mathematics, provide apperception, and provide guiding questions during discussions, but has not been able to explore students' answers. (3) Subject knowledge about students: subject 12 has been able to understand student characteristics in terms of understanding material and organizing group discussions, but has not given students the opportunity to draw conclusions.

The PCK profile for subject 13 when viewed from the learning practices carried out is as follows: (1) Subject knowledge about learning models: the subject has been able to open learning, is able to invite students to participate in solving problems, provide feedback in the form of explanations or guiding questions, provide conclusions and reflections, but has not been able to provide guiding questions for conclusions. (2) Subject 13's learning content knowledge has been able to present contextual problems that connect science and mathematics, provide apperception, and provide guiding questions during discussions, explore student answers but has not been able to provide students with the opportunity to draw conclusions. (3) Subject knowledge about students: subject 13 has been able to understand student characteristics in terms of understanding the material and organizing group discussions, but has not given students the opportunity to draw conclusions.

The PCK profile for subject 14 when viewed from the learning practices carried out is as follows: (1) Subject knowledge about learning models: the subject has been able to open learning, explain the provisions for solving problems, reflect, but has not been able to provide guidance questions during discussions and conclusions and has not provided conclusions.. (2) Subject 14's learning content knowledge has been able to explore students' answers, but has not been able to provide guidance questions and feedback to students and has not provided apperception. (3) Subject knowledge about students: subject 14 has been able to involve students in learning activities, but has not been able to guide students in discussions and ensure students' understanding of the material.

The PCK profile for subject 16 when viewed from the learning practices carried out is as follows: (1) Subject knowledge about learning models: the subject has been able to open learning, provide apperception, explain the provisions for solving problems, and reflect, but has not been able to provide guidance questions during discussions and conclusions and has not provided conclusions.. (2) Subject 16's learning content knowledge has been able to provide apperception, present problems related to science and mathematics, confirm conclusions, but has not been able to explore students' answers. (3) Subject knowledge about students: subject 16 has been able to understand the characteristics of students in understanding the problems given, but has not been able to understand the difficulties experienced by students.

The PCK profile for subject 17 when viewed from the learning practices carried out is as follows: (1) Subject knowledge about learning models: the subject has been able to open learning, is able to invite students to participate in solving problems, provides feedback in the form of explanations or guiding questions, provides conclusions and reflections, but has not been able to provide learning benefits. (2) Subject 17's learning content knowledge has been able to present contextual problems that connect science and mathematics, provide apperception, and provide guiding questions during discussions, explore students' answers and are able to confirm students' answers. (3) Subject knowledge about students: subject 17 has been able to understand the characteristics of students in terms of understanding the material and organizing group discussions, but has not been able to understand students' abilities in understanding problems.

The PCK profile for subject 18 when viewed from the learning practices carried out is as follows: (1) Subject knowledge about learning models: the subject has been able to open learning, is able to invite students to participate in solving problems, provide feedback in the form of explanations or guiding questions, provide conclusions, but has not been able to provide guiding questions for conclusions and conduct reflection. (2) Subject 12's learning content knowledge has been able to present contextual problems that connect science and mathematics, remind students of experiences related to activities, and provide guiding questions during discussions, explore students' answers but has not been able to provide apperception related to the material. (3) Subject knowledge about students: subject 12 has been able to understand student characteristics in terms of understanding the material and organizing group discussions, but has not given students the opportunity to draw conclusions.

To categorize PCK ability based on the results of the students' observations, the following were done by the researcher: setting a score of 3 when the subject's PCK ability indicator was assessed as being visible and optimal, a score of 2 when the subject's PCK ability indicator was assessed as being visible but not yet optimal, and a score of 1 when the PCK ability indicator was not visible. Furthermore, the researcher did the following: (1) Adding up the students' scores based on the PCK indicators, (2) calculating the average and standard deviation, and (3) determining the boundaries for each group (Arikunto, 2010). The group boundaries set were as follows: high category when the score obtained was greater than the average plus the standard deviation, medium category when the score obtained was between the average plus the standard deviation and the average minus the standard deviation and low category when the score obtained was smaller than the average minus the standard deviation. From this process, the researcher found that the PCK ability of S1 Mathematics Education students in the micro teaching course of class C in the 2023/2024 academic year in teaching junior high school mathematics material using the PBM model and the STEAM approach, out of 19 students, there were 3 students who were in the high category, 13 students were in the medium category and 3 students were in the low category. From the results of the indicator analysis, the researcher can conclude that (1) the abilities that all students can have are the ability to carry out problem orientation, and the ability to create contextual problems that combine science with mathematics; and (2) the ability that is difficult for students to do is the ability to provide guidance questions that can help students solve problems, overcome student difficulties, and draw conclusions.

4 Conclusion

From the results of the data analysis obtained by researchers from the observation process of each practitioner when they were practicing, the following are: (1) Pedagogical Content Knowledge (PCK) of undergraduate students of Mathematics education who took micro-teaching courses in the 2023/2024 academic year in the practice of teaching Mathematics and Science using the STEAM approach and the PBM model for junior high school students, the majority of whom were in the moderate category, (2) the abilities that have been mastered well by students are creating and presenting contextual problems that connect mathematics and science, and (3) the abilities that students do not yet have are the ability to guide students in solving problems, overcoming student difficulties, and making conclusions.

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Implementation of Ignatian Pedagogy Paradigm on The Topic of Probability at Seminary Mertoyudan Senior High School Magelang

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Abstract. Education in Indonesia should be able to help students achieve complete personal development. One approach that can be taken is to apply the Ignatian Pedagogical Paradigm. This study aims to describe mathematics learning using the PPI-style Problem Based Learning (PBL) learning model for learning the material on opportunities for SMA Seminari Mertoyudan students. The research method used is qualitative research which analyzes learning observation data, student reflections, and student learning outcomes. The study subjects were 20 grade XII students of SMA Seminari Mertoyudan Magelang in the 2024/2025 academic year. The results of this study are that the learning process using the Ignatian Pedagogical Paradigm on the material on opportunities is carried out in addition to helping students understand the material on opportunities, it also helps the development of student's personal interests as a whole in terms of the 4C aspects (Competence, Conscience, Compassion, and Commitment). The dynamics of the Ignatian Pedagogical Paradigm applied in the learning process are context, experience, reflection, action, and evaluation. The learning process, of course, is also supported by a personalist attitude and discernment of teachers and students.

1 INTRODUCTION

Education should be able to develop all students' potential harmoniously, be it intellectual, emotional, social, or spiritual. As written in Law Number 20 of 2003, the goal of Indonesian national education is to develop the ability and form the character and civilization of a dignified nation in order to educate the life of the nation, aiming to develop the potential of students to become human beings who believe and fear God Almighty, have noble morals, are healthy, knowledgeable, capable, creative, independent and become democratic and responsible citizens.[1]. The government emphasizes the development of the whole person not only in the cognitive aspect but also in overall development including character.

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In line with this policy, the Indonesian Government has implemented the Independent Curriculum which emphasizes holistic education to develop students as a whole. The Independent Curriculum does not only emphasize academic or cognitive aspects but also emphasizes the importance of character education. The Independent Curriculum aims to realize meaningful and effective learning in increasing faith, devotion to God Almighty, and noble morals as well as developing the creativity, feelings, and intentions of students as lifelong learners with Pancasila characters.[2]. The concept of lifelong learners with the Pancasila character is stated in the Pancasila Student Profile.

The Pancasila Student Profile includes the main characteristics that students are expected to have, such as having faith, being devoted to God Almighty, being globally diverse, working together, being independent, being critical thinkers, and being creative.[3]. The six dimensions of the Pancasila student profile need to be seen as a whole so that each individual can become a lifelong student who is competent, has character, and behaves in accordance with Pancasila values. Therefore, educators are expected to be able to develop these six dimensions in classroom learning activities.

This holistic education concept is in line with whole person education which is the basis of the Ignatian Pedagogical Paradigm (PPI). PPI is an educational method that was born from Jesuit education and emphasizes the development of students not only in cognitive aspects but also developing into individuals who are sensitive to the goodness and needs of others.[4]. With this PPI style of learning, it is expected that students can develop into whole individuals, namely people for and with others. In the Ignatian Pedagogical Paradigm, what is meant by becoming a whole person is when you have achieved 4C (Competence, Compassion, Conscience, Commitment).

To achieve the 4C, the Ignatian Pedagogical Paradigm has learning dynamics that include context, experience, reflection, action, and evaluation.[4].

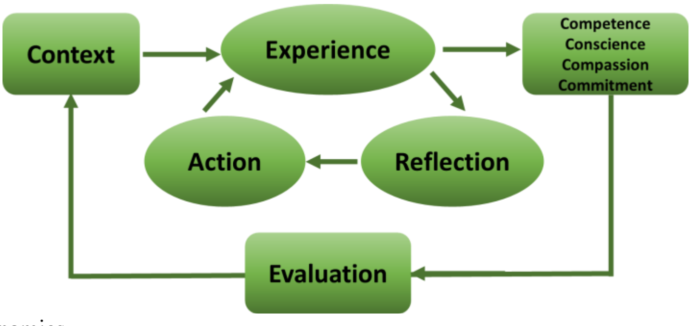

Fig 1. PPI Dynamics

In the context stage, educators are expected to be able to explore individual experiences and needs. Furthermore, educators encourage students to engage with real situations to deepen their understanding. In the reflection stage, students are encouraged to think critically about their experiences and the lessons they learn from the situation. Students are expected to be able to find real actions to be taken by considering the results of previous reflections. In the final stage, students are invited to conduct an evaluation that is comprehensive in terms of knowledge, attitude development, priority setting, and actions that are in line with the principles of men and women for and with others.

These dynamics, allow students to experience learning as a journey that is not only oriented toward the end result but also toward a meaningful process.[4]. Through learning that integrates PPI, students are expected to not only master mathematical knowledge but also have deep moral and social awareness.

As previously conducted research by Andri Anugrahana and Cintya Hasthiolivia [5] which shows that PPR learning not only improves understanding of mathematical material but also develops student character, especially in aspects of compassion and commitment. In addition, several previous studies also showed the same thing as the study conducted by Anna Fitriati [6] with the title Ignatian Pedagogy: An Alternative to Improve Students' Competence, Conscience, and Compassion and Hilda Hakim [7] with the title Reflective Pedagogy Paradigm (RPP) as an Alternative for Online Mathematics Learning to Improve Students' Competence, Conscience and Compassion.

Mertoyudan Seminary High School is a special school for prospective priests. As a formation house for prospective church pastors, Mertoyudan Seminary emphasizes the development of the whole person. As stated in the vision of the Seminary, namely the Saint Peter Canisius Mertoyudan Middle Seminary, it is intended to be a formation house for prospective church pastors who persistently and happily love Jesus Christ, thirst for knowledge, and have a great desire to serve [8]. In the vision, it is seen that the focus of mentoring is not only on the knowledge aspect but also on the service work where the formation of good personal character is needed. Therefore, the learning process carried out by the formators must reach mentoring to foster sensitivity, concern, and action for others.

This is certainly a challenge for educators at SMA Seminari Mertoyudan. Educators are expected to be able to develop intelligent individuals who care about others. Therefore, there needs to be awareness among educators to be able to design learning that triggers students to care about others. One of them is in mathematics learning. Mathematics is a learning that is still considered difficult by students, especially at Seminari Mertoyudan. Students consider that mathematics learning is less relevant to everyday life. This perspective ultimately makes students reluctant to deepen their mathematics learning. From this challenge, mathematics subject teachers are expected to be able to design mathematics learning that is relevant to student‘s lives and is expected to be in line with the school's vision, namely mentoring to foster sensitivity, concern, and action for others.

This study aims to describe mathematics learning using the PPI-style Problem Based Learning (PBL) learning model for learning probability material at Mertoyudan Seminary High School students.

2 METHOD

This research is qualitative. The subjects of the research were 20 students of class XII Merdeka 2 SMA Seminari Mertoyudan. The researcher conducted Mathematics learning for the material of Opportunity using the Problem-Based Learning model and applying the Ignatian Pedagogy Paradigm. Data were obtained through learning observations, student reflections, and student learning outcomes. The study's results will descriptively present the implementation of mathematics learning using the Problem-Based Learning model and applying the Ignatian Pedagogy Paradigm.

Observation instruments are used to determine the implementation of learning. The dynamics of PPR that must be implemented include the stages of context, experience, reflection, action, and evaluation. While for the syntax of Problem-Based Learning (PBL) used is student orientation to the problem, organizing students to learn, guiding individual and group investigations, developing and presenting work results, analyzing and evaluating the problem-solving process.

The reflection instrument provided contains the following questions:

- Describe the dominant feeling you are feeling right now!
- Do you understand the concept of the material studied today?
- What was interesting about today's learning?
- What difficulties did you encounter in today's learning dynamics?

- What do you do to overcome that?
- How does the discussion process work in groups? What is your role?
- What are your intentions for the future?

The evaluation for achieving the 4Cs used in this study is:

Table1. 4C Assessment Indicators

Aspect	Indicator	Information
Competence	Probability of Event	Determining the probability related to the counting rules (multiplication rules, addition rules, permutations, or combinations)
Conscience	Responsibility	Responsible for tasks in the group.
	Honest	Be honest in following the dynamics of learning.
Compassion	Value	Speak politely and do not interrupt other students
	Care	Care about the difficulties of his group of friends
Commitment	Consistent	Consistency plays a role in groups, especially in helping friends who are having difficulties.
	Committed	Always make sure and help your group mates.

3 RESULT AND DISCUSSION

3.1. Implementation of the Ignatian Pedagogical Paradigm

The implementation of the Ignatian Pedagogical Paradigm in mathematics learning using the Problem-Based Learning learning model will be analyzed through the dynamics of PPI as follows.

3.1.1. Context

In this stage, educators are expected to understand the context and background of students. Education is tailored to individual experiences and needs. Therefore, in this process, researchers explore the context of students by starting with the seminary context, the global context, and the context of mathematical material.

The institutional context takes information about the context of Mertoyudan Seminary High School. Mertoyudan Seminary High School is a formation house for prospective church pastors. Therefore, Mertoyudan Seminary emphasizes the development of the whole person. In the development of mentoring, not all students will become priests but will also continue their education in other areas.

The global context takes information about the rampant eradication of online gambling in Indonesia. Students are invited to see how the phenomenon of online gambling develops in society, even among young people. Students are asked to see the impact of online gambling in society and analyze what the causal factors are until they find the concept of opportunity in the phenomenon.

The global context regarding the impact of online gambling is presented in the form of a video taken from the page:https://www.youtube.com/watch?v=2mKISkqswoY. In the video, it is reported that there is a child who is cruel to abuse his mother because he was not given money to gamble online. From the video, there is a discussion process in the class about the negative impacts of online gambling. There are students who express whether online gambling creates excessive dependence. When observed from the video, it can be seen that there is a child who is cruel enough to abuse his own mother in order to be able to gamble.

Some children also liken online gambling to other online games that have actually made them addicted to playing during school holidays.

The context of understanding the material takes information about students' understanding of the concept of probability of events in mathematics. Teachers are expected to be able to know how students understand the concept of probability so that they can accompany them in understanding the concept more optimally.

3.1.2. Experience

The experience conducted in this study is direct experience. The direct experience of students is in the form of a simulation of a number guessing game and group discussion activities to deepen the experience of determining the probability of an event.

In the simulation experience of guessing numbers, students are invited to experience a simulation game that has a similar impression to number gambling. The simulation of guessing numbers is carried out in four sessions with each session having a different level of difficulty. In the first session, the level of difficulty in guessing numbers is quite high because no clues are given but the prizes are quite large. In the last session, namely the 4th session, the level of difficulty is relatively easy because you only guess one number but the prizes you get are small. This simulation experience aims to provide a dependency effect on students such as the initial assumption that online gambling is popular with young people because it causes dependency.

In the second experience, students were invited to have a discussion in groups. Students were asked to discuss the chances of winning in each session of the simulation game. This process was carried out to prove the student's statement, namely "Session one has a small chance of winning, while session 4 has a big chance of winning"

Fig 2. Group Discussion Process

3.1.3. Reflection and Action

In the reflection stage, the teacher invites students to look back at the experiences and learning processes that have been done before. The teacher can provide question guidance that helps students interpret previous experiences.

This reflection process is expected to trigger the emergence of real action personally. The following are the results of student reflection and action reviewed from the student reflection journal:

✓ Reflection and action 1

Students can understand the given opportunity material. He actively participates in group discussions. Initially, he found it difficult to understand the material, but he tried to ask and tried to work using the given formula. On the other hand, he was happy because he could help his friends who had difficulty understanding the material. From this experience, the intention arose to continue to try to be humble by not being ashamed to ask questions in order to understand the material well.

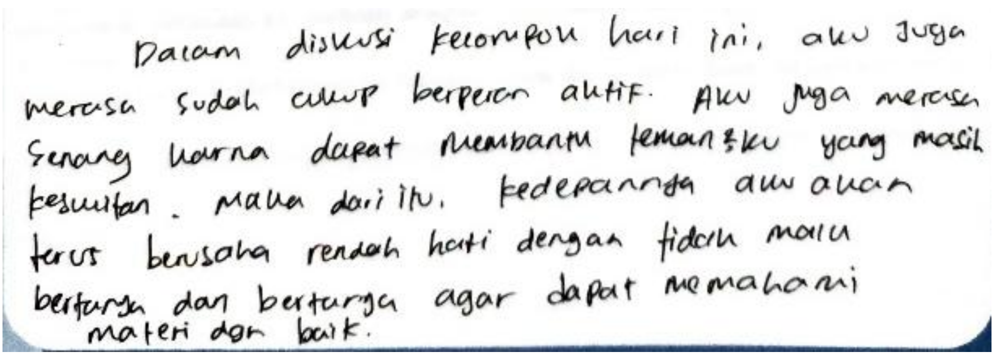
Dalam diskusi kelompok hari ini, aku juga
merasa sudah cukup berperan aktif. Aku juga merasa
senang karna dapat membantu teman2ku yang masih
kesulitan. Maka dari itu, kedepannya aku akan
terus berusaha rendah hati dengan tidak malu
bertanya dan bertanya agar dapat memahami
materi dgn baik.

Fig 3. S1 Reflection Result

✓ Reflection and action 2

The student stated that he knew that everything that happened had a chance. The student did not yet fully understand the material on chances, especially those related to permutations. The student also admitted that he did not yet understand the basic concept of chances. However, after conducting a group discussion, he began to understand the material. From this experience, an intention arose to pay attention to learning and ask questions if they had difficulties.

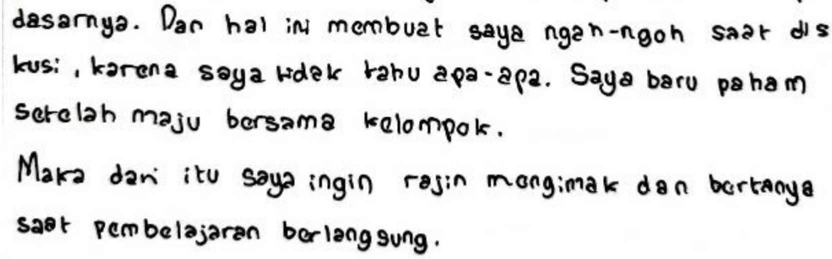
Kesulitan yang sama alami: yaitu belum paham konsep
dasarnya. Dan hal ini membuat saya ngah-ngoh saat dis-
kusi, karena saya tidak tahu apa-apa. Saya baru paham
setelah maju bersama kelompok.
Maka dari itu saya ingin rajin menyimak dan bertanya
saat pembelajaran berlangsung.

Fig 4. S2 Reflection Result

✓ Reflection and action 3

Students can understand the material of probability. Students feel happy, nervous, and excited when doing the simulation of guessing numbers. When doing group discussions, they had difficulty solving the questions given. However, after discussing, he and his group were able to solve them. This student was seen making sure his friends understood the material given. From his experience, he did not want to gamble but rather focused on building learning motivation by challenging himself and his friends regarding the results of the test scores.

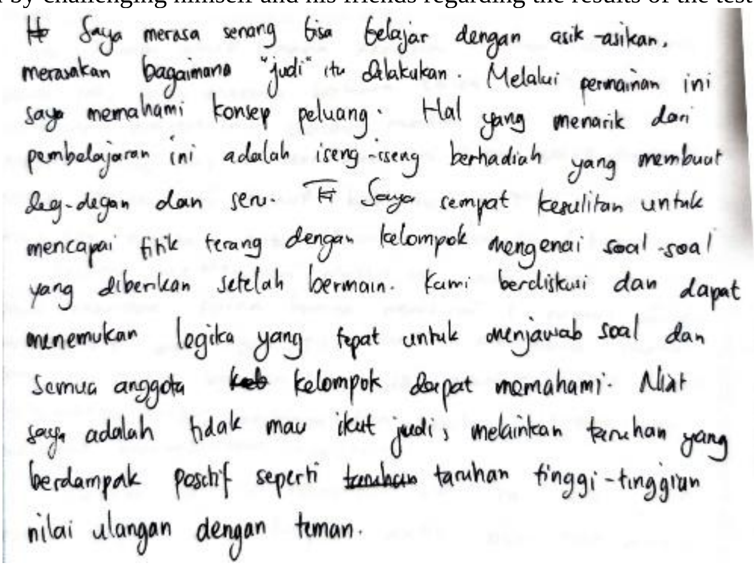
~~Ho~~ Saya merasa senang bisa belajar dengan asik-asikan, merasakan bagaimana "judi" itu dilakukan. Melalui permainan ini saya memahami konsep peluang. Hal yang menarik dari pembelajaran ini adalah iseng-iseng berhadiah yang membuat deg-degan dan seru. ~~K~~ Saya sempat kesulitan untuk mencapai titik terang dengan kelompok mengenai soal-soal yang diberikan setelah bermain. Kami berdiskusi dan dapat menemukan logika yang tepat untuk menjawab soal dan semua anggota ~~keb~~ kelompok dapat memahami. Niat saya adalah tidak mau ikut judi, melainkan taruhan yang berdampak positif seperti ~~taruhan~~ taruhan tinggi-tinggian nilai ulangan dengan teman.

Fig 5. S3 Reflection Result

✓ Reflection and action 4

Students said that gambling leaves a sense of dependence and irregular attachment. From this experience, the intention that emerged was not to gamble and to be aware of attachment.

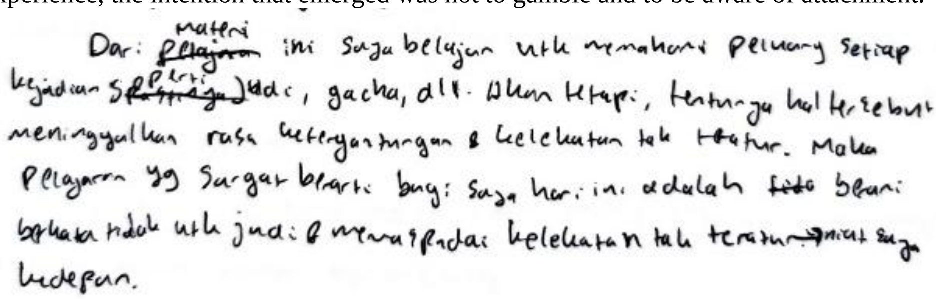
Dari ~~pelajaran~~ materi ini saya belajar utk memahami peluang setiap kejadian ~~seperti~~ seperti judi, gacha, dll. Akan tetapi, tentunya hal tersebut meninggalkan rasa ketergantungan & kelekatan tak teratur. Maka pelajaran yg sangat berarti bagi saya hari ini adalah ~~tido~~ berani berkata tidak utk judi & menyadari kelekatan tak teratur → niat saya kedepan.

Fig 6. S3 Reflection Result

✓ Reflection and action 5

From previous experience, students have the intention to understand the material by asking for help from other friends or by studying with friends.

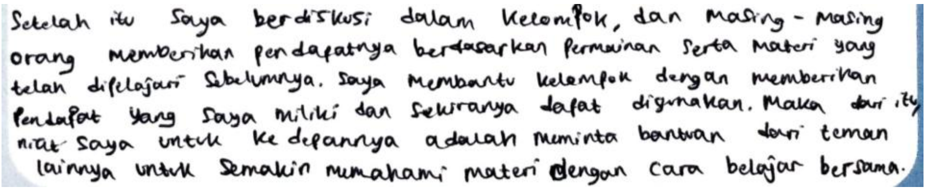
Setelah itu Saya berdiskusi dalam Kelompok, dan masing - masing
orang memberikan pendapatnya berdasarkan Permainan Serta Materi yang
telah dipelajari Sebelumnya. Saya Membantu kelompok dengan memberikan
Pendapat Yang Saya miliki dan Sekiranya dapat digunakan. Maka dari itu,
niat Saya untuk Ke depannya adalah meminta bantuan dari teman
lainnya untuk Semakin memahami materi dengan cara belajar bersama.

Fig 7. S5 Reflection Result

Based on the results of the reflection, there has been no concrete action taken for others. If reviewed again, the purpose of PPR learning should lead to real action taken for others.

3.1.4. Evaluation

After the learning process is carried out, teachers and students carry out an evaluation process. The evaluation carried out will review the achievement of 4C. This 4C assessment is carried out by looking at the results of group discussions in LKPD and observations of the learning process in the classroom.

In the competence aspect, the assessment is carried out on the results of problem solving in LKPD where students are asked to determine the chances of each session in the number guessing game. In general, students have shown mastery of the material in groups. However, in the results of personal assessments, personal assistance is still needed for several students to help them understand the concept of chances.

In the aspect of conscience, the assessment is carried out by observing the dynamics of the learning process. The aspects observed are how students are responsible for their tasks in groups and honest attitudes in following the dynamics of learning, especially in guessing number games. In general, it is quite good, especially in the aspect of honesty. In the aspect of responsibility, there is still a need for guidance for several children to be able to play an active role in the dynamics of group discussions.

In the compassion aspect, the assessment is carried out by observing the dynamics of the discussion process in the group. The aspects observed are respecting and caring for others. In general, students have been able to show an attitude of respecting and caring for others, as seen from how they respect friends who are giving opinions, teachers who are explaining, and are sensitive to helping friends who have difficulty in learning.

In the aspect of commitment, the assessment is carried out by observing the dynamics of the learning process. The aspects observed are how students are consistent in helping friends who are having difficulties and ensuring that group members can follow the dynamics of learning well. In general, in this aspect, more intensive assistance is still needed, especially in bringing up consistency in helping friends who are having difficulties.

In addition to conducting evaluations in the 4C aspects, evaluations were also conducted on the progress of the learning process. The teacher-reviewed the results of observations of the implementation of learning to find out the processes that had been running and the processes that needed to be improved in the future learning process. In general, the learning process had gone according to plan. However, the notes given by the observer were regarding time management.

The learning process will be better if the time given to do the experience can be more flexible so that the process of understanding the concept and developing the character of the students is more optimal. Based on the results of the reflection, it can be seen that there are still students who do not understand the material. If there is more free time, the teacher can provide more optimal guidance.

In addition to time management, it is necessary to re-emphasize the experience that arouses sensitivity to others or in this context is being sensitive to the needs of his friends. It can be seen from the results of the reflection that one tends to still focus on oneself. This process will be even better if it can achieve the spirit of "people with and for others"

3.2. PPR Spirit that Accompanies the Learning Process

3.2.1. The Spirit of Cura Personalis

Cura personalis is attention to the individual. In the context of learning, it is attention to students. Cura personalis carried out by a teacher is to pay attention to the difficulties experienced by students during learning. Teachers can help explain personally to students who are still having difficulty understanding the material.

Cura personalis is not only done between teachers and students but can also be done between students. As seen in Figure 8, it can be seen that there is a spirit of cura personalis between students. Students have the sensitivity to help friends who have difficulty understanding the material on opportunities. In this process, of course, it will require sensitivity in seeing the needs of their friends which raises attention to the individual.

Fig 8. Cura Personalis between students

3.2.2. Spirit of discernment

Discernment is decision making. In decision-making, it is certainly expected to involve the heart and mind to look back at the problems faced. Students are expected to be able to make decisions in their lives maturely and wisely, not just following the flow but having strong and good life principles.

In the context of this learning, students are faced with the problem of online gambling. Based on the results of discussions and reflections in class, it can be seen that students have decided not to play online gambling. This decision-making is certainly not spontaneous. Students collect information about online gambling, the impacts felt after online gambling, determine the factors that cause dependence, ensure the chances of winning, and finally make the decision to stay away from online gambling.

4 CONCLUSION

In this study, the dynamics in the Ignatian Pedagogical Paradigm have been implemented well. In terms of context, experience, reflection, action and evaluation have been implemented as planned. However, for the development of future learning, better time management and stimulus development can be carried out. Teachers can provide more free time for students to experience so that there is a process of understanding concepts and developing student character that is more optimal.

In addition, the selected stimulus should be able to emphasize more attention to others so that students do not only focus on themselves. This study found that students' attention is still centered on themselves. In the context of the Ignatian Pedagogical Paradigm, students need to be helped to become men for and with others.

In the dynamics of learning, the spirit of the Ignatian Pedagogical Paradigm that emerges is cura personalis and discernment. The spirit of cura personalis does not only occur between teachers and students but also appears in the dynamics of discussions between students. In groups, there is a process of helping each other with friends who have difficulty working on questions.

Discernment attitude is seen in learning when students respond to the rampant online gambling. After studying the material of opportunities and their implementation, they have the intention not to fall into online gambling.

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The Relationship Between Mathematical Thinking and Resilience in Number Sequence Lesson Through Ethnomathematics Among Pre-service Primary School Teachers

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Abstract. This research investigates the relationship between mathematical thinking and resilience in 31 pre-service primary school teachers, explicitly focusing on a number sequence lesson using ethnomathematics. Mathematical thinking, crucial for effective teaching, is a cognitive process that depends on overcoming challenges and is closely connected to resilience. By incorporating ethnomathematics, which integrates mathematics into cultural practices and real-world situations, the study examines how this approach supports the development of mathematical thinking and resilience. Participants were evaluated on their resilience and mathematical thinking during lessons. The findings revealed that the relationship between mathematical thinking assessment (MTA) and resilience scale of mathematics (RSM) using the Pearson Correlation test is $r_{MTA*RSM} = 0.116 > 0.05$. These results show no significant relationship between resilience and mathematical thinking. These results highlight the importance of including culturally relevant pedagogies like ethnomathematics in teacher education programs to better equip future educators for diverse classroom settings.

1 Introduction

Mathematical thinking is essential to successful teaching and learning in primary education [1]. Thus, it is also a crucial competency for pre-service primary school teachers (PPST). This construct is beyond mere comprehension of concepts, involving exploration, questioning, visualization, and problem-solving in diverse contexts [2]. These components are essential for future educators, as they need to understand mathematical concepts and facilitate students learning challenges [3], [4], [5], [6]. Stacey, Burton, and Mason [7] describe mathematical thinking as specializing, generalizing, conjecturing, justifying, and convincing skills. Specializing is a simple technique that everyone can use when they cannot proceed with a question. Generalizing moves from a few instances to making guesses about

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a broad class of cases. Conjecturing arises automatically by carrying out the familiar processes of specializing and generalizing. Justifying involves providing logical arguments or evidence to support a conjecture or solution. Convincing involves communicating reasoning to persuade others of the validity of a solution or argument.

Number sequence is a mathematical topic promoting these mathematical thinking skills. Research by Mor et al. [8] found that designing activities and tools that allow students to construct and share models of number sequences promotes mathematical thinking. That is also confirmed by Pasnak [9] because understanding number sequences leads to understanding mathematics concepts. In particular, pre-service primary school teachers should thoroughly understand number sequences because these fundamental concepts underpin essential arithmetic skills, pattern identification, and cognitive development [10],[11]. Number sequences can assist pre-service elementary teachers in linking additive and multiplicative structures in number sets [12], developing logical thinking, and understanding the fundamentals of mathematics [13].

However, it is essential to acknowledge that the competencies of PPST students differ from those of prospective mathematics educators. PPST has a background characterized by insufficient engagement with mathematics. So, resilience is essential for comprehending number sequence concepts. Mathematical resilience is the ability to overcome challenges and thrive in mathematics education, which is gaining recognition as a crucial factor for success. Resilience is significantly related to academic performance in mathematics [14]. Some research also explained that resilience is crucial for success in mathematics education [15]. Resilience is closely connected with cultural identity and how culture influences mathematical thinking, abilities, and learning trajectories. Then Owen [16] continues exploring how the cultural setting of mathematics enhances a teacher's identity as a mathematically thinking teacher. He concludes that an activity that links culture and school mathematics plays a role in building values and identities.

Numerous examples exist of real-life applications that integrate with the topic of number sequences. Cultural context is one of the relevant topics that resonates with students. Additionally, Yogyakarta, Indonesia, is characterized by rich traditional cultures. This context can be a starting point for learning mathematics, particularly in number sequences. Pangestuti and Prahmana [17] assert that the intersection of culture and mathematics represents a starting point of mathematics learning. In other words, ethnomathematics provides a cultural perspective for analyzing mathematical cognition by relating school mathematics to students' cultural and daily experiences, promoting their learning and resilience [18]. Ethnomathematics focuses on the contextual and cultural significance of mathematical topics, potentially improving comprehension and involvement. Integrating an ethnomathematics approach into number sequence courses can benefit primary school teachers in the early phases of their professional growth. This technique promotes both advanced mathematical thinking and the ability to solve pedagogical problems. In this study, we choose "sedekah bumi" (charity to the earth) traditional ceremony as a cultural context for several reasons: 1) this traditional ceremony is usually performed in Yogyakarta, Indonesia; 2) Students from outside the region are invited to learn about the culture in the area where they study; 3) the concept of number patterns that emerge in traditional ceremonies.

The interconnection of mathematical thinking, resilience, and cultural integration emphasizes a wide area for exploration within pre-service primary school teachers. Prior studies have examined the impact of culture on cognition and teacher self-perception [19], [16]; however, further emphasis should be placed on resilience within a context to enhance mathematical thinking in classes centred on number sequences. This study seeks to investigate two research questions: (1) is there a relationship between mathematical thinking and resilience of pre-service primary school teachers when engaged in a number sequence lesson with cultural integration? (2) how do culturally integrated number sequence lessons support pre-service primary school teachers' mathematical thinking skills and resilience? This research aims to provide literature regarding the importance of integrating cultural context into mathematics education and its role in fostering resilience and mathematical thinking skills in future educators.

2 Research Methods

This study employed a mixed-methods approach, integrating quantitative and qualitative methods. While quantitative methods investigate the relationship between mathematical thinking and resilience within an ethnomathematics framework, the qualitative method investigates how culturally integrated number sequence lessons support mathematical thinking skills and resilience. The study involved 31 first-year pre-service primary school teachers in a private university in Yogyakarta, Indonesia. Participants were selected based on their enrolment in a required mathematics education course, including number sequence lessons. Informed consent was obtained from all participants, and the university granted ethical approval for this study. It was conducted in one meeting in 90 minutes in the middle of the semester. The participants were given a number sequence lesson through ethnomathematics. Then, they worked on mathematical thinking tests, resilience questionnaires, and open-form reflection.

We use a mathematical thinking assessment (MTA) instrument adapted from Mason and Stacey [7]. The assessment included tasks that required specializing, generalizing, conjecturing, justifying, and convincing skills. It was conducted at the lesson's end to measure students' mathematical thinking. The second instrument is the resilience scale for mathematics (RSM). The RSM was adapted from the Adversity Quotient (AQ) by Stoltz [20] and was designed to evaluate the participants' resilience in the context of mathematics learning. Venkatesh [21] also states that AQ is a scientifically grounded tool that can measure and strengthen human resilience. We adapted the questions to consist of 4 items as follows: (1) Realizing math is not as difficult as imagined, (2) Feeling challenged by math subjects, (3) Experiencing anxiety when receiving math assignments, (4) Preferring to copy friend's work on math assignments. Based on validation test data, the sig value (2-tailed) for Q1: $0.008 < 0.05$; sig value (2-tailed) for Q2: $0.006 < 0.05$; sig value (2-tailed) for Q3: $0.00 < 0.05$; and sig value (2-tailed) for Q4: $0.00 < 0.05$. Based on these data, the four aspects of RSM are valid. Instrument reliability testing was carried out using Cronbach's Alpha test. The obtained coefficient was 0.417. The r table value for alpha is 0.05, and n is 31, resulting in an r table of 0.355. The RSM instrument is reliable because Cronbach's Alpha value is $0.417 > 0.355$. The last instrument is an open questionnaire conducted to gain deeper insights into how culturally integrated number sequence lessons support pre-service primary school teachers' mathematical thinking skills and resilience. The method of data collection using a questionnaire was chosen because data on mathematical thinking and the resilience scale of mathematics for all research subjects will be obtained. Data collection was supplemented with reflection from students to obtain more in-depth data.

The MTA and RSM scores were analyzed using correlation analysis to explore the relationship between mathematical thinking and resilience. The study aims to determine the relationship between the MTA and RSM variables. The correlation test is used because the study wants to determine to what extent changes in the RSM variable affect the MTA variable. Correlation analysis can also identify whether there is a positive, negative, or no relationship. Before conducting a correlation test, a prerequisite test is conducted for the variables to be tested. The prerequisite tests conducted are 1) normality test, 2) homogeneity test, and 3) linearity test. If the prerequisite test is met, the next correlation test is carried out using the Pearson Product Moment method. Meanwhile, if the prerequisite test is not met, the hypothesis test will be carried out using Spearman Rank Correlation. The open questionnaires were analyzed using a case study to examine the influence of cultural context on mathematical thinking and resilience among pre-service primary school teachers.

3 Result and Discussion

3.1 Result

Before we analyzed the correlation between mathematical thinking and resilience, we conducted normality and homogeneity tests. To find out the normality of the data, this research used SPSS 29 for the normality test. All research variables have a normal distribution using Liliefor significance correction. Based on validation test data, the sig value (2-tailed) for Q1: 0.008 < 0.05; sig value (2-tailed) for Q2: 0.006 < 0.05; sig value (2-tailed) for Q3: 0.00 < 0.05; and sig value (2-tailed) for Q4: 0.00 < 0.05. Based on these data, the four aspects of resilience are valid**.** The homogeneity test results using the Levene test show that Sig. on the RSM variables based on mean: 0.802 > 0.05 and Sig. based on trimmed mean: 0.785 > 0.05. Thus, it can be concluded that the RSM variables have the same variance or are homogeneous. The following prerequisite test was conducted: the linearity test. This research used SPSS 29 for the linearity test. The results of the linearity test between the MTA and RSM variables in the deviation from the linearity section obtained Sig.: 0.703 > 0.05. Based on these results using the ANOVA test, it can be concluded that there is a linear relationship between the MTA variable and the RSM variable. After all prerequisite tests, the t-relationship test was carried out with the following results: The correlation test was carried out on the MTA and RSM variables.

Hypothesis:
Ho: There is no relationship between the MTA variable and RSM ($r_{MTA*RSM} = 0$)
Ha: There is a relationship between the MTA variable and RSM ($r_{MTA*RSM} \neq 0$)

Based on the Pearson correlation test, Sig. (2-tailed): 0.116> 0.05. These results show no significant relationship between the MTA and RSM variables. The correlation coefficient results of 0.228 strengthen these results. Based on Table 1, the interpretation of the correlation coefficient means that the MTA and RSM variables have a weak correlation.

Table 1. Correlation Test

		MTA	RSM
MTA	Pearson Correlation	1	0.288
	Sig. (2-tailed)		0.116
	N	31	31
RSM	Pearson Correlation	0.288	1
	Sig. (2-tailed)	0.116	
	N	31	31

The qualitative data highlighted that participants felt more confident and enjoyed the lesson due to the culturally grounded approach to the lessons. This increased resilience was particularly noted in their willingness to tackle complex problems and reduce reliance on copying others' work. Key themes of thematic analysis in the open questionnaire included "cultural connection enhancing engagement (for example, as stated by S16, “learning is more interesting than before because it uses coloured stickers in culture context, which makes it more engaging") "reduction in math anxiety" (for example, as stated by S16 “I can learn mathematics easily without anxiety and it turns out mathematics is not always difficult”), and "increased confidence in problem-solving (for example, as stated by S12, “While working on ethnomathematics questions, I realized that cultural motifs contain mathematical elements, so I felt confident to do that”). These themes reinforce the finding that ethnomathematics significantly boosted resilience, even though it did not significantly impact mathematical thinking.

3.2 Discussion

This study examines the relationship between mathematical thinking and resilience in the context of an ethnomathematics teaching approach. The results show no significant correlation between better mathematical thinking abilities and the resilience of pre-service primary school teachers. The findings indicate that ethnomathematics influences resilience, offering significant insights for mathematics teaching. However, there is no correlation between cognition and resilience; this lack of relationship suggests that these two variables grow independently. Utilizing culturally aligned teaching approaches yields results comparable to Faradilah's research [22], indicating that mathematical resilience does not influence critical thinking. The slight rise in documented cognitive activity indicates that, while ethnomathematics may improve engagement and relevance, additional work is required to foster sophisticated mathematical thinking within the limited study period. Carrillo et al. [23] assert that high-achieving students enhance their cognitive abilities and resilience, suggesting a correlation between these two constructs. Improving thinking requires thinking logically, solving problems, and achieving significant progress; it may require much effort and various approaches. This is especially true when considering the important factors of the cultural environment. A substantial improvement in resilience highlights ethnomathematics' advantages in making mathematics more accessible and less intimidating for students. When participants approach mathematics from a cultural context, they express greater confidence and reduced anxiety. Qualitative data shows that cultural context can encourage attitudes toward mathematics for sustained engagement. As stated by Iqbal [24], ethnomathematics can promote a mathematical perspective for sustained engagement and achievement. The participants demonstrate resilience because of the connection between mathematical concepts and their cultural backgrounds. This result influenced learning to be more pertinent and less abstract. It is aligned with research highlighting the significance of responsive teaching in building student motivation and engagement [18]. Ethnomathematics situates mathematical problems into contexts that enhance student engagement and motivate them to confront challenges.

The results show that although ethnomathematics can contribute to increased resilience, more efforts may be needed to enhance mathematical thinking. For example, improving problem-solving skills and logical thinking within an ethnomathematical culture may be necessary. Additionally, research can investigate how various combinations of materials can enhance mathematical thinking and resilience. The research results show that culturally relevant mathematics education methods are crucial for fostering student resilience.

4 Conclusion

This study investigated the relationship between mathematical thinking and resilience in pre-service primary school teachers using the lens of ethnomathematics-based teaching. The findings indicate that there is no relationship between mathematical thinking and resilience among pre-service primary school teachers. It means that while ethnomathematics promotes resilience, it does not result in an improvement in mathematical thinking. These findings showed that improving resilience through culturally relevant training does not always result in enhanced mathematical thinking. As a result, educators should consider mixing ethnomathematics with other instructional strategies that focus more directly on the development of mathematical thinking and problem-solving abilities.

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Implementation of Mathematics Teaching Module Based on Reflective Pedagogy Paradigm (RPP) at Taruna Nusantara High School

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Abstract. This study aims to explore the application of teaching modules based on the Reflective Pedagogy Paradigm (PPR) in learning mathematics, especially on the topic of geometric sequence at Taruna Nusantara High School. The focus of the research is to assess the academic and character aspects of students based on the 4C components (Competence, Conscience, Compassion and Commitment). This is a qualitative approach with Problem Based Learning (PBL) model. The research subjects are 33 X grade students who took part in learning geometric sequence through PPR teaching modules. Data were collected using cognitive diagnostic tests, observation of the learning process, 4C assessment, and reflections form. The results of the study showed that the application of PPR teaching modules not only contributed to the improvement of students' understanding on the topic of geometry sequence, but also effective in assessing students' character development in accordance with 4C. The average score for the competence aspect reached 78.74, while for the conscience, compassion and commitment aspects obtained an average score of 8.03, 8.25 and 8.16 respectively. This finding indicates that character education implemented at Taruna Nusantara High School can run well through the integration of PPR in the mathematics learning process.

1 Introduction

A quality education not only emphasizes the development of students' competencies, but must also pay attention to the development of their characters. In line with the vision of Taruna Nusantara High School which is to produce national leaders with quality and character, the education implemented in this institution should focus on two main aspects, namely student competence and character. To achieve these goals, a learning approach that integrates character education in the process is required. Currently at Taruna Nusantara High School, character education is implemented through special exercises such as marching, leadership lessons, state defense, scheduled projects such as: General Sudirman's footsteps, Environmental Care Community Exercises, and others. While in learning character education is still not implemented and measured clearly. Taruna Nusantara High School teachers currently face challenges in conducting student character assessments, especially in the 3C aspects (Conscience, Compassion, and Commitment), because so far character assessments

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are often only based on guidelines provided by the government. The guidelines do not contain clear details of what should be achieved and how the assessment rubric.

Learning that integrates character education has the potential to maximize the development of all dimensions of students, including cognitive, social-emotional, physical, creativity, and spiritual domains [1]. The Merdeka Curriculum currently implemented by schools also emphasizes the development of a holistic education through the Pancasila student profile. The Pancasila student profile has the aim of shaping student character in accordance with the values in Pancasila such as social, spiritual, and moral values. This is in line with the thinking in the Reflection Pedagogy Paradigm (RPP), which states that to form learners into fully developed individuals, it is necessary to pay attention to cognitive, psychological, physical, emotional, affective, moral, social, and spiritual aspects. [2]. Character education is very important for improving the quality of human resources (HR) in Indonesia. Indonesia really needs quality human resources so that the implementation of development programs runs well and the country can progress [3], [4], [5], [6]. As a form of follow-up to the importance of character education and school needs, teachers can create a teaching module that can facilitate the development of competence and character development of students and contain rubrics that can assess 4C (Competence, Conscience, Compassion, and Commitment), namely RPP-style modules [7], [8], [9], [10]. The teaching module can integrate various learning models and also aspects of the Reflection Pedagogy Paradigm such as the spirit that accompanies PPR, PPR Principles, PPR-style assessment, and the PPR cycle.

In this study, the term “Reflective Pedagogy Paradigm (PPR)” has the same meaning with “Ignatian Pedagogy Paradigm (IPP).” The term “Reflective Pedagogy Paradigm (PPR)” is considered more inclusive. Although the concepts of “Ignatian Pedagogy Paradigm” already inclusive and can be used in general education, but the term “Ignatian” is not well known for many people. Since the heart of IPP is reflection, then the word “reflective” is used.

The RPP cycle according to Suparno includes context, experience, reflection, action, and evaluation [2]. Context is related to all factors that support or hinder the learning process such as students, environment and school. Experience can be defined as feeling or appreciating something deeply. Reflection is a process carried out by learners so as to make the learning experience their own (appropriation), obtaining meaning and meaning from the learning experience for themselves and others. Action is an activity that contains attitudes, priorities, commitments, habits, values, ideals, internal growth of people so that they act for others. The process of discovering a context is commonly known as context mining. The term 'action' refers to internal human growth based on experience that has also been reflected as its external manifestation. Action is both related to oneself and related to what one will do outside oneself. Evaluation as a cycle in RPP is to form a human being with a whole personality, intellectually competent, willing to always develop, religious, and full of love and determination to do justice in sincere service to fellow human beings. Meanwhile, evaluation as an assessment can be carried out after providing experience to students by conducting general assessments. The following is an overview of the RPP cycle.

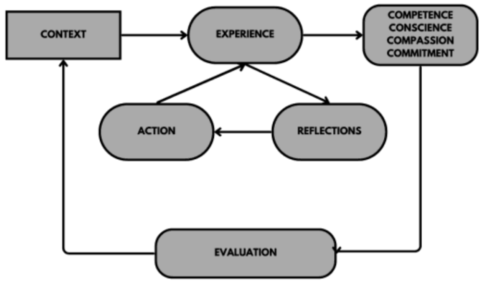


Fig 1. RPP cycle

There are several researchers who have implemented PPR teaching modules such as those conducted by Pradita, namely the implementation of PPR with the topic of pollution at SMA Seminari Mertoyudan [11], then research conducted by Nanga on how the implementation of PPR in trigonometry classes to review the 3C aspects [12], Hartana's research on the application of PPR strategies to improve learning outcomes and motivation to learn science in elementary school students. [13], Melisa's research on the implementation of PPR to develop number theory learning outcomes [14], Pratini's research on the implementation of the reflective pedagogy paradigm in mathematics learning to improve students' competence, conscience, and compassion [15], and Kristanto's research on the implementation of PPR in computer-based algebra and trigonometry courses to improve collaboration and communication between students [16]. The difference between this research and previous research is that the subjects chosen are students who attend semi-military schools that have different character education from schools in general. In addition, the selected learning is geometric sequences.

Based on this background, this study aims to determine the implementation of PPR-style teaching modules in learning mathematics geometry sequence material to help assess student character education at Taruna Nusantara High School.

2 Methods

This is a qualitative study involving 33 grade X students at Taruna Nusantara High School. In this study, mathematics learning will be implemented focusing on the topic of geometric sequence using the Problem Based Learning (PBL) model that integrates the reflective pedagogy paradigm. This study aims to present a description and evaluation of the implementation of learning using the PBL model based on the reflective pedagogy paradigm. The data obtained in this study include cognitive diagnostics, observation of the learning process, learning outcomes, and reflections from students. The following is a cognitive diagnostic test instrument used in this study.

Table 1. Cognitive diagnostic test instrument

No	Question
1	Given two consecutive numbers in a sequence, how do you determine the ratio between them?
2	Given a sequence of numbers 2, 4, 8, 16, ... can students determine the next number? Explain how to find it.
3	Do you understand how to determine the nth term in a geometric sequence?
4	Can students clearly distinguish between geometric and arithmetic lines?

5	If you know that a geometric sequence has a first term of 3 and a ratio of 2, can you determine the 4th term of the sequence?

The following is the 3C (Conscience, Compassion, Commitment) assessment instrument in this study.

Table 2. 3C assessment instrument

Assessment	Aspec
Conscience	Integrity
	Responsibility
	Honesty
	Discipline
Compassion	Leadership
	Cooperation
	Caring
	Patience
Commitment	Critical Thinking
	Creative
	Mutual Cooperation

The reflective questions used in this study integrate the 4C components presented in the following table

Table 3. Reflection Guidelines

4C Component	Questions
Competence	1. What are the main concepts you learned about geometric sequences today? What are the main differences you found between geometric sequences and arithmetic sequences? Give an example 2. What challenges or difficulties did you face when learning and solving geometric line problems? How did you overcome them?
Conscience	3. Do you feel confident that you can calculate the nth term or the sum of the first terms of a geometric sequence? Explain the steps you took to solve the problem.
Compassion	4. How do you help a friend in your group who has difficulty understanding the concept of geometric sequences?
Commitment	5. What have you learned about yourself while studying geometry lines? Is there any attitude or way of learning that you want to improve in the future?

3 Results and Descriptions

Learning Implementation Description

In this study, the classroom learning was conducted for 90 minutes or the equivalent of two lesson hours on the topic of geometric sequence. In this meeting, students will learn how to determine the ratio and the n^{th} term in a geometric sequence. The method used is Problem-Based Learning (PBL) which emphasizes a constructivistic approach. Through this approach, learners will be guided to build their knowledge independently, especially regarding the concept of geometric sequence s. The learning objectives are formulated based on the four dimensions of 4C as follows.

- **Competence**: Learners are expected to be able to solve various problems related to geometric sequences, including the ability to determine the n^{th} term in a geometric sequence appropriately.
- **Conscience**: Learners are expected to understand the importance of the concept of geometric sequence in everyday life. In addition, they are expected to show integrity in completing tasks, a sense of responsibility in the learning process, honesty in problem solving, and discipline in following each stage of the work.
- **Compassion**: Learners are expected to show leadership in the group, cooperate effectively in solving problems related to geometric sequences, and have concern for the opinions and difficulties experienced by their peers. They should also be patient in dealing with differences in understanding and be ready to help group members who need support.
- **Commitment**: Learners are expected to show commitment to complete the project given in PBL learning by thinking critically, creatively, and working together in mutual cooperation in finding solutions to problems related to geometry sequences.

3.1 Context

Learning begins by exploring the students' context. Context mining is important to find out how the condition of students before learning. The process of extracting this context is done by using a cognitive diagnostic test that contains questions that can explore the basic understanding of students related to geometric sequence material. The results of this cognitive diagnostic test will provide an overview of how the level of understanding of students before proceeding to the next stage of learning. The following graph shows the results of the cognitive diagnostic test conducted.

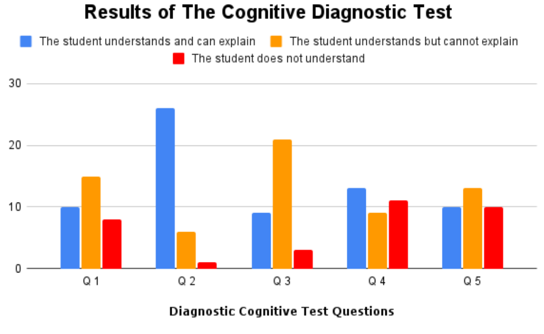


Fig 2. Results of the cognitive diagnostic test

The Figure 2 shows different results for each question asked. In questions number 1 and number 3, it can be seen that the number of students who understand the material but have difficulty explaining is less than the number of students who claim to understand but cannot explain the concept. Conversely, in question number 2, the number of students who not only understood but could also explain was more than those who only understood but could not express their understanding. In question number 4, the number of students who did not understand was slightly more than the students who understood and could explain, but the number was less than the students who understood but could not explain clearly. Whereas in question 5, the number of students who did not understand was the same as the number of students who understood and could explain, although the number of students who understood but could not explain was slightly higher.

3.2 Experience

In the first stage of Problem Based Learning (PBL), the teacher focuses students' attention on the issue that needs to be solved. The teacher asks questions related to the geometric sequence such as the division of bacteria or the decay of radioactive substances every once in a while. The PPR spirit of a positive world and finding God in everything is seen when the teacher explains that both phenomena occur by God's will. Next, the teacher organizes students to learn. Students are divided into small groups of two, and each group is given a Learner Worksheet (LKPD) to help them write the solution to the problem given. During the group discussion process, the teacher's task is to assist students in the learning process and also guide students who have difficulty in finding solutions to the problems given. The spirit of RPP which is Cura Personalist can be seen in the teacher accompanying and guiding students during the discussion.

Group discussion activities in this lesson are designed to provide direct experience to students in understanding the concept of geometric sequences. Through the activity of working on LKPD in groups, students not only gain a deep understanding of the concept, but also engage in an active and fun learning experience. This process encourages students to construct their own knowledge, thus strengthening the understanding of geometric sequences in a more applicable and interesting way. During the student discussion process there are several PPR spirits that can be seen, namely the magical spirit and discernment. Magical spirit exists when students are serious in solving the problems given. While discernment is seen when students determine the solution strategy and determine the solution of the problem given.

Fig 3. Group Discussion

3.3 Reflection

In the early stages of learning, when exploring the context, students are asked to reflect on their understanding of geometric sequences that they have learned at the previous level of education, namely in Junior High School (SMP). Through group discussions, students are given the opportunity to reflect on how they solve the problems contained in the LKPD, by utilizing the available information and knowledge they have mastered. In addition, students can also evaluate the extent of their contribution to the success of the group in solving the problem.

At the end of the learning session, each student is given a sheet to write their self-reflection. This reflection includes a number of questions related to their experience while learning the geometry sequence material. These reflective questions also integrate the 4C components.

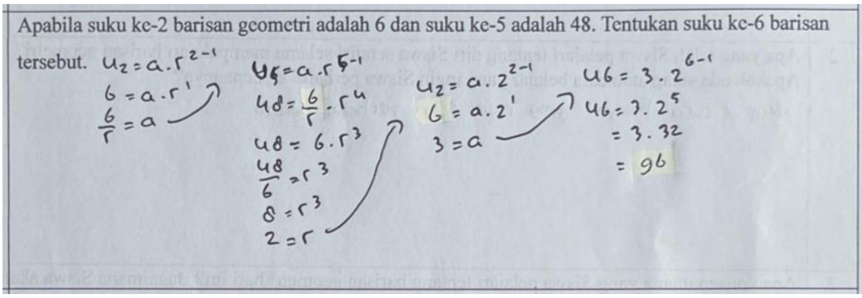
In the competence aspect, the results of students' reflections showed that they were able to understand the main concept of geometric sequences by comparing it with the concept of arithmetic sequences. In the conscience aspect, the reflection results show that students understand the material well, which reflects their integrity and responsibility in the learning process. In the compassion aspect, the reflection results show that students can understand the concept of geometric sequences and know the best way to explain the concept to their friends. This reflects a leadership attitude, where they strive to complete their own understanding, as well as a caring attitude, where they think of the best way to convey their knowledge to their friends. In the commitment aspect, the reflection results show that students have an awareness to improve their learning and practice, which reflects their determination to be lifelong learners.

3.4 Action

After the group discussion, the teacher selects a number of students to be representatives in presenting the results of the discussion and the solutions they find related to the problems faced. In the Problem Based Learning (PBL) framework, this activity is included in the development and presentation of work. Several group representatives then presented the results of their thinking in front of the class. Through this presentation, students not only explain the concepts they have learned, such as the concept of geometric sequences, but also train their ability to convey ideas in a clear and systematic way.

During the presentation process, students are faced with the challenge to not only understand the material well, but also to convey that knowledge in a way that their peers can understand. This requires them to communicate effectively, consider their audience, and adjust their delivery so that the message is truly understood. The ability to convey ideas clearly and persuasively is essential, both in academic contexts and in everyday life. These presentation skills also equip students with the ability to influence others, build confidence, and manage pressure that may arise when speaking in public. Thus, this activity not only develops students' communication skills, but also prepares them for professional challenges, such as project presentations, work meetings, or even leading a team in more complex situations.

After students have gained the knowledge and experience to work on problems related to geometric sequences, students are asked to use their knowledge to work on problems individually. In this situation, the spirit of PPR that emerges is repetition, the spirit of repetition arises when students repeat what they learned before to deepen and strengthen their skills in solving future problems.


Apabila suku ke-2 barisan geometri adalah 6 dan suku ke-5 adalah 48. Tentukan suku ke-6 barisan tersebut.

$u_2 = a.r^{2-1}$
$6 = a.r^1$
$\frac{6}{r} = a$

$u_5 = a.r^{5-1}$
$48 = \frac{6}{r}.r^4$
$48 = 6.r^3$
$\frac{48}{6} = r^3$
$8 = r^3$
$2 = r$

$u_2 = a.2^{2-1}$
$6 = a.2^1$
$3 = a$

$u_6 = 3.2^{6-1}$
$u_6 = 3.2^5$
$= 3.32$
$= 96$

Fig 4. Sample student answer

3.5 Evaluation

Assessment in this reflective pedagogy is based on four main aspects, namely the 4C: Competence, Conscience, Compassion and Commitment. Competence assessment is presented in the following figure.

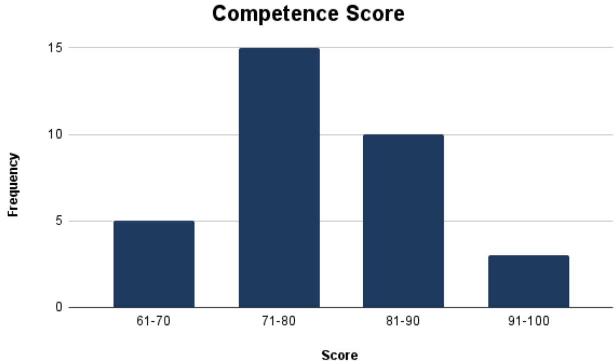


Fig 5. Competency assessment results

Figure 5 shows that the score for competence has an average of 78.74. The Minimum Completion Criteria (MCQ) is 70, so the number of students who passed is 84.85% and the number of students who did not pass is 15.15%. The assessment for the conscience aspect is presented in the following figure.

Fig 6. Conscience assessment results

Figure 6 shows that the score for the conscience aspect reached an average of 8.03. This shows that the students' character contained in the components of the conscience aspect is good. The assessment for the compassion aspect is presented in the following figure.

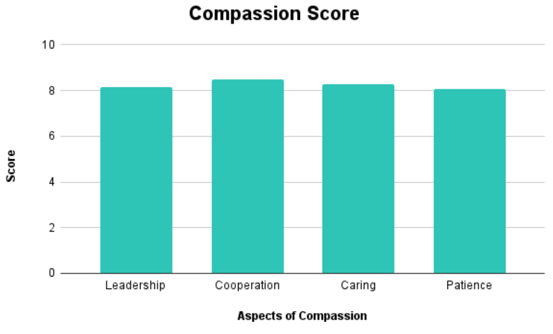


Fig 7. Compassion assessment results

Figure 7 shows that the score for the compassion aspect reached an average of 8.25. This shows that the student character contained in the components of the compassion aspect is good. The assessment for the commitment aspect is presented in the following figure.

Fig 8. Commitment assessment results

Figure 8 shows that the score for the commitment aspect reached an average of 8.16. This shows that the student character contained in the components of the commitment aspect is good. Based on the assessment results from the aspects of conscience, compassion and commitment all show good results. The assessment indicates that so far the character education in the semi-military style Taruna Nusantara High School can shape student character well. The character education has been well implemented and can be maintained.

4 Conclusion

Based on the results of the analysis and discussion, it can be concluded that the mathematics teaching module with the PPR approach is effectively used to assess the implementation of character education based on the 3Cs at Taruna Nusantara High School, in accordance with the stated research objectives. The data obtained showed that the average score for the competence aspect of students reached 78.74, the average score for the conscience aspect of students was 8.03, the average score for the compassion aspect of students was 8.25, and the average score for the commitment aspect of students was 8.16. The reflection activity carried out is an interesting experience for students because they rarely or never do it. Reflection makes students realize what they have gained, how they gained it and what to do in the future. This encourages students to be fully present for every activity they do. In addition, reflection also shows the affective side of students where students who already understand the material can develop a step-by-step plan that they will do to teach their friends who cannot yet.

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Ethnomatematics Study on Ikat Woven Fabric of The *Kwatek* Lamalera

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Abstrak. Ethnomathematics is a mathematical activity that grows and develops in a culture and customs in a particular region. The purpose of this research is to examine the deepest fundamental activities in the ikat woven fabric of the Kwatek Lamalera based on the fundamental activities proposed by Bishop, namely counting, locating, measuring, designing, playing, explaining. This research used a qualitative research method with an ethnographic approach and The data collection method used were observation and interviews with several weavers in Lamalera Village. The data of this research will be divided into four classes, namely: (1) history and development; (2) manufacturing process; (3) motif; and (4) philosophical meaning. The results showed that the activities of counting and explaining were found in all classes, while the activities of measuring and locating were found in the first, second, and fourth classes. In addition, playing activities were found in the second and fourth classes, and designing activities were found in the second classes.

1 Introduction

Indonesia is one of the countries located in the Southeast Asia region which is an archipelago because it has 17,504 large and small islands and has a diversity of ethnicities, races, religions, beliefs, and community ideas from an early age which is a form of culture that exists in each region (Kiswahni, 2022). One of the cultures found in Indonesia and is now a heritage developed to the current generation is in the form of traditional fabrics such as songket, batik and weaving.

One of the provinces that has woven fabrics with diverse motifs is the province of East Nusa Tenggara (NTT). Each tribe living in the province of NTT has its own cultural characteristics and this colors the shape of the traditional woven fabric motifs they make (Setiohardjo & Harjoko, 2014). The motifs of woven fabrics in NTT province are generally a legacy of the ancestors and are still preserved today. One of the woven fabrics originating from NTT is the Kwatek ikat woven fabric made by the community of Lamalera village, Wulandoni sub-district, Lembata Regency. The motifs in the woven fabric are ancestral heritage motifs that identify the village's distinctiveness as a “whale hunter village”.

One of the efforts to develop student creativity is to organize a learning process that integrates mathematics and culture so that the learning experienced by students becomes

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meaningful (Wulandari & Puspadewi, 2016). Ethnomathematics is one of the bridges that can connect the integration process between culture and mathematics (Ledi et al., 2020). (Hasanuddin, 2017) said that Ethnomathematics is the application of mathematics carried out by a cultural group, be it a cultural group that is customary or a professional group. Thus, any cultural activity related to mathematics can be considered as part of Ethnomathematics. The term Ethnomathematics was introduced by D'Ambrosio in 1985 who defined Ethnomathematics as mathematics practiced in a cultural group which includes indigenous peoples, working communities, children of certain age groups, to professionals (d'Ambrosio, 1985). Based on the opinions of several experts, it can be said that Ethnomathematics is a science that examines mathematical elements that exist in the culture of a particular community group.

Bishop (1988) stated that there were six fundamental mathematical activities which can help develop the potential of mathematics in human culture. The six fundamental activities were: (1) counting which developed from people's habit of counting their possessions and possessions so that from this initial goal it can help people to compare their belongings with other similar items; (2) locating which originally aimed to help people choose locations for various activities such as hunting and navigating. Nowadays, locating activities include the position of oneself and other objects in the spatial environment; (3) measuring involved sorting, comparing, and judging which all societies recognize the importance of these things; (4) designing which meant activities that involve the shape and design of an object which includes the process of making, applying patterns and art in making things; (5) playing which meant there were certain rules in an activity; and 6) explaining which meant it involved the ability to compose language, classify, explain stories and use reliable sources and of course these activities are universal in both cultural and social terms.

Ethnomathematics has become a research innovation that connects mathematics and culture, and many studies have been conducted in Indonesia that focus on one form of local culture, namely woven fabric. One of them was a study conducted by Mendoca et al. (2021) which examined the woven fabric of the Lamaksenulu village community in the Belu area, NTT province. From this research, it was found that there is a relationship between the motifs of the woven fabric of the Lamaksenulu village community and the concept of flat shapes, such as hexagons, rhombuses, dots, and lines. These motifs could be used as a context to teach the concept of flat shapes for junior high school students. In addition, Sutarto et al.(2021) also examined how to use the motifs in the woven fabric of the Sasak tribe of Central Lombok to teach geometry transformation. From the results of this study, it could be concluded that the wayang, subahnale, keker, four stars, and alang/lumbung motifs could be used to build the concepts of reflection and translation. From the research conducted by Auliani & Suripah (2024), it was found that in the songket Siak woven fabric motif there were fundamental mathematical activities found, namely: measuring, designing, and counting. In this study, researchers will try to answer the following problem formulations: (1) What is the history and philosophy of the ikat woven fabric of the Kwatek Lamalera and the fundamental mathematical activities in the history and philosophy of the the ikat woven fabric of the Kwatek Lamalera? (2) What is the process of making the ikat woven fabric of the Kwatek Lamalera and the fundamental mathematical activities in the process? (3) What are the characteristics and uniqueness of the Kwatek Lamalera ikat woven fabric motifs and the fundamental mathematical activities on these characteristics and uniqueness? and (4) What is the philosophical meaning of the Kwatek Lamalera ikat woven fabric motifs and the mathematical fundamental activities on these meanings?

2 Methodology

The type of research used is qualitative research with an ethnographic approach. Ethnography is a type of qualitative research derived from anthropological methodology that aims to explore society and culture through analyzing human aspects, interpersonal, social, and cultural relationships in their complexity (Wijaya, 2018). In this research, the sources that will be used are the weavers of ikat kwatek woven fabrics in Lamalera village. Because this research is qualitative research, the main instrument in this research is the researcher himself. To help researchers to obtain more complete data, the researchers made auxiliary instruments, namely interview sheets containing questions that would be asked to the sources, and also observation sheets used to record observations during the interview process. Validation of auxiliary instruments in this study was carried out with expert validation techniques. The data validation process in this study was carried out using source and technique triangulation because in this study, researchers will interview two resource persons who are ikat woven fabric weavers and in the data collection process, in addition to conducting interviews with the two weavers, researchers observed all activities carried out by the two weavers. The process of analyzing interview and observation data will be carried out through the stages of data analysis according to Miles and Huberman, namely reducing data, presenting data, and drawing conclusions (Fadli, 2021).

3 Results and Discussions

3.1 History and Development of *Kwatek* Lamalera Ikat Woven Fabric

In Lamalera village, many women start weaving as teenagers (14-20 years old) as a tradition passed down from their ancestors. This tradition aims to reduce the dependence of family income on the women's profession as a pnetang or fish seller as well as to help support the family economy. Over time, the weaving tradition has changed. Changes occur in the yarn and dyes used to make cloth. Nowadays, people use yarns made in factories to replace natural yarns from cotton produced by kapok trees, as well as synthetic dyes that are more practical to use compared to natural dyes. In addition to the basic materials for making fabrics that have changed, changes have also occurred in the looms used. Initially the looms used were planted or settled in one place, but for now the looms used are portable looms so that the placement is more flexible.

The making of the Lamalera kwatek ikat woven fabric was originally not intended to have an economic selling value but only to function as a body warmer. However, nowadays the community utilizes it as a source of livelihood. The woven fabric from Lamalera village consists of two types, namely nowing, which is a woven fabric specifically for men and kwatek, which is a woven fabric specifically for women. The kwatek woven fabric is then further divided into two types based on the part of the woven fabric that is put together into one whole woven fabric, namely Nai telo if the number of woven fabrics put together is three lirang, and Nai rua if the number of woven fabrics put together is two lirang.

Fig 1 *Nowing*

Fig 2 *Kwatek*

Fig 3 *Nai Telo* (Tiga Lirang)

Fig 4 *Nai Rua* (Dua Lirang)

Generally, the development of Lamalera's kwatek woven fabric has not undergone any changes in terms of motifs because the motifs depict the characteristics that characterize Lamalera village as a “whaling village” and this has not changed until now. The motifs in Lamalera's kwatek woven fabric are whale, stingray, and pledang motifs. In addition to these three motifs, there are also woven fabrics that have taru mata and klape motifs. However, these two motifs are rarely found today, because the process of making them is difficult and requires the use of natural materials.

From the exploration conducted by the researcher related to the history and development of Lamalera's kwatek ikat woven fabric, the researcher found four fundamental mathematical activities, namely counting, measuring, locating, and explaining. The following are the details of the research findings for each fundamental mathematical activity in the history and development of Lamalera's kwatek ikat woven fabric.

a. *Counting*
The activity of counting, involves the process of quantification that indicates the quantity of an object. Here are the research findings for the fundamental mathematical activity of counting:
1) Some weavers from Lamalera village started weaving as teenagers, between the ages of 14-20.
2) Ikat woven fabric from Lamalera village is divided into two types, namely nowing for men and kwatek for women.
3) This kwatek woven fabric is divided into two types based on the part of the woven fabric that is put together into one whole woven fabric, namely Nai telo if the number of woven fabrics put together is three lirang, and Nai rua if the number of woven fabrics put together is two lirang.

b. *Measuring*
The measuring activity involves measuring a certain size of an object in a certain unit of time. The research findings for the fundamental mathematical activity of measuring are as follows: Some weavers in Lamalera village started weaving as teenagers, between 14 and 20 years old.

c. *Locating*
Locating activity involves identifying the position of a particular object. Here are the research findings for the fundamental mathematical activity of locating:
1) Some weavers in Lamalera village started weaving as teenagers, between 14 and 20 years old.
2) Nowadays, weavers can easily obtain the desired color by simply buying yarn at the yarn shop in Lewoleba Town.

d. *Explaining*
The explaining activity involved explaining the development and types of woven fabrics from Lamalera Village. The following are the researcher's findings for the fundamental mathematical explaning activity:
1) Weaving is one of the traditions passed down by the ancestors of the Lamalera Village community to women in the village, with the aim that they are not dependent on the profession of fishmonger and can help increase their respective family incomes.
2) Changes in the weaving tradition from the past to the present can be seen in the use of basic materials. In the past, the ancestors used natural yarn obtained from cotton produced by the fruit of the kapok tree.

3) In addition to changes in basic materials, there are also changes in the looms used.
4) Nowadays, the work as a weaver has been considered as the main job for most weavers, because the income earned from weaving can better help the family economy.
5) The ikat woven fabric from Lamalera village is divided into two types, namely nowing and kwatek.
6) *Kwatek* is further divided into two types based on the number of lirangs joined together, namely Nai telo which consists of three lirangs joined together, and Nai rua which consists of two lirangs joined together.
7) The development of ikat woven fabrics in Lamalera Village can only be seen in changes in the use of yarn and the coloring process.
8) In terms of motifs, there is generally no change because the motifs in the ikat woven fabric of Lamalera Village from ancient times to the present are still the same and are characteristic of the village.
9) Currently Setiohardjo, woven fabrics of Lamalera Village with taru mata and klape motifs are increasingly rare, because the process of making them is difficult and they are only used for “balas belis” purposes.

The following summarizes the basic mathematical activities in the history and development of Lamalera's kwatek ikat weaving fabric:

Table 1 Fundamental Mathematical Activities in the History and Development of Kwatek Lamalera Ikat Woven Fabrics

Counting	Measuring	Locating	Explaining
Calculating the age at which weaving began, the type of woven fabric, and the number of lirang in the making of ikat woven fabrics in Lamalera Village.	Measuring the age range of weavers.	Identify the age range of weavers and the location of yarn sales.	Explain the changes in weaving traditions in Lamalera Village, including the purpose of cultural heritage, changes in basic materials and looms, and the development of woven fabrics and motifs.

3.2 The Making Process of Kwatek Lamalera Ikat Woven Fabric

The making of Lamalera kwatek ikat woven fabric requires special tools and materials. The tools used are as follows: tenane which is used for weaving, selaga which is used to design the pattern, vulo gulokajo which is used to hold the pattern, faniduang which is used to tidy up the pattern, huri which is used to tighten the thread, kdaje which is used to support the front thread, sligu which is used to support the weaver's body, paso which is used to soak the thread, kduke which is used to rotate the thread so that it does not tangle when woven, and mnue which is used to untangle the thread. Natural or artificial dyes and raffia are also used in the process of making motifs so that colors do not mix.

The process of making Kwatek Lamalera ikat woven fabric begins with untangling the threads on the selaga to form the motifs that have been designed, then woven into a complete cloth. The spacing between motifs has no fixed rules, but usually the longest spacing is 40-60 strands of yarn, while the shortest spacing is around 5-10 strands. Distinctive motifs such as whales, stingrays and pledangs are arranged sequentially with a certain spacing pattern until the fabric is composed, and at the end of the fabric a distance of 180 strands of yarn is added before reaching the final length.

The stages of making motifs begin with designing the design, tying certain parts of the outlined yarn using Vulo Gulokajo, then coloring it with natural or artificial dyes. After the basic coloring is complete and the yarn is dried in the sun, the ties in the motif section are opened to provide color according to the desired design. This process is continued until the Kwatek Lamalera ikat woven fabric is fully formed. Making Kwatek Lamalera ikat woven fabric requires 50-60 skeins of yarn for one whole cloth measuring 80 × 160 cm or 100 × 200 cm. The process usually takes about a week, while motif making takes 1-3 days, depending on the age, energy, and busyness of the weaver. Weavers can produce 3-4 pieces of kwatek per month if they only focus on kwatek, but if interspersed with other types of weavings such as nowing or shawl, production decreases to around two pieces. The fabric is sold to tourists and local people.

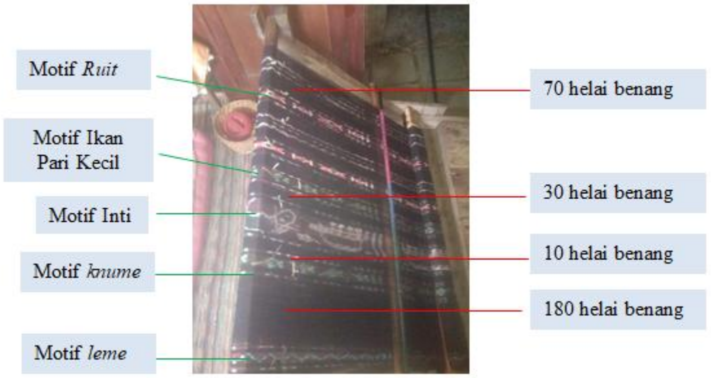

Fig 5 Pattern for Making *Kwatek* Lamalera Ikat Woven Fabric

All fundamental mathematical activities are present in the process of making Lamalera kwatek ikat woven fabric. The following are the details of the research findings for each mathematical fundamental activity in the process of making Lamalera kwatek ikat woven fabric

a. *Counting*

The following are the researcher's findings for the fundamental mathematical activity of counting:

1) The distance between motifs on the fabric is usually 40-60 strands of yarn for long lengths and 5-10 strands for short lengths.
2) In the whale, stingray, and pledang motifs, the first row consists of 40 strands of thread followed by the knume motif with a distance of 10 strands of thread to the next motif.
3) The pattern continues with a distance of 10 strands to the knume motif, 30 strands to the next motif, and repeat until the fabric making process is completed into a woven fabric that is ready for use.
4) Before reaching the final limit of the length of the fabric, the end is spaced 180 strands of yarn.
5) For whale, stingray and pledang motifs, six sections are tied: three for the front and three for the back of the cloth.

6) If the weaver makes this type of weave, only two pieces of kwatek can be completed in a month.

b. *Playing*
The following are the researcher's findings for fundamental mathematical playing activities:
1) The stage of making motifs begins with untangling the yarn on the vulo gulokajo and tying the two ends of the untangled yarn.
2) After tying, the yarn is removed from the vulo gulokajo, then dipped into the base color of the fabric such as black, brown or other colors, and dried in the sun to dry.
3) After drying, each part is opened and given color according to the motif.
4) This process is done for all motifs until the desired color and pattern can be formed properly.

c. *Measuring*
The following are the research findings for the fundamental mathematical activity of measuring:
1) The making of Lamalera kwatek requires 25-30 spools of thread per piece, so for one whole sheet it takes about 50-60 spools of thread.
2) The size of the kwatek varies, but generally the size is $80 \times 160\ cm$ or $100 \times 200\ cm$.
3) Making one kwatek sheet takes about one week.
4) Working on motifs takes 1-3 days, depending on age which can affect accuracy and time.
5) Weavers can produce 3-4 pieces of kwatek per month if they only make kwatek without other types of weavings.
6) In addition, difficulties arise in obtaining yarn from Lewoleba City, which takes 2-4 hours to travel.

d. *Designing*
The following are the research findings for the fundamental mathematical activity of designing: The making of Lamalera's kwatek ikat woven fabric begins with untangling the threads in a selaga to form the specified motif.

e. *Explaining*
The following are the research findings for the fundamental mathematical explaining activity:
1) Tools for making Lamalera kwatek ikat woven fabric include *tenane, selaga, vulogulokajo, faniduang, huri, kdaje, sligu, paso, kduke, mnue.*
2) Additional tools for making motifs are natural or artificial dyes and raffia.
3) There is no standard rule in determining the distance between motifs in the making of Lamalera kwatek ikat woven fabric.
4) The size of the kwatek varies according to the wishes of the buyer or weaver, but is generally $80 \times 160\ cm$ or $100 \times 200\ cm$.
5) Making one piece of Lamalera kwatek ikat woven fabric takes about one week.
6) The ikat *kwatek* woven fabric is marketed to tourists, both local and foreign, as well as the community around the village.
7) Making *kwatek* with real yarn is difficult because natural dyes such as daunt arum are only available during the rainy season.

8) The making of *kwatek* ikat weaving does not involve special ceremonies because weaving is a common activity like those in other areas.
9) Each weaving motif has a meaning, such as whale and stingray motifs that depict marine products, and pledang which means whaling boat.
10) There are no specific rules in the use of kwatek, but tradition teaches that kwatek is only for women who are used for traditional ceremonies or other arts.

f. *Locating*
The following are the researcher's findings for the mathematical fundamental activity of locating: Weavers have difficulty obtaining shop yarn quickly because they have to buy it in Lewoleba Town, which is a 2-4 hour drive from Lamalera.

The following presents a summary of the mathematical fundamental activities in the process of making Lamalera kwatek ikat woven fabric:

Table 2 Fundamental Mathematical Activities in the Making Process of Kwatek Lamalera Ikat Woven Fabric

Counting	Playing	Measuring	Designing	Explaining	Locating
Calculating the distance between motifs, the number of strands of yarn for each row of motifs, and the total number of parts tied on the Lamalera kwatek ikat woven fabric.	Determine the sequence of making kwatek woven fabric motifs from untangling yarn, coloring, to forming patterns.	Measuring the number of threads, fabric size, time of making, and number of weaves in making kwatek Lamalera.	Designing motifs for Lamalera's kwatek ikat woven fabric.	Describe the tools, materials, and processes in the making of woven fabrics including motif making, size, timing, marketing, as well as the meaning of motifs and traditions of use.	Location for obtaining basic materials for making woven fabrics.

3.3 Distinctive Characteristics and Uniqueness of the Kwatek Lamalera Ikat Woven Fabric Motifs

The difference between Lamalera's ikat kwatek woven fabric and similar woven fabrics lies in the motif of the woven fabric which uses whale, stingray, and pledang motifs that illustrate the identity of Lamalera Village as a “Whale Hunter Village”. The whale symbolizes the ability to conquer rare mammals. The pledang depicts a whaling boat, and the stingray is a sea product that is often obtained by local fishermen. In addition to these motifs, there are other motifs used, namely: ruit motifs that are used as decoration or additional motifs.

Fig 6 Whale motif

Fig 7 *Pledang* motif

Fig 8 Stingray motif

Fig 9 *Ruit* motif

Motifs that are difficult to make are taru mata and klape motifs. Because the making of these two motifs requires original yarn and natural dyes. Making woven fabrics using original yarn and natural dyes will take longer because it requires more care and longer time for the drying process.

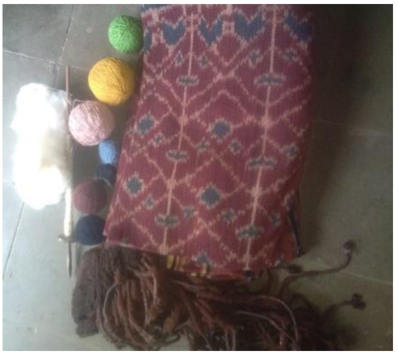

Fig 10 *Taru Mata* motif

Fig 11 *Klape* motif

There are two fundamental mathematical activities in the characteristic motifs of Lamalera's ikat kwatek woven fabric, namely: counting, and explaining. The following is a breakdown of the researcher's findings for each fundamental mathematical activity in the characteristics of the Lamalera kwatek ikat woven fabric motif.

a. *Counting*
The following are the research findings for the fundamental mathematical activity of counting:
1) In addition to the three main motifs, there are additional motifs such as taru mata and klape on the original yarn-based *kwatek*.
2) Making *taru mata* and *klape* motifs is more difficult due to the limited natural dyes and the time-consuming drying process.

b. *Explaining*
The following are the researcher's findings for the fundamental mathematical activity of explaining:
1) Lamalera's *kwatek* woven fabric has whale, stingray, and pledang motifs that distinguish it from other regions' *kwatek.*
2) The choice of motifs is based on the symbolization of Lamalera Village as a "Whale Hunter Village".
3) The whale motif depicts the ability to conquer rare mammals, the pledang as a whaling boat, and the stingray as a sea product often obtained by Lamalera fishermen.
4) In addition to the three main motifs, there are also other motifs on Lamalera kwatek woven fabrics such as the taru mata and klape motifs for the original yarn, and the ruit motif as an additional decoration.
5) The difficulty in making taru mata and klape motifs is caused by the need for more natural dyes and long drying time.
6) Making whale, stingray, and pledang motifs with took yarn has no difficulty because these motifs are familiar to the weavers.

The following presents a summary of the basic mathematical activities in the characteristics found in the Lamalera kwatek ikat woven fabric motif:

Table 3 Fundamental Mathematical Activity in Characteristic Motifs of *Kwatek* Lamalera Ikat Woven Fabric

Counting	Explaining
Calculating the number of motifs and the difficulty of making taru mata and klape motifs on Lamalera kwatek ikat woven fabric.	Explains the motifs in Lamalera kwatek woven fabric, the symbolism of the village, and the difficulties in making the motifs.

3.4 Philosophical Meaning of the Kwatek Lamalera Ikat Woven Fabric Motifs

Lamalera's kwatek ikat woven fabric has distinctive motifs such as whales, stingrays, and pledangs that reflect the whale hunting tradition that has existed for hundreds of years. These motifs also depict sea products that are often obtained by Lamalera fishermen. The choice of motif depends on the taste of the weaver or consumer, with a collaboration between whale, stingray and pledang motifs often chosen as the main motif. Although these motifs are mandatory in Lamalera kwatek, not all types of motifs must be present in one piece of cloth, and weavers can choose motifs as they wish.

Lamalera kwatek ikat woven fabrics are used by the community, especially women, for various daily purposes, including traditional activities and ceremonies such as leva mass and the opening of the whaling year. There are no fixed rules regarding the motifs that should be used in traditional ceremonies, but the community tends to choose motifs that depict the distinctiveness of Lamalera village. The choice of motifs is based on custom and respect for the inherited culture, with whale, stingray and pledang collaboration motifs being the most popular.

The skill in making kwatek ikat woven cloth is also a source of additional income for the Lamalera community. The selling price of kwatek varies depending on the materials used, size, and quality. Kwatek made from real yarn is sold at a higher price, between Rp 5,000,000 and Rp 10,000,000, while those made from shop yarn are sold at between Rp 800,000 and Rp 2,500,000. Sales are made manually to tourists or through technology promotion by younger weavers. The sales process involves an explanation of the qualities and advantages of each type of weave, followed by price haggling and cash or installment transactions.

There are five fundamental mathematical activities in the philosophical meanings of Lamalera's kwatek ikat woven fabric motifs, namely: counting, measuring, explaining, playing, and locating. The following are the details of the researcher's findings for each fundamental mathematical activity in the philosophical meaning of the Lamalera kwatek ikat woven fabric motifs.

a. *Counting*

The following are the research findings for the fundamental mathematical activity of counting: Each motif in Lamalera's kwatek ikat woven fabric has a philosophical meaning derived from the whale hunting tradition that has lasted hundreds of years.

b. *Measuring*

The following are the research findings for the fundamental mathematical activity of measuring:

1) Each motif in kwatek woven fabric has a philosophical meaning related to the whale hunting tradition that has existed for hundreds of years.
2) The price of Lamalera kwatek varies, with kwatek with original yarns priced between Rp. 5,000,000 to Rp. 10,000,000, and kwatek with taken yarns sold between Rp. 800,000 to Rp. 2,500,000.

c. *Explaining*

The following are the researchers' findings for the fundamental mathematical activity of explaining:

1) Each motif in the kwatek woven fabric has a meaning related to the whale hunting tradition that has existed for a long time.

2) Motifs on Lamalera kwatek must be present in the making of every type of kwatek originating from Lamalera village, but the type can be chosen according to the taste of the weaver or consumer.
3) The making of kwatek has no fixed rules, only the order and distance between motifs.
4) Women in Lamalera use kwatek for daily activities such as going to the market, church, or traditional events.
5) Lamalera *kwatek* woven fabric must be used in every traditional ceremony.
6) *Kwatek* making can be a source of additional income, with prices depending on the material, quality, motif, size, and number of lirang.
7) Promotion of *kwatek* sales is done directly or through technology.
8) Transactions can be made through cash or installments, but installments are only allowed for the closest people.

d. *Playing*
The following are the researcher's findings for fundamental mathematical playing activities:
1) The sales process begins with the weaver explaining in general terms the woven products offered.
2) After prospective customers choose, the bargaining process is then carried out until they find a price agreement.

e. *Locating*
The following are the researchers' findings for the mathematical fundamental activity of locating: Manual sales are carried out by offering directly to consumers or displaying woven products in front of the house to attract the attention of visiting tourists.

The following is a summary of the mathematical fundamental activities in the philosophical meaning of the Lamalera kwatek ikat woven fabric:

Table 4 Fundamental Mathematical Activity on the Philosophical Meaning of Kwatek Lamalera Woven Ikat Fabric Motifs

Counting	Measuring	Explaining	Playing	Locating
Counting the number of motifs that represent the tradition of whale hunting on Lamalera kwatek woven fabric	Measuring the price of Lamalera kwatek based on the type of thread and number of lirang.	Explain the meaning of motifs, the process of making, using, and how to promote and transact Lamalera woven fabrics.	Describe the transaction process sequence of Lamalera kwatek woven fabric.	Determine a location to promote handwoven or traditionally woven products.

4 Conclusion

Lamalera ikat kwatek woven fabric is one type of woven fabric that is specialized for women and originates from Lamalera village, Wulandoni sub-district, Lembata Regency.

There are three main motifs used when making ikat kwatek woven fabrics, namely whales, stingrays, and pledangs. The research data obtained are grouped into four classes, namely: (1) the history and development of woven fabrics; (2) the development process of woven fabrics; (3) the characteristics of the motifs in woven fabrics; and (4) the philosophical meaning of the motifs in woven fabrics. Based on the analysis of fundamental mathematical activities, (1) counting activities exist in every class; (2) playing activities exist in two classes, namely the second and fourth classes; (3) measuring activities exist in three classes, namely the first, second, and fourth classes; (4) designing activities exist in only one class, namely the second class; (5) locating activities exist in the first, second, and fourth classes, and (6) explaining activities exist in all classes.

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Visual mathematics: an implementation to students in an indigenous community and a sub urban area

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Abstract. Visual mathematics is a mathematical exploration involving the senses and provides students with models and images for concepts, enhancing mathematics learning. Hands-on activities are central to educe and nurture students' logic, observational skills, and habits. Visual mathematics may also be useful for communicating mathematical concepts that minimizes the dependence on culture and language. We propose two modules on visual mathematics for year 4 elementary school students, the first in the arithmetic operation of fractions, and the second in basic planar figures, their perimeters, and areas. The construction is not only appealing figuratively but more importantly, it maintains precise mathematical constructions.

1 Introduction

Mathematics, at its core, is a visual and spatial discipline. The fundamental concepts and relationships in mathematics are often best understood through visual representations and intuitions (see for example Boaler [5], Presmeg[15]). In accordance to Zimmermann and Cunningham (1991) [25], as well as Hershkowitz et al. (1989) [10], Arcavi (2003)[2] defines visualization as follows: "Visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings". Furthermore, one of the strong justifications for visual mathematics can be grounded to modern brain theory. According to some results in neuroscience research about the ways human brains work when they study and learn mathematics, the brain activity is spread out across a widely-distributed network, which include two visual pathways: the ventral and dorsal visual pathways. Neuroimaging has shown that even when people work on a number calculation, such as 12×25, with symbolic digits (12 and 25) our mathematical thinking is grounded in visual processing [6].

From the geometric proofs of ancient Greece to the complex visualizations used in modern mathematical research, the ability to effectively communicate and reason about mathematical ideas through visual means has been a crucial aspect of the field. Diagrams,

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charts, and other math-based visualizations can convey complex quantitative information in a way that transcends the need for written or spoken translation. A graph illustrating economic trends, for example, can be interpreted and understood by people from vastly different cultural backgrounds.

Although ideally mathematical ideas are universal, and therefore they transcend cultures and languages, in real world, especially in teaching and learning mathematics, linguistic and cultural differences can often be barriers for students to communicate and understand. To overcome these barriers, visual mathematics is proposed. The teaching and learning of mathematics have increasingly incorporated more visual elements in recent decades. From number lines to geometric proofs, the visual components of math make the subject more accessible and intuitive for students. This visual approach has been especially beneficial for students who may be learning math in a language that is not their native tongue. The diagrams and symbolic representations allow them to grasp the concepts without being hindered by the vocabulary.

For young learners, visual mathematics is a vehicle for them to learn mathematics, and the world around them as well. It should be an integral and strategic part of learning mathematics in schools. Visual mathematics is an integral part of the proposal for a STEAM education framework for schools in indigenous communities [20]), exactly for the reason of its capacity to transcend potential language and cultural barriers.

Proficiency in visual representation is essential for success in advanced mathematical and scientific courses, as these subjects heavily rely on visualization and spatial reasoning to solve complex problems (Zhang et al [22]). Thus, it is not surprising that many authors emphasize the significance of visual thinking in the learning process and provide compelling arguments regarding the pivotal role of visualization in the reform of mathematics teaching [23,2,13]. However, they also acknowledge the potential challenges and constraints associated with visualization, as indicated by other studies [14,3,4].

Despite the significance of visualization in mathematics, research by Presmeg & Bergsten [15] and Healy & Hoyles [7] reveals that students generally exhibit reluctance to engage in visualization in mathematics. Hence it raises an intriguing question: how can students best be encouraged to employ this way of thinking?

This article is a preliminary report of our ongoing project to construct visual mathematics modules for elementary school students. The objective of the project is to write and construct visual mathematics modules to serve two purposes, those are:

1. As an accompanying module in traditional math teaching, so the teacher can deliver the topics in a proper logical order. Also, the students can be motivated to learn the topics due to the figures used in the module. For this purpose, the teacher should carefully pick the figures to be discussed, to meet the time limit and also to enable the students to learn independently.
2. As an independent learning material that can be used by the students to learn the topics independently, regardless of the language used in the module. Hence the concepts in this module are slowly but rigorously built using a series of figures that are quite common or familiar to most of the students. For this purpose, the students should be conditioned to use the module. Keen observation is one of the key points required for learning the topics using the module. Some games involving observations can be given in the beginning.

In designing our modules, the major steps are:

1. Construct an algorithm to introduce mathematical concepts using pictures. The pictures should not be only illustrative, but furthermore it should carry the mathematical idea correctly, and also suitable to the cognitive development of students,

2. The objects in the pictures can be chosen to be universal or common objects in most of all communities, such as trees, fruits, leaves, stones. The pictures can be made more interesting by taking more relevant objects to the students, or the communities, and thus, the modules designed using visual mathematics can be adopted with minimum effort to be used in various communities.

The first module which covers the arithmetic of fractions was trialed in the elementary school in the indigenous community of Kasepuhan Ciptagelar, in South Sukabumi, in August 2023. The second module covers the geometry of simple planar shapes and was trialed in elementary schools in Lembang, West Bandung Regency in September 2024. The first location was chosen to represent students in remote and isolated communities. Kasepuhan Ciptagelar is an indigenous community based on its sustainable rice-paddy agriculture. One of the unique features of the Kasepuhan Ciptagelar is their openness to technology (including somewhat regulated internet access), despite their strict adherence to their culture. Their school facilities and infrastructures are still minimal, such as the poor ratio of students-teachers. However, their limited bandwidth of internet and low ownership of devices, still enables them to limited exposure to various learning material. The second location was selected to represent students in schools in sub urban areas. Lembang is a suburb of Bandung City. Despite its vicinity to large city, Lembang still maintains the mix characteristics of a city and kampong, in terms of socio-economic-cultural values and conditions, access to modernity, and etc. These two choices represent somewhat contrasting background conditions for the implementation of visual mathematics. The second module was created with feedback from the first trial.

2 The modules

In making visualizations of concepts in mathematics, a detailed process of building or describing the concepts should be done. It is needed to make the students learn the concepts rigorously, comprehensively, and effectively.

In the beginning of the process, it is important to make the students familiar with this method is by guiding them directly in the class or using some preliminary activities. The students are asked to look closely at the series of figures given in the module and describe the process that is represented by those figures.

When a concept is introduced, we have to find the basic ideas so that the advanced topic can be built based on these basic ideas while retaining consistency. The visual objects used in the modules were tailored to students' backgrounds and environments. Exercise problems that follow, inquire students to show their understanding by sketching series of pictures related to the concepts. This type of exercise not only shows the students' understanding, but also let them express their creativity. The freedom in expressing their understanding may help the students develop the joy of learning mathematics.

2.1 Fractions and their arithmetic

Fractions are fundamental concepts in mathematics, used to represent the concept of parts of a whole. Since fractions form the basis for more advanced mathematical concepts, mastery of fractions and their arithmetical operations is crucial for students [1]. It is crucial to their capacity to successfully grasp new ideas like calculus, algebra, probability, and trigonometry [5]. Even though fractions are taught starting in elementary school, many adults, including high school students, continue to struggle with fractions misconceptions [19,21,16]. Consequently, it is imperative to find an alternative approach.

2.1.1 Arithmetical Operations of Fraction

Elementary school students often find arithmetic operations, such as addition, subtraction, multiplication, and division of fractions, to be challenging concepts to grasp [10,9] Hence, we consider it very critical to find an alternative method to explain these operations, which helps students build conceptual understanding [1].

There are several methods of approaching the problem and some have proved to be successful [9]. There is still the need to minimize the difficulty of understanding the concept. By employing a visual approach to introduce mathematical concepts, it is expected that students can avoid the misconceptions that sometimes arise. However, studies have also shown that young students can successfully divide a given number of items among persons and fail to do so when the same problem is presented simply with symbolic notation and no visual clues [17].

Therefore, employing visual representation is highly likely to help students build a solid understanding of the concept.

2.1.2 Fractions

Some researchers claimed that the process of dividing a unit into equal parts is crucial for developing a comprehensive understanding of rational numbers [6,11,18]. Thus, the idea behind our visualizations is based on the notion that a fraction represents an object or collection of objects that are equal in part. For instance, a pizza or a pie is cut into four equal slices or parts. Each slice represents $\frac{1}{4}$ of the whole. Thus, the fraction $\frac{3}{4}$ represents a collection of three of them.

Fractions can have the same value; in that case, we refer to them as equal or equivalent. We need this notion when we compare two fractions or to add two fractions that have different denominators.

2.1.3 Least Common Multiples

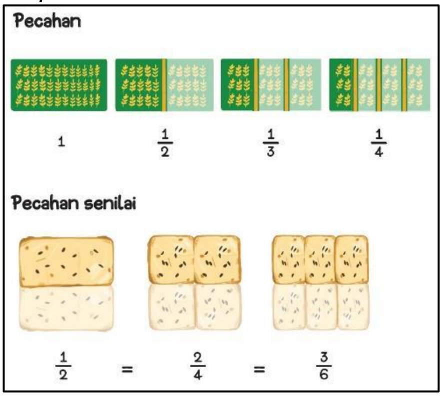

Fig. 1. Equivalent fractions

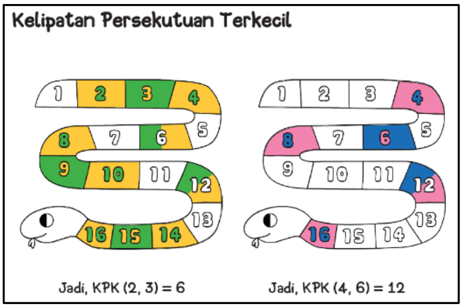

Fig. 2. Least Common Multiple

In order to determine the common denominator, students must possess an understanding of common multiples, but the least common multiple (lcm) is the most efficient method because usually it avoids the need to simplify the result. The concept of least common multiples can be visualized as seen in the following.

2.1.4 Addition

When fractions have the same denominator, the addition operation is in general straightforward. Every fraction addition problem is known to be reducible to a fraction addition problem with the same denominator.

1. Check if the denominators are the same. If not, reduce the problem into a problem of fractions addition with the same denominator, say $\frac{n}{m} + \frac{p}{m}$.
2. Cut each two pies (or rectangles) into m equal parts. For each pie, shade the appropriate number of slices to reflect the fractions.
3. The total number of shaded slices is the numerator of the fraction that results from the addition process. The denominator is m.

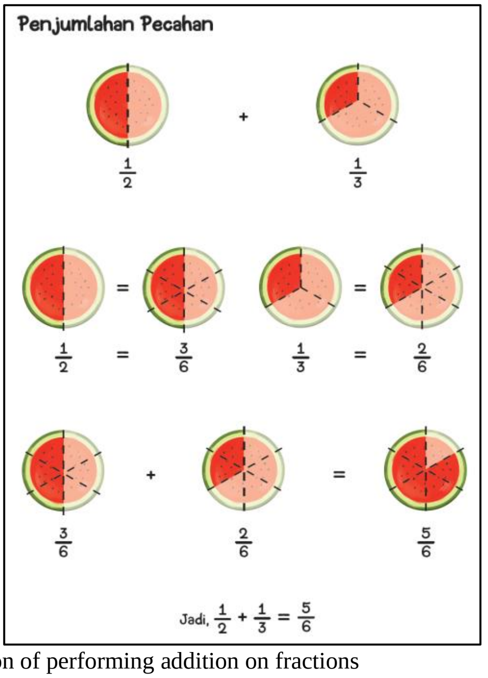

Fig. 3. Visual representation of performing addition on fractions

2.1.5 Multiplication

Compared to addition, the concept of fractions multiplication is more challenging, even though the operation is more straightforward. Making the concept relatable to students is important to help students understand it. Multiplication of fractions focuses on finding a portion of a portion. Multiplication of fractions asks, “What part of the whole do you get when you take a part of a part?” Therefore, it is very sensible to start with the most straight forward case, which is multiplications involving unit fractions. We can find the multiplication $\frac{n}{m} \times \frac{p}{k}$ as follows.

1. Cut a rectangle vertically into m equal parts and then shade the n slices to represent the $\frac{n}{m}$ fraction.
2. Cut each slice of the rectangle into k parts equally. Thus, each thin slice corresponds to $\frac{1}{mk}$ of the rectangle.
3. For each slice, shade p thin slices.
4. Then the number of shaded thin slices is the numerator of the fraction that arises from the multiplication process. In the meantime, mk is the fraction’s denominator.
5.

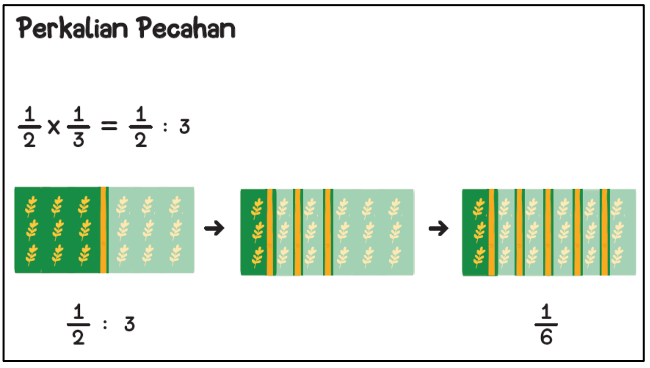

Fig. 4. Unit fractions multiplication

2.1.6 Division

The division is essentially the inverse of multiplication. To put in words, division asks “How many of these fit into that?” Consequently, it is best to go over the division's verbal meaning to the students before moving into the division operation.

We may ask students to relate the division problem $\frac{1}{2} : \frac{1}{4}$ to the question "How many quarters of bread fit into a half of bread?" using this verbal interpretation. The visual explanation in Figure 4 should help students to find that the answer is 2.

However, the answers to division problems are not necessarily integers. Nevertheless, the same meaning still holds in this case. A question like “How much of this fits into that?” can be used to verbalize the division $\frac{f_1}{f_2}$, if f_1 and f_2 are fractions such that $f_1 < f_2$. For instance, $\frac{1}{6} : \frac{1}{3} = \frac{1}{2}$ indicates that half of a third of a pizza fits into a sixth of a pizza. It would be easier to answer this question if the fractions were expressed using the same denominator. The expression of the fractions using the same denominator enables us to visualize that a sixth pizza is half of a third pizza.

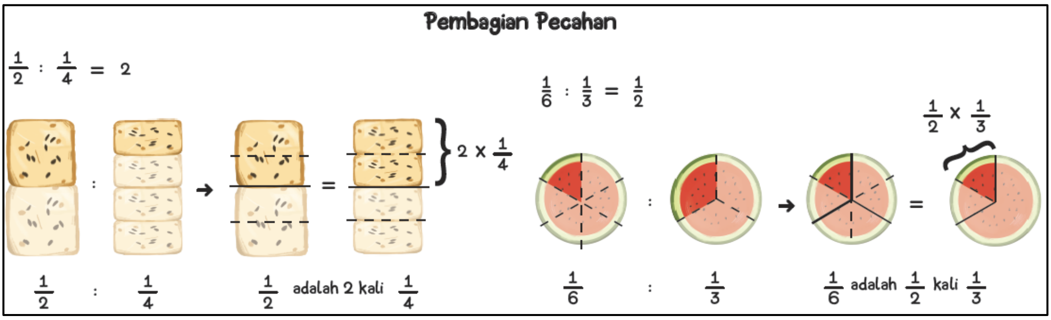

Fig. 5. Division of fractions

2.1.7 Ordering Fraction

Sometimes we need to find out which of two fractions is greater than the other one. Therefore, we need to know how to order two fractions. When ordering fractions, the most important thing to remember is that the denominators must be the same, thus the order is determined by the numerators. The same method is used in some arithmetic operations on fractions, those are addition, subtraction, and division.

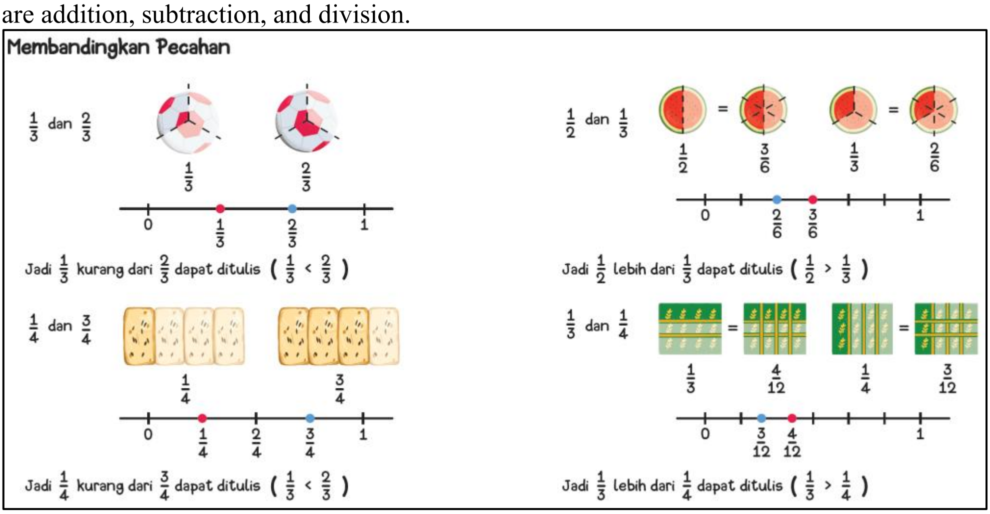

Fig. 6. Ordering fractions.

2.2 Shape and their measurements

According to the National Council of Teachers of Mathematics (NCTM), a key objective of learning geometry is to develop students' visual awareness and make them visually literate persons. NCTM states that geometry will lead students to analyze the properties of geometric shapes by using spatial thinking and geometrical modeling in solving problems. Therefore, it was emphasized that learning geometry will help students to become more proficient in reasoning and justification [12].

In the twenty-first century, the capacity to analyze digital, visual, and audio information is as fundamental as reading and writing. In this regard, geometry instruction should help students maximize the benefits of the visual materials among the course materials [8].

Thus, the topic discussed in the second module is basic planar shapes, especially their perimeter and area. We use the main ideas as follows.

1. The perimeter of a planar shape is the sum of the length of its sides. It includes introducing the Pythagorean Theorem.
2. The concept of the area of a planar shape is introduced using a decomposition of the shape into right triangles. The area of a planar shape is the sum of the areas of the right triangles inscribed in the shape.

The detailed process of introducing the concepts of perimeter and area of planar shapes is as follows.

1. A line segment is introduced with its length. Some real objects in the shape of line segments are given, with their length.

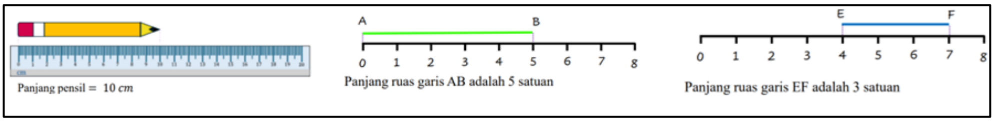

Fig. 7. Length of segments

2. A square is introduced whose sides are line segments. The perimeter of the square is discussed using the length of its sides. The area of a square whose length of each side is 1 unit is defined as one unit square. Then the area of a square is defined according to the number of the unit square inscribed in the square.

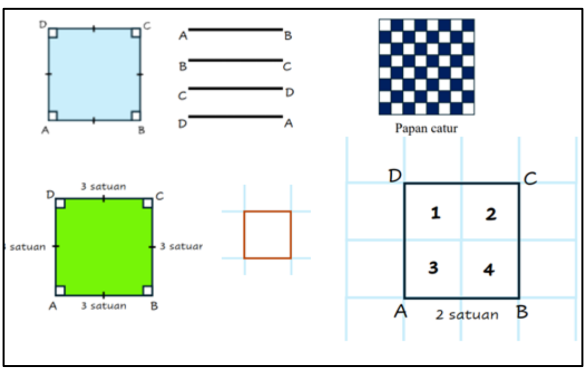

Fig. 8. Perimeter and area of squares

3. The perimeter and area of a rectangle are introduced as the ones for the square.
4. A diagonal of a rectangle is introduced, as well as its length using the Pythagorean Theorem.

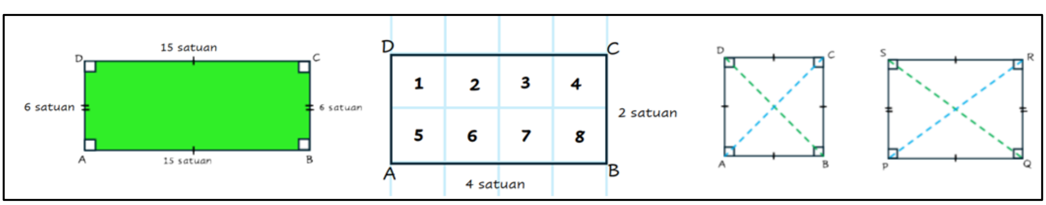

Fig. 9. Perimeter and area of rectangles

6. A right triangle can be obtained by dividing a rectangle into two equal parts along one of its diagonals.

7. The perimeter of a right triangle is introduced as the sum of the length of its sides. The area of a right triangle is defined as half of the area of the corresponding rectangle.

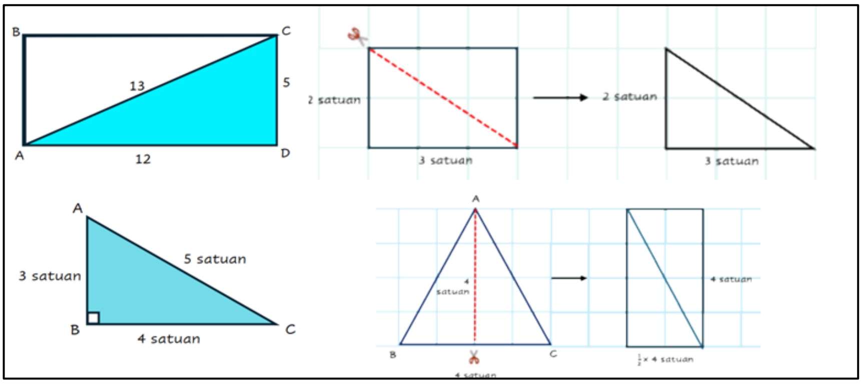

Fig. 10. Perimeter and area of triangles

8. The perimeter of a parallelepiped, trapezoid, kite, and rhombus are introduced as the sum of the length of their sides.
9. The area of a parallelepiped, trapezoid, kite, and rhombus are introduced as the sum of the area of the right triangles inscribed in each shape.

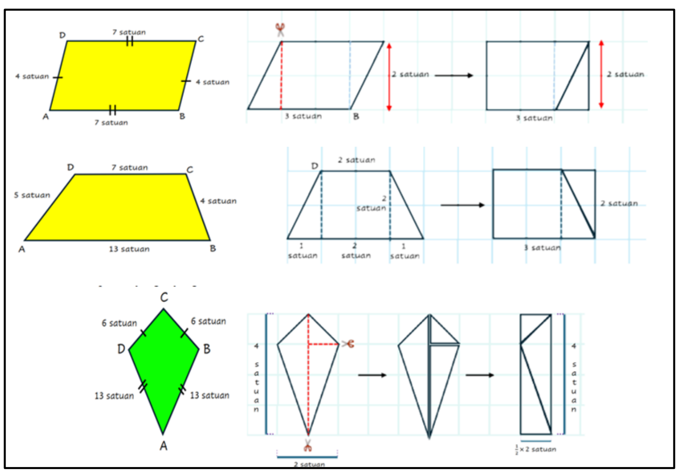

Fig. 11. Perimeter and area of quadrilaterals

10. A series of figures are given to guide the students in finding the formula of the perimeter and area of each shape.
11. Some exercise problems, with increasing levels of complexity, are placed following the given examples.

The figures in the module as examples or in exercise problems are varied. Usually, they begin with some real objects that are familiar to the students. Then gradually they become more abstract as the concepts are introduced for general objects with a certain shape.

3 Conclusion and Further plan

Visual mathematics is proposed to be a method of learning mathematics that can be used directly, or adapted quite easily if required, to various communities with their own cultures and languages. Two modules that are constructed using this method have been trialed to two groups of students. One group of students is in an indigenous community, the other is in the sub urban area.

The modules can be used as a main reference for teachers in the class when they deliver the topic. The modules also help to guide the teacher and students in discussing the topic due to their detailed steps of the process building the concepts in a precise, detailed, rigorous manners. The teachers can help the students to learn more effectively, and the students are quite motivated because of the figures used in the modules. The modules can also be used by the students as an independent study guide, with more time allowance. It requires keen, logical, and critical observations. Some preconditioning exercises for the students are strongly suggested.

Some challenges in making the modules are picking the strategy to deliver the concepts, how detailed the process of building the concepts is shown, and the objects used to describe the process. The strategy is selected based on the compatibility of using figures in explaining the concepts. The flow of presenting the concepts relates to the ability of the students to learn. The objects used in the figure should be familiar and quite common to the students.

A quatitative study to measure the effectiveness of the method is planned as a follow up to this study, as well as trial involving more communities.

We acknowledge the support of the PPMI Grants for Community Empowerment, FMIPA Institut Teknologi Bandung, FY 2023 and FY 2024. We thank Iwan Pranoto whose initiative on Visual Math has led us to this project; Hendri Maulana, Dianing Ratri, and Shanaya Muttaqy who were responsible for designing the graphics for the modules; our collaborators Common Room Network Foundation and Bosscha Observatory Community Outreach; the Elders of Kasepuhan Ciptagelar, School Administrators of SDN III, and SDN XI, Lembang, who welcomed us and facilitated our activities on-site.

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Implementing The Ignatian Pedagogy Paradigm in Mathematics Learning: Annuity Material at The Vocational School Level

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Abstract. The Ignatian Pedagogy Paradigm with a Problem-Based Learning (PBL) approach is expected to provide a meaningful learning experience. This study aims to determine the implementation of PBL using the Ignatian Pedagogy cycle in vocational school students' mathematics learning of Annuity material. The results of the study show that implementing this learning has succeeded in creating meaningful learning experiences. This learning connects the material with real situations as context excavation, group discussions in building learning experiences, learning reflections, and the presentation process as actual actions. This learning was evaluated using the 4C (Competence, Conscience, Compassion, and Commitment) principles. The Ignatian spirituality contained in this learning is Cura personalis and lifelong learning. Implementing the Ignatian Pedagogical Paradigm through PBL helps students understand the topic that they learn and builds a person who is reflective, responsible, and ready to face future challenges.

1 Introduction

Education is very important in improving the quality of human resources (HR), which intends to develop individuals holistically, including aspects of spiritual and social attitudes, knowledge, and skills. More than just transferring knowledge, education has a moral responsibility to create individuals who can think critically, act wisely, and adhere to noble values. However, learning practices are often oriented towards rote memorization and purely academic achievement, losing meaning and relevance. Especially in mathematics learning, where abstract concepts are frequently taught without a clear connection to the real world, students need help understanding the significance of this science in everyday life. Therefore, an approach is required to provide a meaningful learning experience while shaping student character.

The Ignatian Pedagogical Paradigm (IPP) offers a holistic approach that emphasizes the development of competence, awareness, empathy, and commitment to learning. With the 4C (Competence, Conscience, Compassion, and Commitment) principles, the Ignatian Pedagogical Paradigm integrates cognitive, emotional, and spiritual aspects [1]. The Ignatian Pedagogy Paradigm is applied through a cycle of context, experience, reflection, action, and evaluation to accompany the learner in understanding truth and exploring human values. This

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pattern is central to Jesuit education aimed at forming competence, responsible, and compassionate individuals. With this approach, teachers can help students develop holistically through a meaningful learning process involving the dynamics of Ignatian Pedagogy, as shown in the following scheme [2].

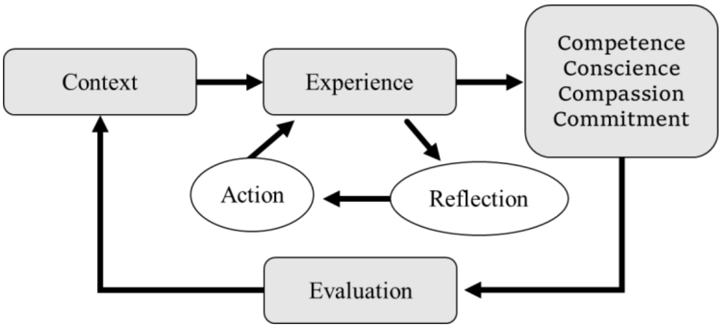

Fig. 1. Dynamics in the Ignatian Pedagogy Schema

One learning model that aligns with Ignatian Pedagogy is Problem-Based Learning (PBL). The model also focuses on reflective, contextual, and meaningful learning. In PBL, students are invited to solve real problems relevant to their lives to develop critical and analytical thinking skills and practice values such as responsibility, cooperation, and empathy. This model encourages hands-on learning experiences, deep reflection, and concrete actions, which align with the dynamics in the Ignatian Pedagogy, such as reflection, action, and evaluation, to form competent individuals with integrity and oriented toward the common good. PBL is divided into five phases that can be observed in the following syntax scheme. [3]

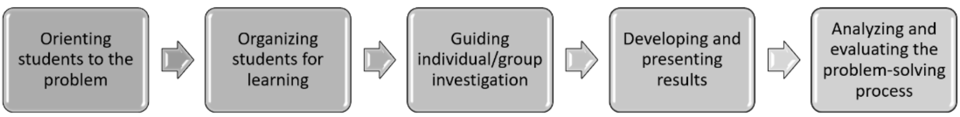

Fig. 2. PBL Syntax Schema

In their research, Putri et al. concluded that the Ignatian Pedagogical Paradigm approach helps generate feelings of pleasure, encourages learning interest, focuses students' attention, and gets students involved so that students can realize the importance and benefits of learning mathematics, which remains based on 3C (Competence-Conscience-Compassions) [4]. Some researchers in Indonesia use the term Reflective Pedagogical Paradigm (RPP) instead of Ignatian Pedagogical Paradigm. In her research, Pratini shows that applying the Reflective Pedagogical Paradigm (RPP) can develop students in terms of 3C [5]. Meanwhile, Sulistyani successfully conducted PBL based on the Reflective Pedagogy approach. The results of her research show that the phases of PBL can be implemented well in Advanced Statistics learning using RPP [6].

Grounded in the presented research, this research will be carried out to implement PBL using the Ignatian Pedagogical Paradigm approach to mathematics learning. The material taught is Annuity, where this material will not only be taught cognitively but will also understand the value of its application in real life. This research aims to determine the implementation of PBL using the Ignatian Pedagogy cycle in mathematics learning Annuity material in vocational school students.

2 Method

This research is a qualitative study. The research subjects were 28 students of grade X of Visual Communication Design (DKV) at Bunda Mulia Vocational School, Jakarta. This study will conduct mathematical studies for the Annuity material using the PBL model by applying the Ignatian Pedagogical Paradigm. Data from this study is obtained through learning observations, learning outcomes, and student reflection outcomes. This research will describe and evaluate the implementation of learning using a PBL model based on the Ignatian Pedagogical Paradigm.

3 Results and Discussion

3.1 Description of Learning Implementation

Learning was carried out for 2 hours of lessons or 90 minutes. The material taught was Annuity, especially the definition of annuities. This material was further material from the Simple Interest and Compound Interest Chapters. This learning used the PBL learning model with a Constructivist approach. Students were guided to build their knowledge, which is the definition of Annuity. Learning objectives are formulated according to 4C.

- **Competence**: Students can understand and apply annuity concepts, including calculating annuity values, interest, and payment amounts in real situations, such as home credit or investment.
- **Conscience**: Students can recognize and differentiate actions based on Conscience in learning.
- **Compassion**: Students care about each other between group members and can help members who have difficulty understanding the material.
- **Commitment**: Students demonstrate a commitment to taking clear roles and responsibilities in completing independent or group tasks.

3.1.1 *Context Excavation*

Learning began with excavating the student context. The teacher displayed a motorbike credit brochure, often found or given on the street (Figure 3). The teacher asked how to read the information and asked several students for an opinion. Some students could express their opinions and understand how to read and interpret information in a brochure.

TIPE MOTOR	UANG MUKA	ANGSURAN PER BULAN / ARREAR			
		11	23	35	47
PCX 160 CBS 34,560,000.00	3,500,000	3,549,000	1,969,000	1,492,000	1,270,000
	4,000,000	3,496,000	1,940,000	1,470,000	1,251,000
	4,500,000	3,443,000	1,911,000	1,448,000	1,233,000
PCX 160 ABS 37,810,000.00	3,800,000	3,864,000	2,144,000	1,624,000	1,382,000
	4,000,000	3,843,000	2,133,000	1,616,000	1,375,000
	4,500,000	3,790,000	2,103,000	1,594,000	1,356,000
ADV 160 CBS 36,810,000.00	3,700,000	3,768,000	2,091,000	1,584,000	1,348,000
	4,000,000	3,736,000	2,073,000	1,571,000	1,337,000
	4,500,000	3,683,000	2,044,000	1,549,000	1,318,000
ADV 160 ABS 39,650,000.00	4,000,000	4,039,000	2,242,000	1,698,000	1,445,000
	4,500,000	3,986,000	2,212,000	1,676,000	1,426,000
	5,000,000	3,933,000	2,183,000	1,654,000	1,408,000
CB150 Verza SW 23,270,000.00	2,400,000	2,437,000	1,354,000	1,024,000	870,000
	2,500,000	2,427,000	1,348,000	1,019,000	867,000
	2,750,000	2,400,000	1,333,000	1,009,000	857,000
CB150 Verza CW 23,930,000.00	2,400,000	2,508,000	1,393,000	1,053,000	895,000
	2,500,000	2,497,000	1,387,000	1,049,000	892,000
	2,750,000	2,471,000	1,372,000	1,038,000	882,000
SONIC 150R 26,740,000.00	2,700,000	2,775,000	1,541,000	1,165,000	990,000
	2,750,000	2,770,000	1,538,000	1,163,000	988,000
	3,000,000	2,743,000	1,523,000	1,152,000	979,000
SONIC 150R HRR 27,150,000.00	2,800,000	2,808,000	1,559,000	1,179,000	1,002,000
	3,000,000	2,786,000	1,547,000	1,170,000	995,000
	3,250,000	2,760,000	1,533,000	1,159,000	985,000
CB150R Streetfire Std 31,810,000.00	3,200,000	3,261,000	1,810,000	1,369,000	1,163,000
	3,250,000	3,256,000	1,807,000	1,367,000	1,162,000
	3,500,000	3,229,000	1,793,000	1,356,000	1,152,000
CB150R Streetfire SE 32,820,000.00	3,300,000	3,358,000	1,864,000	1,410,000	1,198,000
	3,500,000	3,337,000	1,852,000	1,401,000	1,190,000
	3,750,000	3,310,000	1,838,000	1,390,000	1,181,000
CRF150L 37,670,000.00	3,800,000	3,849,000	2,136,000	1,618,000	1,366,000
	4,000,000	3,828,000	2,124,000	1,609,000	1,359,000
	4,250,000	3,801,000	2,110,000	1,598,000	1,350,000

Fig. 3. Example of Motorbike Credit Brochure

From the picture provided, the teacher asked students' opinions regarding credit, such as what objects are usually purchased with credit and students' responses regarding credit payments. Most students thought valuable objects like cellphones, motorbikes, cars, houses, etc., were usually purchased with credit. There was also a student who believed that all goods today could be purchased on credit; in simple terms, the student explained the existence of a Paylater method on several shopping platforms, which states that any item can be purchased on credit. Furthermore, some students also argued that credit methods cost money more than cash purchases; they felt that credit methods are quite disadvantageous to them. Through this process, teachers found the student context that students already understood the concept of credit, and purchasing goods using the credit method was also common for them to use or encounter in everyday life.

At the beginning of the lesson, the teacher also asked how students bought an item with their own money. Some said that they saved by setting aside pocket money. Some bought goods if they got more money; for example, during Chinese New Year, they got red envelopes. Through this process, teachers could find the context in which students aged 15-16 are able to manage pocket money.

3.1.2 *Experience*

In the first PBL phase, orienting students to the problems, teachers asked how to determine the money that needs to be deposited monthly (in installments) when students buy goods on credit. In the second phase, organizing students for learning, teachers divided students into groups of three students and distributed Student Worksheets to solve problems. The Student Worksheet gave them experience finding definitions of annuities. Student Worksheets were deliberately designed to guide students in constructing their knowledge and finding definitions of annuities. Through working on the Student Worksheet in this group, students built an active and enjoyable learning experience.

3.1.3 *Reflection*

At the beginning of learning, when excavating context, students also reflected on how they managed their finances. Students reflected on whether they have been able to manage the money they have well. They also reflected on how to buy the items they wanted with their money.

In the group discussion, students reflected on how they solved problems with the Student Worksheet using the information provided and their knowledge. In addition, they could reflect on how much they contributed to the group's success in answering questions.

At the end of learning, students were given a small paper to write their reflections on. The reflections consisted of several questions regarding their learning experience in this material. The reflection questions are also integrated with the 4C Principles, as in Table 1. The results of the reflections are shown in Table 2.

Table 1. Reflection Question Guide

4C Principles	Reflection Questions
Competence	- What did you learn today? - Do you understand today's lesson? If not, which part do you not understand?
Conscience	- Why is it important to understand the concept of interest rates and annuities when making credit-related decisions?
Compassion	- How did you contribute to the group discussion? - Describe your feelings today in one sentence!
Commitment	- In your opinion, what was the most interesting thing from today's lesson?

Table 2. Summary of Reflection Results

Competence	Conscience	Compassion	Commitment
All students understood what was learned today. Some felt confident, while some still struggled with the calculations.	Some students understood the importance of learning about credit and annuities, particularly to prepare for the future and make wiser credit decisions.	Some students contributed during group discussions, some helped with calculations, while others assisted their peers. However, some students felt they did not contribute much. Some students were happy with today's lesson, while others felt tired from a full day of activities.	Most students found the lesson interesting because they learned about the calculations involved when taking credit.

3.1.4 *Action*

After group discussion, teachers selected several student group representatives to present the discussion results and their results in answering problems. In PBL syntax, this is the phase of developing and presenting results. Several group representatives presented their answers in front of the class. Through this presentation, students explained the concepts excavated, namely the definition of annuities in communication skills. They shared an understanding that can then be the subject of class discussions.

When making a presentation, students were not only required to understand the material (competence) but also considered how to convey it clearly and care about their friends' understanding (compassion). This demands real action in communicating effectively. In addition, this condition resembles a real-world situation where one must present an idea or solution to another person. It trains students to take actions relevant to their professional lives. The following image is documentation of students presenting in front of the class.

Fig 4. Students Taking Action

3.1.5 *Assessment*

Evaluation or assessment in this Ignatian Pedagogy is carried out based on 4C. Competence assessment in learning at this meeting was carried out by assessing Student Worksheet performance. Each group's Student Worksheet results were based on preciseness and completeness. Meanwhile, Conscience, Compassion, and Commitment assessments were carried out by observing students' learning experiences and contributions to group discussions. Each aspect comprises six indicators with value provisions: Score 4 for 5-6 indicators is met, score 3 for 3-4 indicators is met, score 2 for 1-2 indicators is met, and score 1 if no indicator is met.

Table 3. Summary of Evaluation Results

Competence	**Conscience**	**Compassion**	**Commitment**
All groups have completed the Student Worksheet. Most answers are correct, but the calculations still have some errors.	Most students were able to recognize actions based on Conscience during group dynamics. They demonstrated honesty, respect, fairness,	Some students were able to show care among group members and help friends who had difficulty understanding the	Most students showed Commitment by taking roles and responsibilities in completing tasks. All students were willing to work on the tasks,

	professionalism, and responsibility.	material. During the process, some students voluntarily helped without looking down on their friends' abilities and backgrounds.	but some still lacked initiative in asking about material they did not understand.

3.2 Ignatian Pedagogical Spirit in the Learning Experience

The Ignatian Pedagogy Paradigm aims to help students develop into individuals who are devoted to God [1]. The goal of life in this learning process is to become a complete human being. Being complete means developing one's personality and becoming an advanced human being willing to live and collaborate with others. The principles or spirit that accompany this goal include Personalist Cura and Lifelong Learning.

3.2.1 *Cura Personalis*

Cura personalis, which means care for the whole person, focuses on creating meaningful and personalized learning experiences. *Cura personalis* means reviewing daily life [7]. By linking annuities to real situations, such as home credit simulations or Paylater, students felt that learning directly impacts their lives. Meanwhile, through reflection questions such as "What was the most interesting thing from today's lesson?" students were invited to see the impact of learning on their personal development.

In the discussion and presentation process, students were encouraged to try and not be afraid of failure. Learning is a growing process that builds self-confidence so students feel their potential is recognized. Teachers provide opportunities for exploration so students can take initiative in learning.

3.2.2 *Lifelong Learning*

Lifelong education is a concept that can be carried out anytime, anywhere, and without age restrictions. Kenneth H. Silber [8] argues that everyone is a learner throughout their life. Learning related to annuities certainly does not only limit mathematical concepts but also connects them with real applications such as credit and savings planning. Students learned that this knowledge can be applied to financial management decision-making, a skill they will need in the future.

In a reflection, a student wrote, "Knowledge of this material can help me in the future." Through this writing, the student saw learning as fulfilling academic obligations and investing in developing valuable abilities at various stages of life. This belief builds a lifelong learning mindset, in which learning is seen as a means of growing fully throughout life.

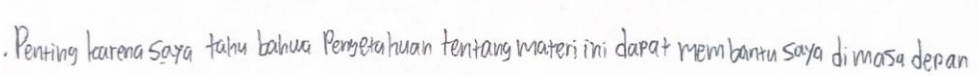

Fig 5. A Student's Reflection

4 Conclusion

The results of this study lead to the conclusion that the Ignatian Pedagogy Paradigm applied through the Problem-Based Learning (PBL) model has successfully created a meaningful learning experience for students learning Annuity materials. This learning process builds students' cognitive abilities, such as understanding and applying annuity concepts. It integrates the values of conscience, care, and commitment in line with the 4C (Competence, Conscience, Compassion, and Commitment) principles. The context of this lesson relates this material to the actual situation by reading the information in the credit brochure. Then, through the discussion process and working on the Student Worksheet, students can build an active and enjoyable learning experience. Processes of deep reflection and practical action, such as group discussion, help build students' social skills and self-confidence, allowing them to develop academically and personally.

This learning instills the spirit of lifelong learning by motivating students to view learning as a continuous process. By linking Annuity materials with practical applications such as financial management, students learn that the knowledge they acquire has long-term benefits in their personal and professional lives. This approach aligns with the spirit of Cura personalis, which emphasizes attention to each student's needs, potential, and individual experiences. Therefore, applying the Ignatian Pedagogy Paradigm through PBL helps students understand academic material and builds a person who is reflective, responsible, and ready to face future challenges.

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Validity Analysis of Digital Puzzle Game Media with a Realistic Mathematics Education Approach on Arithmetic Operations Material for Early Childhood

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Abstract. Early childhood in the era of digital development is in a unique process of growth and development. Children have growth and development patterns, thinking power, creativity, language and communication, which are included in intellectual intelligence, emotional intelligence, spiritual intelligence, according to the child's growth and development level. The purpose of the results of this study is to produce a product in the form of a digital puzzle game media with arithmetic operations material approach Realistic Mathematics Education for Kindergartens. The method in this article uses 4 stages, namely 1) potential and problems, 2) data collection, 3) product design, 4) design validation. The output that has been achieved in this study is that a valid digital puzzle game based of RME product is produced on validity tests from material experts and media experts. Based on the results of the average assessment obtained from material experts that is equal 89,43% and the average assessment of media experts is equal 91,64%, It can be concluded that the results of the digital puzzle game media assessment of material experts and media experts are classified as very worth it and the digital puzzle game based of RME media is suitable for use by kindergarten students at Kekancan Mukti and IT Harapan Bunda.

1 Introduction

The education that is really needed at this time is education that can integrate character education with education that can optimize the development of all dimensions of children (cognitive, physical, social-emotional, creative and spiritual). Education with this educational model is oriented towards the formation of children as complete human beings [1].

In the learning context in the Early Childhood Education (PAUD) environment, all activities require tools as learning media. The most effective tools for early childhood are educational games (APE). Playing is a very important way of learning for young children. [2] Through play, the activities that are expected to take place in PAUD are fun, interesting

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and meaningful for young children. For children, playing is their duty and also a means of learning.

Play activities help children to explore every object they encounter in the natural environment through observing objects, both living and inanimate objects. The closest way to a child's developmental phase to understand their world is through playing. Through play, children's cognitive development as one of the determinants in developing children's skills can be stimulated, so that children have the ability to solve every problem, think logically and symbolically, and can manage their minds. In Piaget's theory, play not only reflects the attitude of cognitive development, but also the child himself while contributing to the child's cognitive development [4]. The formation of knowledge, attitudes and skills as an implication of early childhood cognitive development cannot be separated from the role of teachers as educators. If the learning process through play is carried out by the teacher using game tools that are varied, fun and interesting, it will have strong potential for children's cognitive development, conversely if the teacher is not creative and innovative then the target for achieving children's cognitive development will not develop according to expectations.

Furthermore, according to the aim of puzzle games, namely that they can train accuracy, can develop children's cognitive, patience and concentration. Thus, the puzzle game is considered suitable for use by children aged 0-6 years, with different levels of difficulty. Apart from its unique shape, it also contains aesthetic value which makes children feel interested. According to Supartini, one of the benefits of playing puzzles is that it trains and helps [5] cognitive skills. If interpreted briefly, a puzzle game is a game of disassembly.

In the current digital era, puzzle games that can develop aspects of cognitive development can be done digitally using cellphones, laptops, tablets and other electronic devices. One of them is a puzzle game that can be played on a cellphone or laptop via the Children's puzzle game application which has been downloaded on an Android/cellphone.

As a result of observations that researchers have made, it is known that students do not understand the use of mathematics or what mathematics will be used for, especially in their daily lives. Apart from that, there are several things that influence education in schools, one of which is the learning method/approach/model [6]. It can be seen that teachers who still carry out learning use conventional learning, namely direct learning. This causes students to appear passive and not understand mathematics. This condition is then thought to be the cause of low student learning outcomes. Based on these apparent problems, the solution offered is using game-based media.

One of the approaches to learning that can be used by mathematics teachers is the Realistic Mathematic Education (RME) approach [7]. RME or Realistic Mathematics Education is an approach that was first introduced and developed in the Netherlands in 1970 by the Freudhenthal Institute [8]. This theory says that mathematics must be linked to reality and mathematics is a human activity. This means that mathematics taught by teachers should be related to the realities of life experienced by their students so that the knowledge taught is embedded in students and can be used to solve problems related to their daily lives or solve problems related to that knowledge in the field. Others [9] argue that the RME approach has the potential to improve students' mathematical understanding. If students' understanding of mathematics improves, their learning outcomes will also automatically increase, especially in the cognitive aspect.

The results of research by [10] show that the realistic mathematics learning approach is effective for student learning outcomes. Therefore, the Realistic Mathematic Education (RME) approach is an appropriate learning alternative that can combine real problems known to students and involve students' active role in learning so that it can improve students' mathematics learning outcomes.

Based on the problems explained above, the researcher formulated a problem, namely analysis of digital puzzle game media with a realistic mathematics education approach on arithmetic operations material for early childhood.

2 Methodology

Participants in this research were students at Kekancan Mukti Kindergarten and Harapan Bunda IT Kindergarten in Semarang. The population for this study included all Class B students, while the sample comprised students from conventional and media classes, with each sample consisting of 23 students.

The research design and procedure outlined in this article utilized a research and development approach. This study is classified as a type of learning media development [11]. The steps of the development research in this study are presented in the following chart:

Fig 1. Schematic of Research and Development Design

From the diagram above, the research steps consist of six stages there are potential and problems, data collection, product design, design validation.

This test was conducted with students from Kekancan Mukti Kindergarten and Harapan Bunda IT Kindergarten in Semarang, focusing on a limited set of arithmetic operations. A total of 23 children were selected for the initial trial of this learning media. This research was conducted over a period of eight months, from February to September 2024.

The data analysis technique used in this research is quantitative data analysis technique. The assessment data obtained from the validator is analyzed descriptively qualitatively and used as a reference for revising the product, thereby producing a viable product. The product design developed is assessed by validators using a validation sheet. The results of the assessment of all aspects are measured using a Likert Scale. The Likert scale is a number of positive or negative statements about an attitude object. The basic principle of the Likert scale is to determine the location of a person's position on a continuum of attitudes towards an attitude object ranging from very negative to very positive [12].

In this study, the answers to the instrument items were classified into five choices. Each indicator measured is given a score on a scale of 1-5, namely:

Table 1. instrument indicator measured

Indicator Score	Qualification Category
5	very good/very suitable/very appropriate/very clear

4	good/suitable/decent/clear
3	not good/not suitable/not suitable/not enough clear
2	not good/not appropriate/not appropriate/unclear
1	very not good/very inappropriate/very inappropriate/very unclear

The next step is to assess the feasibility of a learning media to be implemented in the arithmetic operations material at Kekancan Mukti Kindergarten and Harapan Bunda IT Kindergarten in Semarang. After the data is obtained, then look at the weight of each response and calculate the average score using the following formula.

$$\bar{x} = \frac{\sum x}{n} \quad (1)$$

Information:
$\bar{x}$: Average score
n : Total evaluation
$\sum x$: Total score of each

Then the formula for the percentage of results can be calculated using the following formula.

$$\text{Result} = \frac{Total\ score\ obtained}{maximum\ score} \times 100\% \quad (2)$$

Eligibility categories are based on the following criteria [13].

Table 2. Media eligibility criteria

No	Score in percent (%)	Qualification Category
1	< 21 %	Not really worth it
2	21 – 40 %	Not feasible
3	41 – 60 %	Decent Enough
4	61 – 80 %	Worth it
5	81 – 100 %	Very Worth It

3 Results and Discussions

The results achieved in this research are a digital puzzle game based on arithmetic operations material which has been validated by material experts and media experts. This product is the result of media innovation for early childhood because previously schools only used conventional puzzle media. The following are the results of digital puzzle game media products using the RME approach:

Fig 2. Digital puzzle game cover product with arithmetic operations material

Cover game puzzle digital menu display consists of play, on and off music and animation character. First the play button functions to see what material is on the media application. The two on and off music buttons function to play and turn off the instrumental music that is on the media application. The merging of the game puzzle digital display design using Corel Draw software is then processed by adding a script using adobe animate CC.

Fig 3. Material puzzles about getting arithmetic operations

In the material arithmetic operations, there are 5 puzzles to choose from. Each puzzle contains different material and questions. So that children don't get bored playing games. In the digital puzzle game arithmetic operations at the bottom right there is an exit button.

Fig 4. Digital puzzle game display arithmetic operations

In the game display there are 9 boxes containing puzzle pieces, each puzzle has numbers 1 to 9 and there are different colors. Children can practice and learn to name the numbers and colors in each box. At the bottom right there is also a menu button to select another puzzle again.

Fig 5. Display of digital puzzle game on arithmetic operations material

Display examples of puzzle results arithmetic operations. In the puzzle display there are 3 exercises that children can do. This question contains children's introduction to transportation and counting exercises.

The discussions results media application digital game puzzle has been validated using an questionnaire. The questionnaire obtained responses from material experts and media experts. The following assessment percentages are presented in the form stem diagram below:

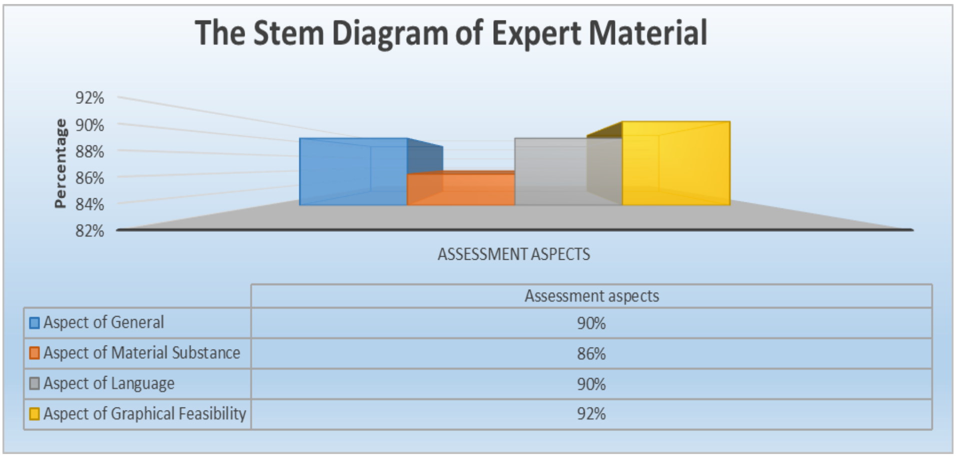

Fig 6. The stem diagram of expert material

Based on the percentage picture above, the material expert evaluates the application product that is 90% general aspect, which means that arithmetic operations materials is easy enough to be understood by students, 86% learning presentation aspect means that this teaching material is quite complete, there are examples of questions, practice questions and summaries, aspects of language 90%, which means the use of language in this teaching material is in accordance with the level of intellectual development of students, the feasibility aspect of the graph is 92%, which means the sentences and images in the arithmetic operations material are quite clear.

Based on the results of interviews with material experts, it is suggested that the material in digital puzzle games contains the practice questions in the digital puzzle game contain questions that are straightforward and easy for users to understand. The digital puzzle game already has a variety of practice questions on arithmetic operations about transportation that are quite good. In the arithmetic operation material, it is necessary to add a story about introducing transportation according to its type so that it will be more interesting. There needs to be additional animation about more transportation so that it will look more interactive. Furthermore, the following is results of the media expert questionnaire. The grading percentages are presented in the form stem diagram below:

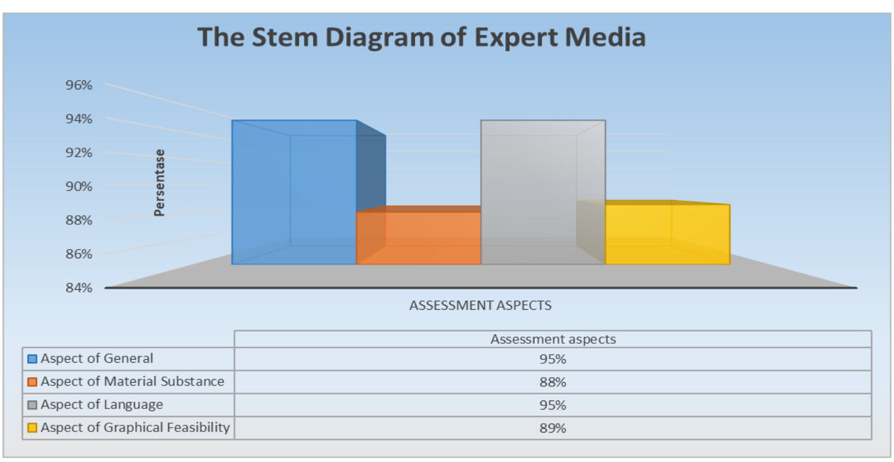

Fig 7. The stem diagram of expert media

Based on the diagram of the results of the adoption of media experts, the assessment of game puzzle digital media products is a general aspect of 95%, which means that the media of the application is classified as creative and innovative media, 88% of the learning presentation aspect means that the systematics and media presentation of the game puzzle digital media are good, the aspect of language feasibility is 95%, it means the level the use of language and language norms is reasonable, the feasibility aspect of 89% means that the appearance of the layout and color elements match the background requirements.

Based on the results of interviews with media experts, it was suggested that the use of color and animation in digital puzzle game media is sufficient and good. In the media there is instrumental music so that it will add variety to children's learning. The game is in apk format, so it can be applied to android mobile phone users. There should be rules for using the game so that users know the steps for using the game. There should be next and back buttons on each page so that users can go directly to the next page.

From the description of the assessment and interview above, the average result of an digital puzzle game media assessment obtained by material experts is 89,43%. While the average digital puzzle game media assessment by media experts is 91,64%. It can be concluded that the results of the digital puzzle game media assessment of material experts and media experts are classified as very worth it. Where as based on the results of the interview there needs to be revisions related to relating to the appearance and material but for the whole it is appropriate to be used on students. It can be concluded that the digital puzzle game media arithmetic operations material is valid and deserves to be tested on students.

4 Conclusions

The validation results by material and media experts assessment and interview above, the average result of an digital puzzle game media assessment obtained by material experts is 89,43%. While the average digital puzzle game media assessment by media experts is 91,64%. It can be concluded that the results of the digital puzzle game media assessment of material experts and media experts are classified as very worth it, which indicates that the digital puzzle game with arithmetic operations material application is categorized as feasible to be tested, according to the findings of the research and discussion of the issue. Based on the recommendations of media experts and subject matter experts, digital puzzle game media apps are suitable for usage by students are Kekancan Mukti kindergarten and IT Harapan Bunda kindergarten. These apps should include more exercise and display modifications animation to ensure that the maximum number of students can benefit from the media.

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The Mathematics of Finance: Pricing Volatility derivatives

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Abstract. In the increasingly complex world of financial markets, the scope of mathematical finance has expanded beyond traditional stock trading to include derivatives on various financial indices. The trading of stock derivatives has become commonplace across global markets. Furthermore, volatility derivatives, which are based on the volatility Index (VIX), have gained significant popularity in recent years. These instruments have been actively traded since the early 2000s. The objective of this article is to review some fundamental results on the pricing of basic volatility derivatives, under the Black-Scholes framework and other mathematical models.

1 Introduction

Financial mathematics focuses on the development of mathematical models and techniques to address complex problems in finance, to understand the dynamics of financial markets, and to devise strategies for decision-making. The main study of financial mathematics focuses on (i) pricing and (ii) portfolio optimization. Pricing involves determining the fair value of financial instruments, while portfolio optimization focuses on maximizing returns or minimizing risks under specific constraints. Together, these themes form the foundation for a wide range of applications, from portfolio management to risk assessment and beyond. An asset is any resource with economic value that an individual, corporation, or country owns, expecting it to provide future benefits. Examples of assets include real estate, gold, commodities, stocks, bonds, and intellectual property. People trade assets for numerous reasons, primarily to achieve financial goals. Among the numerous types of assets, stocks are particularly popular. A stock represents a share in the ownership of a company and entitles its holder to a portion of the company's profits, often distributed as dividends. Stocks are traded worldwide in stock markets such as the New York Stock Exchange (NYSE), the NASDAQ in the United States, the Tokyo Stock Exchange (TSE) in Japan, the Stock Exchange of Thailand (SET) in Thailand, and the Jakarta Composite Index of the Indonesia Stock Exchange (JCI) in Indonesia. To reflect the overall performance of markets, a stock index is commonly used as a statistical measure that represents the market's performance. It serves as a benchmark for analyzing market trends, evaluating economic conditions, and comparing the performance of individual investments or portfolios.

In addition to stocks, the financial market also deals with a class of financial instruments known as derivatives. A derivative is a contract whose value is derived from the performance

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of an underlying asset, such as stocks, bonds, commodities, or currencies [1, 2]. Common types of derivatives include options, futures, forwards, and swaps. Derivatives serve multiple purposes: they allow traders and institutions to hedge against price fluctuations, speculate on future price movements, and improve market efficiency by facilitating risk management. They enable market participants to manage exposure to various risks, such as interest rate changes, currency fluctuations, and commodity price volatility. For example, a farmer might use a futures contract to lock in the price of a crop, ensuring financial stability regardless of future market conditions. Similarly, investors can use options to gain leveraged exposure to an asset's potential upside while limiting downside risks.

Volatility, typically measured by the standard deviation of returns, reflects the extent of price fluctuations in a market index over a designated period and is a critical indicator of market risk and uncertainty. Increased volatility indicates higher price variation and a greater likelihood of significant market movements. Similar to stock indices, there is also a volatility index. The CBOE Volatility Index (VIX), introduced in 1993 by the Chicago Board Options Exchange (CBOE), gauges the market's expectations for near-term volatility. The original VIX was based on the implied volatility of eight at-the-money options on the S&P 100 Index (OEX). In 2003, the methodology was updated to focus on the broader S&P 500 Index (SPX). The VIX became a tradable financial instrument in 2004, with the introduction of VIX futures on the CBOE Futures Exchange (CFE). This marked the first time investors could directly trade on volatility expectations, transforming the VIX from a market indicator into a versatile tool for hedging and speculation. In 2006, VIX options were launched, further increasing its popularity among institutional and retail investors. The index gained significant global attention during the 2008 financial crisis, when it reached an all-time high of 89.53, reflecting extreme fear and market instability. The VIX has remained a key benchmark for assessing market volatility and managing financial risk. The white paper on the VIX, which includes how the index is calculated, can be found in [3].

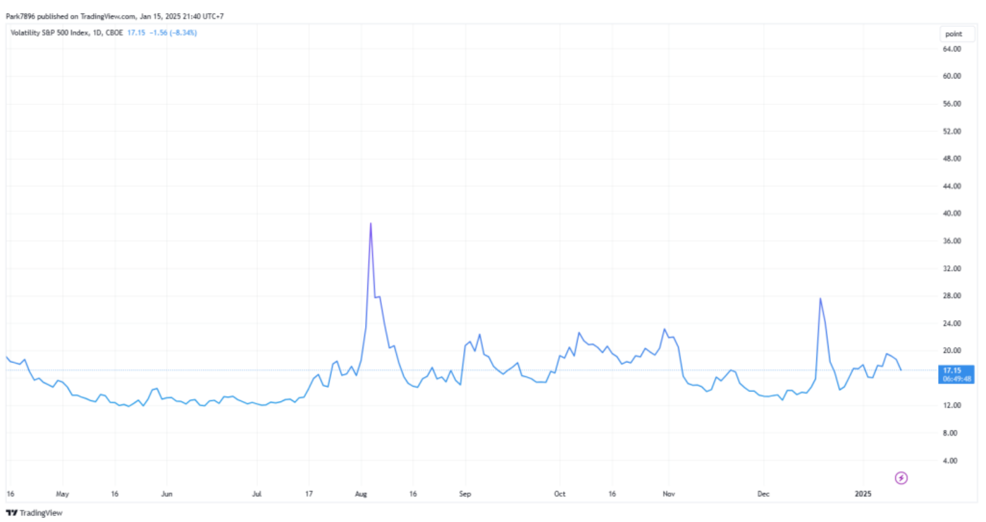

Fig. 1. CBOE Volatility index from www.tradingview.com [4]

The aim of this article is to provide the overview of volatility derivatives under VIX index. In Section 2, we provide fundamental results under Black-Scholes framework. In Section 3, we give the definitions of two fundamental volatility derivatives, variance swap and volatility swap, which are tradable under the CBOE markets. Then, we provide the pricing of the swaps under Black-Scholes assumption in Section 4. Finally, the pricing of volatility derivatives under non-Black-Scholes models are discussed in Section 5.

2 Black-Scholes model and derivative pricings

Black-Scholes model, introduced in 1973 by Fischer Black and Myron Scholes, is a fundamental model in Mathematical finance [1, 5]. The key idea is to assume that the price movement of an underlying asset, S_t, follows the geometric Brownian motion, i.e., the stochastic process $(S_t)_{t\geq 0}$ can be written in the stochastic differential form as:

$$dS_t = \mu S_t dt + \sigma S_t dW_t,\ S_0 = s_0 \tag{1}$$

The assumptions of Black-Scholes model are as follows:

1. The stock price follows geometric Brownian motion behavior with μ and σ constant.
2. The short selling of securities with full use of proceeds is permitted.
3. There are no transaction costs or taxes. All securities are perfectly divisible.
4. There are no dividends during the life of the derivative.
5. There are no riskless arbitrage opportunities.
6. Security trading is continuous.
7. The risk-free rate of interest is constant and the same for all maturities.

By following the Black-Scholes assumptions above, some interesting results follows [1, 2, 5]:

Proposition 1 Suppose that $(S_t)_{t\geq 0}$ follows the Black-Scholes assumptions. Then,

(a) The probability distribution of S_t is

$$S_t = s_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W_t}, \tag{2}$$

where $(W_t)_{t\geq 0}$ is a standard Wiener process.

(b) The probability distribution of ln (S_t) follows the normal distribution

$$\ln(S_t) \sim N\left(\ln(s_0) + \left(\mu - \frac{\sigma^2}{2}\right)t, \sigma^2 t\right). \tag{3}$$

(c) The probability distribution of S_t follows a lognormal distribution, and the expectation and variance of S_t are

$$E(S_t) = s_0 e^{\mu t},\ var(S_t) = s_0^2 e^{2\mu t}\left(e^{\sigma^2 t} - 1\right). \tag{4}$$

Theorem 2 (Black-Scholes equation) Let $f(t, S_t)$ be a derivative of an underlying asset S_t , and assume that Black-Scholes assumptions are satisfied, then

$$\frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 f}{\partial s^2} + r\frac{\partial f}{\partial s}\ S_t = rf, \tag{5}$$

where r is a continuous compound interest rate.

Theorem 3 (European Call and European Put option pricing formula) Assume that Black-Scholes assumptions are satisfied, then the call and put option prices at time t follows

$$c_t = S_t \Phi(d_1) - K e^{-r(T-t)} \Phi(d_2), \tag{6}$$

$$p_t = K e^{-r(T-t)} \Phi(-d_2) - S_t \Phi(-d_1), \tag{7}$$

where

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \tag{8}$$

$$d_2 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t} \tag{9}$$

and $\Phi(z) \coloneqq P(Z \le z)$ where Z is a standard normal random variable.

3 VIX and volatility derivatives

In this section, we provide the definitions of variance swap and volatility swap which are examples of volatility derivatives. Both derivatives are tradable in real markets and the payoffs of both derivatives depend on the VIX index.

We define the forward contract on future realized variance as *variance swap*. Similarly, the forward contract on future realized volatility is called *volatility swap*. To be precise, we first define the log-return of an underlying asset as

$$R_i = \ln(S_{t_i}) - \ln(S_{t_{i-1}}). \tag{10}$$

Next, the (discretely-sampled) *realized variance* of the return at time T is calculated by

$$RV_d(0,N,T) = \frac{1}{N\Delta t} \sum_{i=1}^{N} (R_i)^2 \times 100^2 \tag{11}$$

Note that $N\Delta t = T$. Theoretically, we may set Δt as a very small number. However, in practice, Δt is normally set to be the length of one day which equals to 1/252.

The payoffs of variance swap and volatility swap [6] are, respectively,

$$V_T = (RV(0,N,T) - K_{var}) \times L, \tag{12}$$

where K_{var} is the fixed delivery amount and L is a notational amount, and

$$VOL_T = \left(\sqrt{RV(0,N,T)} - K_{vol}\right) \times L. \tag{13}$$

By the risk neutral measure technique, it can be seen that the fair prices of the swaps are

$$K_{var} = E_0^Q\big(RV(0,N,T)\big), \; K_{vol} = E_0^Q\left(\sqrt{RV(0,N,T)}\right). \tag{14}$$

Suppose that the log-return of the realized variance and the realized volatility are considered.Then, it can be shown that

$$RV(0,N,T) = \frac{1}{T} \sum_{i=1}^{N} \left(\ln\left[\frac{S_{t_i}}{S_{t_{i-1}}}\right]\right)^2 \times 100^2, \tag{15}$$

$$\sqrt{RV(0,N,T)} = \sqrt{\frac{1}{T} \sum_{i=1}^{N} \left(\ln\left[\frac{S_{t_i}}{S_{t_{i-1}}}\right]\right)^2 \times 100^2}. \tag{16}$$

Hence, the fair prices of variance and volatility swaps are

$$K_{var} = \sum_{i=1}^{N} E_0^Q \left[\left[\frac{1}{T} \times \ln\left(\frac{S_{t_i}}{S_{t_{i-1}}}\right) \right]^2 \times 100^2 \right], \tag{17}$$

$$K_{vol} = E_0^Q \left(\sqrt{\frac{1}{T} \sum_{i=1}^{N} \left(\ln\left[\frac{S_{t_i}}{S_{t_{i-1}}}\right] \right)^2 \times 100^2} \right). \tag{18}$$

Note that the first quantity is linear, while the second quantity is non-linear. It can be seen that the first calculation requires knowledge of the joint PDF of S_{t_i} and $S_{t_{i-1}}$ while the second calculation requires knowledge of the joint PDF of $S_{t_1}, \ldots, S_{t_N}$

4 Pricing Volatility derivatives under Black-Scholes model

Assume that an underlying asset S_t follows Black-Scholes model:

$$dS_t = rS_t dt + \sigma S_t dW_t, \; S_0 = s_0 \tag{19}$$

where r and σ>0 are risk-free interest rate and price volatility respectively. Then, the fair prices of K_{var} and K_{vol} are as follows:

Theorem 4 [6] Suppose that an underlying asset S_t follows the Black-Scholes assumptions. Then,

(a) The fair price for discretely sample realized variance at the return at time $T$is

$$K_{var} = 100^2 \sigma^2 (1 + \delta), \tag{20}$$

where,

$$\delta = \frac{\left(\left(r - \frac{\sigma^2}{2}\right)\Delta t\right)^2}{\sigma^2 \Delta t}. \tag{21}$$

(b) The fair price for discretely sample realized volatility at the return at time $T$is

$$K_{vol} = \sqrt{\alpha} \sqrt{\frac{\pi}{2}} L_{\frac{1}{2}}^{\left(\frac{N}{2}-1\right)}(-\lambda^2/2\,), \tag{22}$$

where,

$$\alpha = \frac{100^2 \sigma^2 \Delta t}{T}, \lambda = \sqrt{\delta\left((r - \sigma^2/2)\Delta t\right)^2 / \sigma^2 \Delta t}, \tag{23}$$

and $L_b^{(a)}$ is the generalized Laguerre function.

Proof(Sketch)

(a) From Proposition 1, the explicit solution of S_t under Black-Scholes model is

$$S_{t_i} = S_{t_{i-1}} e^{\left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\left(W_{t_i} - W_{t_{i-1}}\right)}. \tag{24}$$

This implies that

$$\ln\left[\frac{S_{t_i}}{S_{t_{i-1}}}\right] = \left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\left(W_{t_i} - W_{t_{i-1}}\right) \sim N\left(\left(r - \frac{\sigma^2}{2}\right)\Delta t, \sigma\Delta t\right). \quad (25)$$

Since the square of normal random variable follows a chi-squared distribution. Hence, we have $\ln^2\left[\frac{S_{t_i}}{S_{t_{i-1}}}\right]$ follows chi-squared distribution. In other words, by defining Y_i as

$$Y_i = \frac{1}{\sigma^2\Delta t}\ln^2\left[\frac{S_{t_i}}{S_{t_{i-1}}}\right] \quad (26)$$

Then, we have Y_i follows the chi square distribution with 1 degree of freedom and with non-centrality parameter. That is,

$$Y_i \sim \chi_1^2(\delta), \quad (27)$$

where

$$\delta = \frac{\left(\left(r - \frac{\sigma^2}{2}\right)\Delta t\right)^2}{\sigma^2\Delta t}. \quad (28)$$

Note that Y_i is independent of i. Therefore, the realized variance can be expressed as

$$RV(0, N, T) = \frac{1}{T}\sum_{i=1}^{N}\ln^2\left[\frac{S_{t_i}}{S_{t_{i-1}}}\right] \times 100^2 = \alpha\sum_{i=1}^{N} Y_i, \quad (29)$$

where $\alpha = 100^2\sigma^2\Delta t/T$. Since $E_0^Q(Y_i) = 1 + \delta$, we get

$$K_{var} = E_0^Q\big(RV(0, N, T)\big) = \alpha\sum_{i=1}^{N} E_0^Q(Y_i) = \alpha N(1 + \delta). \quad (30)$$

Replacing the value of α and let $N = T/\Delta t$ provide the required result.

(b)For K_{vol} we have

$$K_{vol} = E_0^Q\left(\sqrt{RV(0, N, T)}\right) = \sqrt{\alpha}E_0^Q\left(\sqrt{\sum_{i=1}^{N} Y_i}\right), \quad (31)$$

where

$$Y_i \sim \chi_1^2(\delta), \delta = \big((r - \sigma^2/2)\Delta t\big)^2/\sigma^2\Delta t\,, \alpha = 100^2\sigma^2\Delta t/T. \quad (32)$$

Using the independent increment property of Brownian motion, we can conclude that $Y_i \sim \chi_1^2(\delta), i = 1, \dots, N$ are independent. This impies $\sum_{i=1}^{N} Y_i \sim \chi_N^2(N\delta)$. Moreover,

$$\sqrt{\sum_{i=1}^{N} Y_i} \sim \chi_N(\delta) \quad (33)$$

is a non-central chi random variable with N degree of freedom and non-centrality parameter $\lambda = \sqrt{\delta}$. By using the property of non-central chi random variables,we have

$$E_0^Q\left(\sqrt{\sum_{i=1}^{N} Y_i}\right) = \sqrt{\frac{\pi}{2}}L_{\frac{1}{2}}^{\left(\frac{N}{2}-1\right)}\left(-\frac{\lambda^2}{2}\right), \quad (34)$$

where $L_b^{(a)}$ is the generalized Laguerre function. Hence,

$$K_{vol} = E_0^Q\left(\sqrt{RV(0,N,T)}\right) = \sqrt{\alpha}\sqrt{\frac{\pi}{2}}L_{\frac{1}{2}}^{\left(\frac{N}{2}-1\right)}(-\lambda^2/2\,). \tag{35}$$

Next, we consider the time-varying parameter Black-Scholes model. Under a risk neutral probability space $(\Omega,\mathcal{F},Q)$ and filtration $(\mathcal{F}_t)_{t\geq 0}$, the model can be written in SDE form as

$$dS_t = r(t)S_t dt + \sigma(t)S_t dW_t,\ S_0 = s_0 \tag{36}$$

where $r(t)$ and $\sigma(t)$>0 are risk-free interest rate and price volatility respectively. To obtain K_{var}, let us follow the proof instruction from Theorem 4. Note that the explicit solution of time-varying Black-Scholes model is

$$S_{t_i} = S_{t_{i-1}}e^{\int_{t_{i-1}}^{t_i}\left(r(s)-\frac{1}{2}\sigma^2\right)ds+\int_{t_{i-1}}^{t_i}\sigma(s)dW_s}. \tag{37}$$

This implies that

$$\ln\left[\frac{S_{t_i}}{S_{t_{i-1}}}\right]\sim N(\mu_i,\sigma_i^2), \tag{38}$$

where,

$$\mu_i = \int_{t_{i-1}}^{t_i}\left(r(s)-\frac{1}{2}\sigma^2(s)\right)ds,\ \sigma_i^2 = \int_{t_{i-1}}^{t_i}\sigma(s)ds. \tag{39}$$

It can be seen that

$$Y_i = \frac{1}{\sigma_i^2}\ln^2\left[\frac{S_{t_i}}{S_{t_{i-1}}}\right]\sim\chi_1^2(\,\delta_i) \tag{40}$$

with 1 degree of freedom and non-centrality parameter $\delta_i = \mu_i/\sigma_i^2$. Note that $Y_i\sim\chi_1^2(\,\delta_i)$ does not depend on *i*. Therefore, we can write

$$RV(0,N,T) = \frac{1}{T}\sum_{i=1}^N\left(\ln\left[\frac{S_{t_i}}{S_{t_{i-1}}}\right]\right)^2\times 100^2 = \sum_{i=1}^N\alpha_i Y_i, \tag{41}$$

where $Y_i\sim\chi_1^2(\,\delta_i)$ and $\alpha_i = 100^2\sigma_i^2/T$. Since $E_0^Q(Y_i) = 1+\delta$, we get

$$K_{var} = E_0^Q\big(RV(0,N,T)\big) = \sum_{i=1}^N\alpha_i E_0^Q(Y_i) = \sum_{i=1}^N\alpha_i\,(1+\delta_i). \tag{42}$$

Therefore, the formula of K_{var} for time-varying Black-Scholes model can be obtained. Next, we consider

$$K_{vol} = E_0^Q\left(\sqrt{RV(0,N,T)}\right) = E_0^Q\left(\sqrt{\sum_{i=1}^N\alpha_i Y_i}\right), \tag{43}$$

Using the independent increment property of Brownian motion, it can be concluded again that $Y_i\sim\chi_1^2(\,\delta_i), i = 1,\ldots,N$ are independent. However, when α_i are not equal, the distribution of $\sum_{i=1}^N\alpha_i Y_i$ is unknown This makes K_{vol} difficult to derive in an explicit form and some advance Mathematical technique are required. The full proof of K_{vol} can be seen in [6].

5 Volatility derivatives under non Black-Scholes model

The pricing of variance and volatility swaps has been extensively studied in various financial models over the years. In 2008, Broadie and Jain [7] analyzed variance swap pricing within the frameworks of Black-Scholes model, Heston stochastic volatility model, Merton jump diffusion model, and Bates and Scott stochastic volatility and jump models. Itkin and Carr (2010) [8] proposed an asymptotic method for pricing variance swaps in the Lévy model. Zhu and Lian (2011) [9] employed a PDE-based approach for pricing variance swaps in the Heston model. Elliott and Lian (2013) [10] extended this by incorporating regime-switching into the Heston model, using forward characteristic functions to price variance and volatility swaps. Zheng and Kwok (2014) [11] examined variance swaps in stochastic volatility models with simultaneous jumps. Yuen *et al.* (2016) [12] focused on pricing variance swaps in the 3/2-stochastic volatility model with jumps. In 2018, He and Zhu [13] explored variance swaps in the Heston–CIR hybrid model, where both volatility and interest rates are stochastic. Liu and Zhu (2019) [14] investigated variance swaps in the Hawkes jump-diffusion process characterized by stochastic volatility and clustered jumps, while He and Zhu (2019) [15] addressed variance and volatility swaps in a two-factor stochastic volatility model with regime switching. Yang *et al*. (2019) [16] considered volatility swaps in stochastic volatility models with jumps and stochastic intensity. More recently, Rujivan and Rakwongwan (2021) [17] analyzed variance and volatility swaps in the Black-Scholes model with a time-varying risk-free interest rate, and Rujivan (2022) [6] extended this work to include time-varying parameters in the Black-Scholes framework.

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Numerical Investigation of β-Amyloid Aggregation Process In Alzheimer's Disease

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Abstract. The process of β-amyloid (Aβ) protein aggregation in Alzheimer's disease can be represented by a mathematical model. The model is built based on several groups representing the concentrations of a number of monomer and the concentration of oligomer, which is presented as a system of nonlinear ordinary differential equations. In this paper, the extinction and interior equilibrium point are also determined. The model will be solved numerically by the fourth-order Runge-Kutta method and it is shown that the solution will lead to the equilibrium point obtained. Some numerical investigations are done by analyzing the effect of monomer number that occur. It can be concluded that the greater the number of monomers considered, the lower the concentration of oligomers, so that it can reduce the symptoms of Alzheimer's disease. Moreover, for the greater capacity of the Aβ protein, the faster monomers aggregation process occurs.

1 Introduction

Degenerative disease is a disease in which the function or structure of tissues or organs deteriorates over time. The disease is classified into three main groups, namely cardiovascular, neoplastic, and nervous system. Some degenerative diseases cannot be cured, even in some countries, degenerative diseases are the main cause of death [1]. These diseases are caused not by bacteria or viruses but rather by protein abnormalities. Proteins are essential for the organism because they participate in virtually every process within the cell. Therefore, if they do not function normally, the consequences can be devastating [2]. One example of a degenerative disease in the nervous system group is Alzheimer. Alzheimer's disease (AD) is the most common form of dementia. This disease causes a gradual decline in cognitive abilities, often starting with memory loss. This disease attacks the elderly who are usually 65 years of age or older, and twice more women than men [3]. Currently, there is no cure for AD or stop its progression. Many clinical trials of drugs aimed at preventing or eliminating symptoms of protein aggregation in the brain have failed to show efficacy. For now, the only treatment for AD is with drugs used to treat the symptoms of AD. This makes the cost of care and treatment quite large for patients and their families, also for the government, and for researchers. For example, in 2015, there were more than 5 million people in the United States with AD and the cost of caring for AD patients in the U.S. was estimated at $226 billion for 2015 [3]. Several factors that trigger the high cost of care and treatment are due to a lack of

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understanding about this disease which results in a lack of resources and training for caregivers of people with dementia.

AD is characterized by several things, one of which is the abnormal aggregation of Aβ protein in the brain. This process of aggregation start when Aβ split and form a single piece (monomer). Then monomers thought to combine and form Aβ oligomers, and finally the oligomers will form insoluble fibrils. These fibrils continue to aggregate and become plaques, which is one of the causes of the symptoms of AD [4]. Therefore, a clearer understanding of the this aggregation process of Aβ is needed, in order to understand the disease and find alternative treatments to reduce the symptoms of the disease.

The process of this aggregation can be studied with a mathematical model. Hao and Friedman developed a mathematical model of AD to investigate the effect of drugs in clinical trials. The model consists of partial differential equations and consider Aβ aggregation and hyperphosphorylated tau proteins [3]. Dayeh, et al. proposed a discrete-time mathematical model for the aggregation of Aβ monomers into oligomers based on chemical kinetics and population dynamics concepts, and construct a formula to derive the number of monomers which produce oligomers [5]. Using the same concepts, Ackleh, et al. developed a continues mathematical model for the aggregation of Aβ, which consits of ordinary differential equations and compare the two-discrete time approximation with numerical solutions of the model [4].

In this paper, we will investigate numerically the continues mathematical model of the Aβ protein aggregation in AD, which only consider the formation of oligomers. The model is solved using the fourth order Runge-Kutta method. The effect of monomer number on oligomer formation and capacity of the Aβ protein are investegeted.

2 Governing Equations

The basic continues model of the Aβ protein aggregation considered in this paper based on some assumptions, as follows: The modeled phase is a slower nucleation phase according to the structured size, considering the concentration of 1-monomer (M_1), 2-monomers (M_2), ..., 6-monomers (M_6); Oligomers are formed from at least 7 monomers; Aggregation (nucleation and elongation) occurs through the addition of monomers; Dissociation (reduction of monomers from natural aggregation or drug effects) is ignored; Fibril fragmentation, which will produce many fibrils and further accelerate their spread, is ignored; Monomer production is represented by a saturation function which is a logistic differential equation [4]; There is monomer degradation with rates $\mu_i, i = 1,2,...,6$ and oligomer degradation with rates μ_O; From these assumptions, the Aβ aggregation process can be illustrated with diagram as follows:

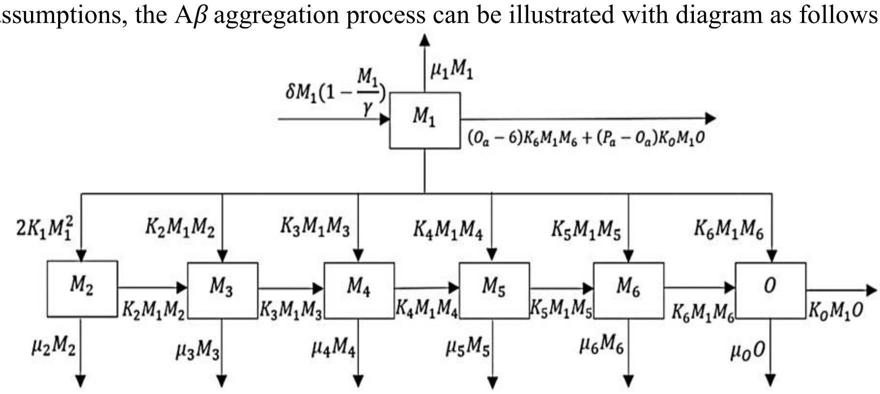

Fig. 1. The Aβ aggregation process.

where $K_i, i = 1,2,\ldots,6$ denote nucleation rate for i monomers, K_O is elongation rate, O_a and P_a are average of oligomer and fibril size, respectively, δ is the growth rate of monomers and γ denotes the carrying capacity.

From Fig. 1, it can be developed a system of ordinary differential equations, i.e.

$$\frac{dM_1}{dt} = \delta M_1(1-\frac{M_1}{\gamma}) - 2K_1M_1^2 - M_1(K_2M_2 + K_3M_3 + K_4M_4 + K_5M_5)$$
$$-(O_a-6)K_6M_1M_6 - (P_a - O_a)K_OM_1O - \mu_1M_1 \tag{1}$$
$$\frac{dM_2}{dt} = K_1M_1^2 - K_2M_1M_2 - \mu_2M_2 \tag{2}$$
$$\frac{dM_3}{dt} = K_2M_1M_2 - K_3M_1M_3 - \mu_3M_3 \tag{3}$$
$$\frac{dM_4}{dt} = K_4M_1M_4 - K_4M_1M_4 - \mu_4M_4 \tag{4}$$
$$\frac{dM_5}{dt} = K_5M_1M_5 - K_5M_1M_5 - \mu_5M_5 \tag{5}$$
$$\frac{dM_6}{dt} = K_6M_1M_6 - K_6M_1M_6 - \mu_6M_6 \tag{6}$$
$$\frac{dO}{dt} = K_6M_1M_6 - K_OM_1O - \mu_OO \tag{7}$$

The factors $(O_a - 6)$ and $(P_a - O_a)$ in Eq. (1) represent the average number of monomers which is needed M_6 to form an oligomer ad the average number of monomers which is needed an oligomer to form a fibril, respectively [4].

3 Existence of Equilibrium Points

Following [4], the existence of equilibrium points of the model is stated in the following theorem:

Theorem 1. Consider the model stated in Eqs. (1)-(7).

(1) If $\delta - \mu_1 \leq 0$, then the model has only the extinction equilibrium.

(2) If $\delta - \mu_1 > 0$, then the model can also have a unique positive interior equilibrium.

The extinction equilibrium point occurs when the monomer stops aggregating. In this condition, there is no more oligomer formation or it can be said that the concentration of oligomer is heading towards zero. Whereas the positive single interior equilibrium point occurs when there is still growth of monomers that aggregate.

The equilibrium points can be obtained by setting the Eqs. (1)-(7) as

$$\delta M_1(1-\frac{M_1}{\gamma}) - 2K_1M_1^2 - M_1(K_2M_2 + K_3M_3 + K_4M_4 + K_5M_5)$$
$$-(O_a-6)K_6M_1M_6 - (P_a - O_a)K_OM_1O - \mu_1M_1 = 0 \tag{8}$$
$$K_1M_1^2 - K_2M_1M_2 - \mu_2M_2 = 0 \tag{9}$$
$$K_2M_1M_2 - K_3M_1M_3 - \mu_3M_3 = 0 \tag{10}$$
$$K_4M_1M_4 - K_4M_1M_4 - \mu_4M_4 = 0 \tag{11}$$
$$K_5M_1M_5 - K_5M_1M_5 - \mu_5M_5 = 0 \tag{12}$$
$$K_6M_1M_6 - K_6M_1M_6 - \mu_6M_6 = 0 \tag{13}$$
$$K_6M_1M_6 - K_OM_1O - \mu_OO = 0 \tag{14}$$

Suppose the extinction equilibrium point is $E^0 = (M_1^0, M_2^0, M_3^0, M_4^0, M_5^0, M_6^0, O^0)$. In the case of extinction, no more M_1 is added to the monomer aggregation, so we get that $M_1 = 0$. Substitute the value into Eqs. (8)-(14) we can obtained $E^0 = (0,0,0,0,0,0,0)$. Suppose the interior equilibrium point is $M^* = (M_1^*, M_2^*, M_3^*, M_4^*, M_5^*, M_6^*, O^*)$ The interior equilibrium point indicates a condition where the monomer concentration is heading towards a value greater than zero. This condition occurs because the monomer continues to increase due to the aggregation process, but also decreases due to the degradation processes. From Eqs. (9), we can derived the component of interior equilibrium for M_2^*, that is

$$M_2^* = \frac{K_1(M_1^*)^2}{(K_2M_1^* + \mu_2)} \tag{15}$$

Substitute Eq. (15) into Eqs. (10)-(14) we can obtain the component of interior equilibrium for $M_3^*, M_4^*, M_5^*, M_6^*, O^*$ as follows

$$M_3^* = \frac{K_1K_2(M_1^*)^3}{(K_2M_1^* + \mu_2)(K_3M_1^* + \mu_3)} \tag{16}$$

$$M_4^* = \frac{K_1K_2K_3(M_1^*)^4}{(K_2M_1^* + \mu_2)(K_3M_1^* + \mu_3)(K_4M_1^* + \mu_4)} \tag{17}$$

$$M_5^* = \frac{K_1K_2K_3K_4(M_1^*)^5}{(K_2M_1^* + \mu_2)(K_3M_1^* + \mu_3)(K_4M_1^* + \mu_4)(K_5M_1^* + \mu_5)} \tag{18}$$

$$M_6^* = \frac{K_1K_2K_3K_4K_5(M_1^*)^6}{(K_2M_1^* + \mu_2)(K_3M_1^* + \mu_3)(K_4M_1^* + \mu_4)(K_5M_1^* + \mu_5)(K_6M_1^* + \mu_6)} \tag{19}$$

$$O^* = \frac{K_1K_2K_3K_4K_5K_6(M_1^*)^7}{(K_OM_1^* + \mu_O)(K_2M_1^* + \mu_2)(K_3M_1^* + \mu_3)(K_4M_1^* + \mu_4)(K_5M_1^* + \mu_5)(K_6M_1^* + \mu_6)} \tag{20}$$

The value $M_1^*$of can be determined by substituting Eqs. (15)-(20) into Eq. (8) and solve it for M_1^*. Moreover, from those equations, it can be concluded that the component of interior equilibrium for $M_2^*, M_3^*, \dots, M_n^*$ have formula:

$$M_i^* = \frac{K_{i-1}M_1^*M_{i-1}^*}{\prod_{j=2}^{i}(K_jM_1^* + \mu_j)} \tag{21}$$

for i = 2,3,...,n.

4 Numerical Simulations and Discussions

4.1 Case of extinction

In this subsection, we will simulate the numerical solution of the mathematical model of Aβ protein aggregation in AD, in case of extinction. The model solved numerically using the fourth-order Runge-Kutta method [6]. For this simulation, we use the initial values as follows: $M_1(0) = 10, M_2(0) = M_3(0) = M_4(0) = M_5(0) = M_6(0) = O(0) = 0$. Since it is expected that the degradation rate will decrease as the monomer stack size increases, while the nucleation rate will increase as the monomer aggregation size increases, to select the values of μ_i and K_i, for i = 2, 3, 4, 5, 6, the relationship that meets these criteria is chosen, i.e. $\mu_i = \mu_{i\text{-}1}$ and $K_i = K_{i-1} + \epsilon$, with $\epsilon > 0$ [4]. Consider $\delta = 10^{-4}$, $\epsilon = 10^{-3}$, $K_1 = 10^{-4}$, $K_O = 10^{-1}$, $\gamma = 75$, $O_a = 10$, $P_a = 100$, $\mu_1 = 10^{-3}$, $\mu_O = 10^{-5}$. The solution of the model can be seen in Fig. 2.

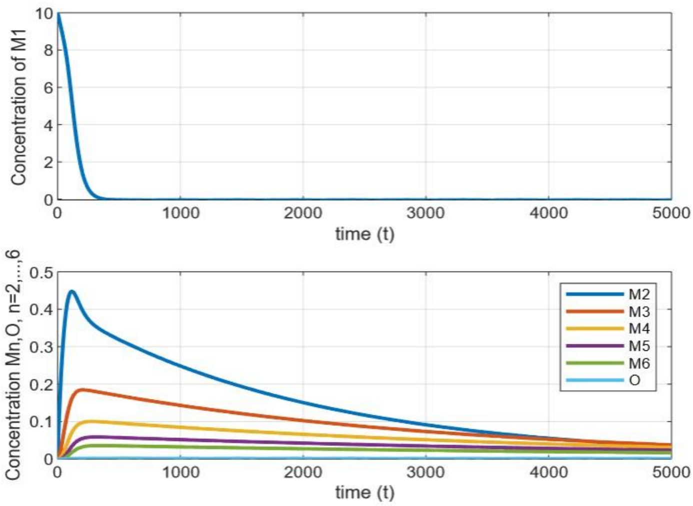

Fig. 2. Numerical solution of Aβ aggregation process, in case of extinction.

From Fig. 2, it can be seen that, with the value of $\delta - \mu_1 < 0$, the concentrations of M_1 decrease relatively quickly towards 0. Whereas, the $M_i, i = 1,2,...,6$ and O concentration all initially increase but then go towards 0, which shows that for a long time there will be no more aggregation of Aβ protein.

4.2 Case of interior equilibrium

Suppose we set the value $\delta = 40$, and the other parameter values are the same as those used in the extinction case. Then $\delta - \mu_1 > 0$. The solution graph for this case can be seen in Fig. 3. Based on Fig. 3(left), concentration of M_1 increase rapidly at $t = 0$ to $t = 0.44$ and a fairly slow decrease when $t = 0.441$ to $t = 100$. This increase occurs because of the aggregation of 1-monomer with other monomers which will form other aggregation. While the decrease occurs due to monomer degradation. After sustain an increase, it will then have a decrease towards the equilibrium point component for the concentration M_1, that is $M_1^* = 73.9$. Furthermore, from Fig. 3(right) it can be seen that the concentrations of M_2–M_6 and O increase until they finally reach their interior equilibrium point, according to Eqs. (15)-(20). This increasing occurs because of the aggregation of 1-monomer with other monomers from each group. It can also be seen that the longer the aggregation of monomers, the lower it will be.

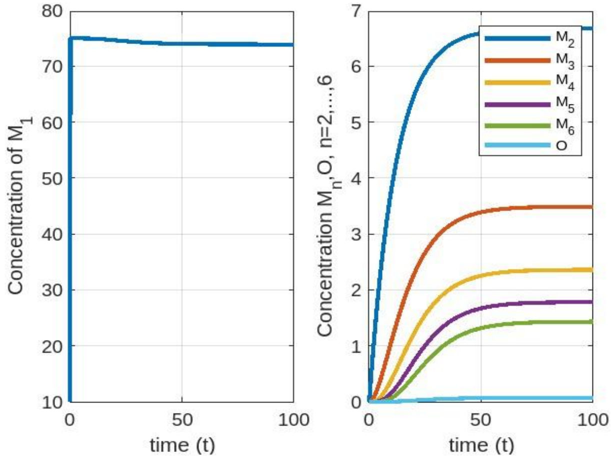

Fig. 3. Numerical solution of Aβ aggregation process, in case of interior equilibrium.

4.3 Number of monomers's effect

In the Aβ aggregation process, the formation of oligomer depends on number of monomers. Reixach, et.al. [7] showed that oligomers are formed from the aggregation of at most six monomers. In this subsection, we investigate the effect of the number of monomers in oligomers forming. The basic model is modified by reducing differential equations related to $M_2 - M_5$, denotes oligomers are formed from the aggregation of 2-monomers until 5-monomers, respectively. The effect of the number of monomers can be seen in Fig. 4. The larger the value of n, the less oligomers will be formed.

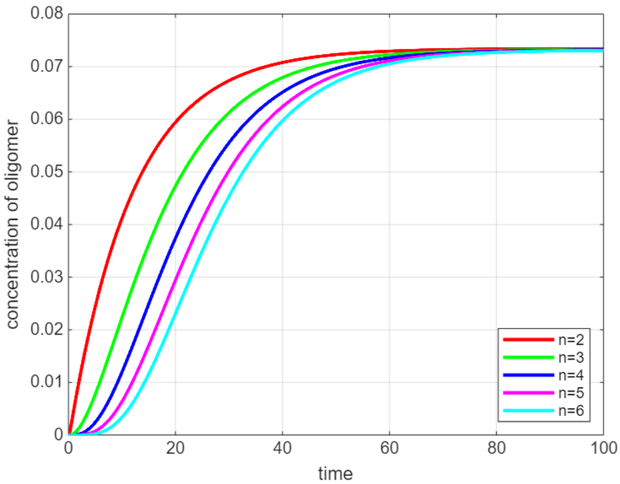

Fig. 4. Concentration of oligomers with various number of monomers.

4.4 Capacity of the Aβ protein

In this section, the effect of the capacity of the β-amyloid protein (γ) on the monomer aggregation process will also be investigated. By using several values of γ, changes in monomer concentration can be observed in Fig. 5. It can be seen that for the greater γ, it makes the concentration of M_1 more higher.

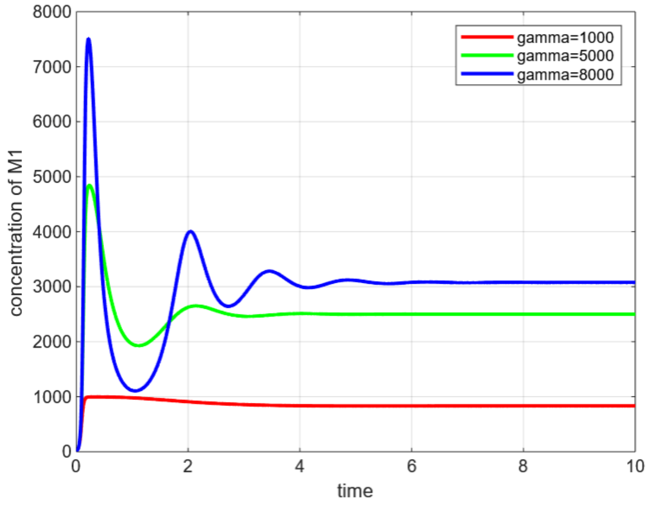

Fig. 5. Concentration of M_1 with various capacity of the β-amyloid.

5 Conclusion

Numerical investigations were conducted on the model of β-amyloid aggregation in Alzheimer's disease. It can be observed that the number of monomers required to form oligomers will affect the concentration of the oligomers themselves. The more monomers required, the lower the concentration of oligomers. In addition, the greater the capacity of the beta amyloid protein, the faster the monomer aggregation process occurs.

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Occupation Times of Fractional Brownian Motion as White Noise Distributions

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Abstract. Occupation times of a stochastic process models the amount of time the process spends inside a spatial interval during a certain finite time horizon. It appears in the fiber lay-down process in nonwoven production industry. The occupation time can be interpreted as the mass of fiber material deposited inside some region. From application point of view, it is important to know the average mass per unit area of the final fleece. In this paper we use white noise theory to prove the existence of the occupation times of one-dimensional fractional Brownian motion and provide an expression for the expected value of the occupation times.

1 Introduction

Technical textiles have attracted great attention to diverse branches of industry over the last decades due to their comparatively cheap manufacturing. By overlapping thousands of individual slender fibers, random fiber webs emerge yielding nonwoven materials that find applications e.g. in textile, building and hygiene industry as integral components of baby diapers, closing textiles, filters and medical devices, to name but a few. They are produced in melt-spinning operations: hundreds of individual endless fibers are obtained by the continuous extrusion of a molten polymer through narrow nozzles that are densely and equidistantly placed in a row at a spinning beam. The viscous or viscoelastic fibers are streched and spun until they solidify due to cooling air streams. Before the elastic fibers lay down on a moving conveyor belt to form a web, they become entangled and form loops due to the highly turbulent air flows. The homogeneity and load capacity of the fiber web are the most important textile properties for quality assesment of industrial nonwoven fabrics. The optimization and control of the fleece quality require modeling and simulation of fiber dynamics and lay-down. Available data to judge the quality, at least on the industrial scale, are usually the mass per unit area of the fleece.

Since the mathematical treatment of the whole process at a stroke is not possible due to its complexity, a hierarchy of models that adequately describe partial aspects of the process chain has been developed in research during the last years. A stochastic model for the fiber deposition in the nonwoven production was proposed and analyzed in [1-4]. The model is based on stochastic differential equations describing the resulting position of the fiber on the belt under the influence of turbulent air flows. In [5] parameter estimation of the Ornstein-

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Uhlenbeck process from available mass per unit area data, the occupation time in mathematical terms, was done.

Definition 1.

Let $X = (X_t)_{t\in[0,T]}$, $T > 0$, be a stochastic process and consider an interval $[a, b] \subseteq \mathbb{R}$. The occupation time $M_{T,[a,b]}(X)$ is defined as

$$M_{T,[a,b]}(X) \coloneqq \int_0^T 1_{[a,b]}(X_t)\, dt = \int_0^T \int_a^b \delta(X_t - x)\, dx\ dt. \tag{1}$$

Here, $1_{[a,b]}$ denoted the indicator function of the interval $[a, b]$ and δ is the Dirac-delta distribution.

Formally, occupation times models the time the stochastic process spends inside the spatial interval $[a, b]$ during the time interval $[0, T]$. In terms of our physical model for the nonwoven production, the occupation time can be interpreted as the mass of fiber material deposited inside the interval $[a, b]$, i.e. the mass per unit area of the final fleece.

Motivated by the above mentioned problem, in [6-7] the occupation times of one-dimensional Brownian motion were studied. In particular, it has been proved that occupation times of one-dimensional Brownian motion is a white noise distribution in the sense of Hida. In the present paper, we extend these results to occupation time of one-dimensional fractional Brownian motion. Although it is possible to study the problem by classical probabilistic method, we use a white noise approach to generalize the results also to higher dimensions in later research. In the next section we provide neccesary background on the white noise theory. The main result together with its proof are given afterward.

2 White Noise Theory

In this section we give background on the white noise theory used throughout this paper. For a more comprehensive discussions including various applications of white noise theory we refer to [8-10] and references therein. We start with the Gelfand triple

$$S(\mathbb{R}) \hookrightarrow L^2(\mathbb{R}) \hookrightarrow S'(\mathbb{R}) \tag{2}$$

where $S(\mathbb{R})$ is the space of real-valued Schwartz test function, $S'(\mathbb{R})$ is the space of real-valued tempered distributions, and $L^2(\mathbb{R})$ is the real Hilbert space of all real-valued Lebesgue square-integrable functions. Next, we construct a probability space $(S'(R), B, \mu)$ where B is the Borel σ-algebra generated by cylinder sets on $S'(\mathbb{R})$ and the unique probability measure μ is established through the Bochner-Minlos theorem by fixing the characteristic function

$$C(f) \coloneqq \int_{S'(\mathbb{R})} \exp(i\langle\omega, f\rangle)\, d\mu(\omega) = \exp\left(-\tfrac{1}{2}|f|_0^2\right) \tag{3}$$

for all $f \in S(\mathbb{R})$. Here $|\cdot|_0^2$ denotes the usual norm in the $L^2(\mathbb{R})$ and $\langle\cdot,\cdot\rangle$ denotes the dual pairing between $S'(\mathbb{R})$ and $S(\mathbb{R})$. The dual pairing is considered as the bilinear extension of the inner product on $L^2(\mathbb{R})$, i.e.

$$\langle g, f\rangle = \int_{\mathbb{R}} g(x)f(x)dx \tag{4}$$

for all $g \in L^2(\mathbb{R})$ and $f \in S(\mathbb{R})$. This probability space is known as the real-valued white noise space since it contains the sample paths of the one-dimensional Gaussian white noise.

In this setting a one-dimensional Brownian motion can be represented by a continuous modification of the stochastic process $B = (B_t)_{t\geq 0}$ with

$$B_t = \langle \cdot, 1_{[0,t]} \rangle \tag{5}$$

In the sequel we will use the Gel'fand triple

$$(S) \hookrightarrow L^2(\mu) \hookrightarrow (S)' \tag{6}$$

where (S) is the space of white noise test functions obtained by taking the intersection of a family of Hilbert subspaces of $L^2(\mu)$. The space of white noise distributions $(S)'$ is defined as the topological dual space of (S). Elements of $(S)$and $(S)'$ are known as Hida test functions and Hida distributions, respectively. Within this framework white noise can be considered as the time derivative of Brownian motion with respect to the topology of $(S)'$. An important tool in white noise analysis is the S-transform which can be considered as the Laplace transform with respect to the infinite dimensional Gaussian measure. The S-transform of $\Phi \in (S)'$ is defined as

$$S\Phi(\varphi) \coloneqq \langle\langle \Phi, : \exp(\langle \cdot, \varphi \rangle) : \rangle\rangle, \quad \varphi \in S(\mathbb{R}) \tag{7}$$

where

$$: \exp(\langle \cdot, \varphi \rangle) : \coloneqq C(\varphi) \exp(\langle \cdot, \varphi \rangle)$$

is the so-called Wick exponential and $\langle\langle \cdot, \cdot \rangle\rangle$denotes the dual pairing between $(S)'$ and (S). We define this dual pairing as the bilinear extension of the sesquilinear inner product on $L^2(\mu)$. The S-transform provides a convenient way to identify a Hida distribution $\Phi \in (S)'$, in particular, when it is hard to find the explicit form for the Wiener-Ito chaos decomposition of Φ.

The following theorem is the main tool to our main result.

Theorem 2. [11]

Let (Ω, A, ν) be a measure space and $\lambda \mapsto \Phi_\lambda$ be a mapping from Ω to $(S)'$. If

1. the mapping $\lambda \mapsto S\Phi_\lambda(\varphi)$ is measurable for all $\varphi \in S(\mathbb{R})$, and
2. there exist $C_1(\lambda) \in L^1(\Omega, A, \nu)$, $C_2(\lambda) \in L^\infty(\Omega, A, \nu)$, and a continuous seminorm $\|\cdot\|$ on $S(\mathbb{R})$ such that for all $z \in \mathbb{C}$, $\varphi \in S(\mathbb{R})$

$$|S\Phi_\lambda(z\varphi)| \leq C_1(\lambda) \exp(C_2(\lambda)|z|^2\|\varphi\|^2) \tag{8}$$

then Φ_λ is Bochner integrable with respect to some Hilbertian norm which topologizing $(S)'$. Hence $\int_\Omega \Phi_\lambda \, d\nu(\lambda) \in (S)'$, and furthermore

$$S\left(\int_\Omega \Phi_\lambda \, d\nu(\lambda)\right) = \int_\Omega S\Phi_\lambda \, d\nu(\lambda) \tag{9}$$

Let $0 < T < \infty$. Recall that fractional Brownian motion with Hurst parameter $H \in (0,1)$ is a centered Gaussian process $B^H = (B_t^H)_{t\in[0,T]}$ defined on some probability space $(\Omega, F, \mathbb{P})$ taking values in $\mathbb{R}$ with covariance function

$$\mathbb{E}(B_t^H B_s^H) = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H}), \quad s,t \geq 0 \tag{10}$$

Fractional Brownian motion is Gaussian extension of the standard Brownian motion which is not Markov process nor semimartingale. It possesses Holder continuous trajectories and long/short-range dependence depending on the value of the Hurst parameter. Within the white noise analysis framework fractional Brownian motion can be represented by

$$B_t^H = \langle \cdot, M_-^H 1_{[0,t]} \rangle \tag{11}$$

where M_-^H is the Weyl fractional integral operator for $H \in \left(\frac{1}{2}, 1\right)$ and is the Marchaud fractional derivative operator for $H \in \left(0, \frac{1}{2}\right)$, see [12] for details. Note that $M_-^{1/2}$ is the identity operator and fractional Brownian motion reduces to the standar Brownian motion. The corresponding Donsker's delta distribution is given by

$$\delta(B_t^H - x) = \delta\left(\langle \cdot, M_-^H 1_{[0,t]} \rangle\right) = \frac{1}{2\pi} \int_{\mathbb{R}} \exp\left(i\lambda\left(\langle \cdot, M_-^H 1_{[0,t]} \rangle - x\right)\right) d\lambda \tag{12}$$

It has been proved that $\delta(B_t^H - x) \in (S)'$. Furthermore, its S-transform is given by

$$S\delta(B_t^H - x)(\varphi) = \frac{1}{\sqrt{2\pi t^{2H}}} \exp\left(-\frac{1}{2t^{2H}}\left(\langle M_+^H \varphi, 1_{[0,t\,]} \rangle - x\right)^2\right) \tag{13}$$

for any $\varphi \in S(\mathbb{R})$. Here M_+^H is the dual operator of M_-^H and for any $\varphi \in S(\mathbb{R})$ the function $M_+^H \varphi$ is continuous. For details and proofs we refer to [12-13]. The Donsker delta distribution is an important research object in the Gaussian analysis. For example, it can be used to study local times, self-intersection local times, stochastic current and Feynman integrals, see e.g. [14-20]. The derivatives of Donsker's delta distribution has been also studied in [21]. In [22] Donsker's delta distribution is analyzed in the context of stochastic processes with memory.

3 Main Result

Now we are ready to prove the main finding of the paper.
Theorem 3.
Let $B^H = (B_t^H)_{t \in [0,T]}$ be a one-dimensional fractional Brownian motion and consider an interval $[a, b] \subset \mathbb{R}$The occupation time

$$M_{T,[a,b]}(B^H) = \int_0^T \int_a^b \delta(B_t^H - x)\, dx\, dt \tag{14}$$

is a Hida distribution.

Proof. It is apparent, at least formally, that occupation times can be obtained by integrating Donsker's delta distribution with respect to the product measure on $[a, b] \times [0, t]$. In this regard we will use Kondratiev-Streit integration theorem (Theorem 2) to prove the statement. Observe that for any $\varphi \in S(\mathbb{R})$ $S\delta(B_t^H - x)(\varphi)$ is a measurable function with respect to the product measure on $[a, b] \times [0, t]$. Now for any $z \in \mathbb{C}$ and $\varphi \in S(\mathbb{R})$ we have
$|S\delta(B\_t\^H - x)(z\varphi)|$

$$= \left| \frac{1}{\sqrt{2\pi t^{2H}}} \exp(-\frac{1}{2t^{2H}}\left(\langle M_+^H\, z\varphi, 1_{[0,t\,]} \rangle - x\right)^2\ ) \right|$$

$$= \frac{1}{\sqrt{2\pi t^{2H}}} \left| \exp(-\frac{1}{2t^{2H}}\left(\langle z M_+^H \varphi, 1_{[0,t\,]} \rangle - x\right)^2) \right|$$

$$= \frac{1}{\sqrt{2\pi t^{2H}}} \left| \exp(-\frac{1}{2t^{2H}}\left(\langle z M_+^H \varphi, 1_{[0,t\,]} \rangle^2 - 2x\langle z M_+^H \varphi, 1_{[0,t\,]} \rangle + x^2\right)) \right|$$

$$
\begin{aligned}
&= \frac{1}{\sqrt{2\pi t^{2H}}} \exp\left(-\frac{x^2}{2t^{2H}}\right) \left|\exp\left(-\frac{1}{2t^{2H}} \langle zM_+^H\varphi, 1_{[0,t\,]}\rangle^2\right) \exp\left(\frac{x}{t^{2H}} \langle zM_+^H\varphi, 1_{[0,t\,]}\rangle\right)\right| \\
&\leq \frac{1}{\sqrt{2\pi t^{2H}}} \exp\left(-\frac{x^2}{2t^{2H}}\right) \exp\left(\frac{1}{2t^{2H}} \left|\langle zM_+^H\varphi, 1_{[0,t\,]}\rangle\right|^2\right) \exp\left(\left|\frac{x}{t^{2H}} \langle zM_+^H\varphi, 1_{[0,t\,]}\rangle\right|\right) \\
&= \frac{1}{\sqrt{2\pi t^{2H}}} \exp\left(-\frac{x^2}{2t^{2H}}\right) \exp\left(\frac{1}{2t^{2H}} |z|^2 \left|\langle M_+^H\varphi, 1_{[0,t\,]}\rangle\right|^2\right) \exp\left(\frac{|x|}{\sqrt{t^{2H}}} \frac{|z|}{\sqrt{t^{2H}}} \langle zM_+^H\varphi, 1_{[0,t\,]}\rangle\right) \\
&\leq \frac{1}{\sqrt{2\pi t^{2H}}} \exp\left(-\frac{x^2}{2t^{2H}}\right) \exp\left(\frac{1}{2t^{2H}} |z|^2 \left|\langle \varphi, M_-^H 1_{[0,t\,]}\rangle\right|^2\right) \exp\left(\frac{x^2}{2t^{2H}}\right) \\
&\quad \exp\left(\frac{1}{2t^{2H}} |z|^2 \left|\langle \varphi, M_-^H 1_{[0,t\,]}\rangle\right|^2\right) \\
&= \frac{1}{\sqrt{2\pi t^{2H}}} \exp\left(\frac{1}{t^{2H}} |z|^2 \left|\langle \varphi, M_-^H 1_{[0,t\,]}\rangle\right|^2\right) \\
&\leq \frac{1}{\sqrt{2\pi t^{2H}}} \exp\left(\frac{|z|^2}{t^{2H}}\ |\varphi|^2 \left|M_-^H 1_{[0,t\,]}\right|^2\right) \\
&= \frac{1}{\sqrt{2\pi t^{2H}}} \exp(\ |z|^2\ |\varphi|^2)
\end{aligned}
$$

The last inequality is obtained by an application of the Cauchy-Schwarz inequality. We have used also the Ito isometry, that is $\left|M_-^H 1_{[0,t\,]}\right|^2 = var(B_t^H) = t^{2H}$**.** Note that

$$\int_0^T \int_a^b \frac{1}{\sqrt{2\pi t^{2H}}}\, dxdt = \frac{(b-a)T^{1-H}}{\sqrt{2\pi}(1-H)} < \infty$$

and $\exp(\ |z|^2\ |\varphi|^2)$ is a bounded function of t and x. Theorem 2 gives the desired result. ■

This is an immediate consequence of Theorem \ref{int} and Theorem \ref{main}.
Corollary 4.
The S-transform of the occupation times of fractional Brownian motion is given by

$$SM_{T,[a,b]}(B^H)(\varphi) = \int_0^T \frac{1}{\sqrt{2\pi t^{2H}}} \int_a^b \exp\left(-\frac{1}{2t^{2H}}\left(\int_0^t M_+^H\ \varphi\,(s)ds - x\right)^2\right)\, dx\ dt \quad (15)$$

for any $\varphi \in S(\mathbb{R})$.

Proof. Since $M_{T,[a,b]}(B^H) \in (S)'$ for every $\varphi \in S(\mathbb{R})$. Theorem 2 gives

$$
\begin{aligned}
&SM_{T,[a,b]}(B^H)(\varphi) \\
&= S\left(\int_0^T \int_a^b \delta(B_t^H - x)\, dx\, dt\right)(\varphi) \\
&= \int_0^T \int_a^b S\delta(B_t^H - x)(\varphi)\, dx\, dt \\
&= \int_0^T \int_a^b \frac{1}{\sqrt{2\pi t^{2H}}} \exp\left(-\frac{1}{2t^{2H}} \left(\langle M_+^H\varphi, 1_{[0,t\,]}\rangle - x\right)^2\right) dx\, dt \\
&= \int_0^T \frac{1}{\sqrt{2\pi t^{2H}}} \int_a^b \exp\left(-\frac{1}{2t^{2H}}\left(\int_0^t M_+^H\varphi(s)ds - x\right)^2\right) dx\, dt
\end{aligned}
$$

Corollary 5.
The expected value of the occupation times of fractional Brownian motion is given by

$$\mathbb{E}_\mu\left(M_{T,[a,b]}(B^H)\right) = \int_0^T \frac{1}{\sqrt{2\pi t^{2H}}} \int_a^b \exp\left(-\frac{x^2}{2t^{2H}}\right) dx\, dt \tag{16}$$

Proof. The expected value with respect to the white noise measure of the occupation times of fractional Brownian motion is obtained by taking the S-transform and evaluating the value at 0:

$$\begin{aligned}\mathbb{E}_\mu\left(M_{T,[a,b]}(B^H)\right) &= SM_{T,[a,b]}(B^H)(0)\\ &= \int_0^T \frac{1}{\sqrt{2\pi t^{2H}}} \int_a^b \exp\left(-\frac{1}{2t^{2H}}\left(\int_0^t 0\, ds - x\right)^2\right) dx\, dt\\ &= \int_0^T \frac{1}{\sqrt{2\pi t^{2H}}} \int_a^b \exp\left(-\frac{x^2}{2t^{2H}}\right) dx\, dt\,.\end{aligned}$$

4 Conclusion

We gave a mathematically rigorous meaning to the occupation times of a fractional Brownian motion as a Hida distribution. An expression for the expected value for the occupation times is also obtained. For the application point of view it is desirable to have a more explicit form for the expected value. This will be done in the future work. We would like also to mention that our present result is limited to one-dimensional setting. For further research we plan to generalize the result to higher spatial dimensions. Moreover, an extension to a more general class of processes (e.g. to the generalized grey Brownian motion) will be also considered.

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Counting The Number of Arrangements of Tatami Mats in a Rectangular Room of Vertical Length 2, 3 and 4

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Abstract. Japanese rooms are measured by the number of tatami mats that will fit inside. The size of a tatami mat can vary by region, but is generally around 180 cm by 90 cm, giving it a 2:1 ratio of length to width. In the following, for simplicity, we suppose that each tatami mat is a rectangle with two adjacent sides of lengths 1 and 2. A typical tea ceremony room is square-shaped and its area is the equivalent of 4 and a half tatami mats. Questions regarding to lay tatami mats are not only fun for elementary school students, but also often included in entrance exams. In this paper, we derive recurrence formulae for determining the number of ways to lay tatami mats in a rectangular room whose vertical length is fixed at four or less, by using the concept of *compartments* or indivisible factors. Since the area of each tatami mat is two, if the area of the room is odd, only one half-sized tatami mat is allowed to be used. Therefore, if the vertical length of the room is three, the results will be different depending on whether the horizontal length of the room is even or odd. A generating function is used in this case, since it is difficult to derive the recurrence formula from direct consideration.

1 Introduction

Japanese rooms are measured by the number of tatami mats that will fit inside. The size of a tatami mat can vary by region, but is generally around 180 cm by 90 cm, giving it a 2:1 ratio of length to width. A typical room for tea ceremony is a square-shaped room of size four and a half tatami mats. A popular living room in a house is of size six. For auspicious reason, they are often arranged so that no four mats meet at a point. Students will recognize that a tatami mat is a rectangle and that half a tatami is a square. Half-sized tatami mats are also available only if it is unavoidable to use. For example, Figure 1 shows four types of arranging tatami mats in a square room: an auspicious room (A), a sorrow room in a temple (B), a tea ceremony room (C), and a room for seppuku (D).

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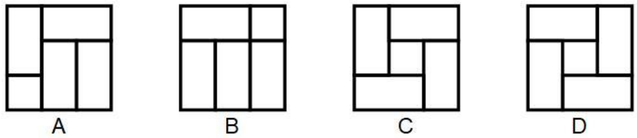

Fig. 1. Types of tatami patterns for a square room of size four and a half.

In the following, for simplicity, we suppose that each tatami mat is a rectangle with two adjacent sides of lengths 1 and 2, and a half-sized tatami mat is a square with a side length of 1. A tatami mat is also called a domino in some literature.

Questions regarding the number of ways to lay tatami mats in a room of given size and shape are not only fun for elementary school students, but often included in entrance exams. For example, the 3rd mathematics problem of early entrance exam for the science students of the University of Tokyo in March 1995 was about the number of ways to cover a room of vertical length 2 and horizontal length n with rectangular mats of size 2 times 1 and square mats of size 2 times 2. Another example is the 2nd part of the 4th question of Mathematics II and Mathematics B of the supplementary examination for the 2021 University Entrance Common Test in Japan was about the ways of covering a rectangular room of vertical length 3 and horizontal length $2n$ with tatami mats.

The mathematical theory of tatami tiling has been studied in detail in recent years, and connections to graph theory, games, and physics are becoming apparent. For more information, consult Read [19], Erickson [4], Alhazov et al [2], Ruskey and Woodcock [20], Kenyon [12] and Erickson et al [5].

In this paper, we derive recurrence formulae for determining the number of ways to lay tatami mats in a rectangular room whose vertical length is fixed at two, three, or four, by using the concept of indivisible factors. We mention that this concept is so natural that it can emerge from discussion in a class of elementary school students of early grades. Since the area of each tatami mat is 2, if the area of the whole room is odd, one must use half-sized tatami mats. In this case, it is interesting to allow to use only one half-sized tatami mat. Therefore, if the room vertical length is fixed to three, the resulting number of tatami arrangements will be somewhat different depending on whether the horizontal length of the room is even or odd. For high school students, it is sometimes convenient and interesting to introduce the concept of generating functions, since it is difficult to derive the recurrence formula from direct geometrical consideration.

This presentation is based on the author's educational practice for 4 years and the goal is to invigorate classrooms and help children better understand the joy of mathematics. Some of the content was presented at annual meetings of the Mathematics Education Society of Japan in the spring of 2022 and 2024 (*Cf.* [24] and [25]).

2 Results and Discussions

In this section, we will discuss three levels of problems regarding counting the number of ways to lay tatami mats in stages. Students are encouraged to use pencil and paper or wooden blocks to simulate the ways they can arrange tatami mats into a room of given size and shape. Throughout this paper, we forget the auspicious condition mentioned in the introduction. Room sizes are given with vertical lengths 2, 3 and 4, and horizontal length n, where n will be an arbitrary integer greater than or equal to zero.

When vertical length is fixed to two and horizontal length n increases as 1, 2, 3 and so on, the number of arrangements varies as 1, 2, 3, 5, 8 and so on to form the Fibonacci sequence f_n (n = 1, 2, 3, …) shown in Figure 2. Students can enjoy constructing the real pattern by wooden blocks, or by drawing pictures on a paper, and then classify them upon discussion.

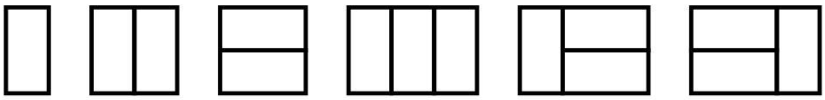

Fig. 2. Some tatami filling patterns for rooms of vertical length two.

After summing up the numbers for several small size rooms, students who can add small whole numbers will recognize the arithmetical law $f_n = f_{n-1}+f_{n-2}$, with $f_1 = 1$ and $f_2 = 2$. Once the increment pattern is obtained, they can calculate as far as they want to get larger digit numbers. This activity, which is intended to the first and second grade elementary school classes, combines the joy of producing new shapes and that of addition to get large numbers.

In my experience, children seem to find intrinsic joy in discovering the Fibonacci numbers. The Fibonacci numbers can also be discovered in other problem settings, such as the number of climbing stairs. Children are influenced by the simplicity and fascination of the discovered patterns, and are motivated to continue adding up considerably far beyond. This activity also meets children's desire to "discover large numbers."

The Fibonacci-type recurrence formula can be derived from the fact that the tatami mats on the left side of the room can be laid out either one vertically or two horizontally, but the younger elementary school students were not observed to discuss this in detail.

Furthermore, if we set $f_0 = 1$, this recurrence formula also holds when $n = 2$. This rule can be interpreted as saying that there is only one way to lay tatami mats in a room of size 2 times 0: to cover it with 0 tatami mats.

By changing the perspective, we can obtain another recurrence formula. Let us classify all the ways in which tatami mats can be laid out in a room of size 2 times n according to the number and position of horizontal tatami mats. If there are no horizontal tatami mats, all the tatami mats are vertical, and there is only one way of laying them out in such a manner. If there are at least one horizontal tatami mats, and we focus on the leftmost one among such tatami mats, then of the n tatami mats, the k-1 tatami mats from the left are vertical, followed by two horizontal tatami mats, and the number of ways of laying them out to the right of those is f_{n-k-1}. This leads to the recurrence formula

$$f_n = 1+ (f_{n-2}+f_{n-3}+\cdots+f_0), \tag{1}$$

where n is greater than or equal to 2.

Next, it is natural to move onto the case where the vertical length is three. When the vertical length is fixed to three, the problem will be more complicated. Indeed, a question to find the recurrence formula in this case was asked in the Common Test for University Admissions in Japan in 2022.

Consider the number b_n of ways to lay tatami mats in a rectangular room of size 3 times n. When n is an odd number, $b_n = 0$. Figure 3 shows that there are just three arrangements for a room of horizontal length 2 (A, B and C in Figure 3), whilst for a room of horizontal length 4, some mat patterns can be divided into right and left parts by a vertical line (D and E), and there is a filling that cannot be divided in such a way (F), which is used in fact in the typical Japanese living room of six tatami mats. For some fixed horizontal length n, students are encouraged to share as many arrangements as possible in a small group and/or the whole class and classify them.

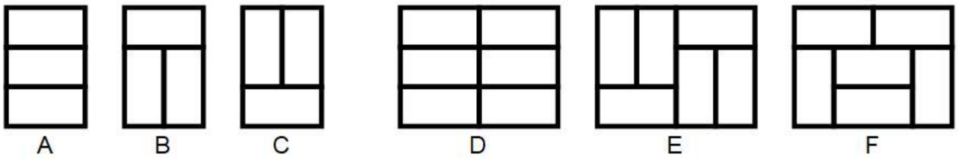

Fig. 3. Some tatami filling for rooms of vertical length three.

An arrangement is called *compartment* if it is impossible to divide by any vertical line. Any arrangement is either a compartment or a combination of more than one compartments. Because there are three compartments of horizontal length two and two compartments of horizontal length four, the number of arrangements of horizontal length four is $3^2+2 = 11$. Here multiplication comes in naturally with an important role. Because there are always 2 compartments of horizontal length n, with n being even and larger than 4, one can calculate that there are $2+2\cdot3\cdot2+2^2 = 41$ arrangements for a room of horizontal length 6, so $b_6 = 41$.

It is possible to continue calculating the number of filling patterns of rooms of vertical length 3 and horizontal length n, with n being even, using the concept of compartments and a recursive formula. First, it is easy to speculate that the number of compartments of any fixed horizontal length greater than 2 is 2. Any layout can be divided into a union of compartments by dividing it with a vertical dividing line. Let us use this fact to find a recurrence formula. Given a layout with horizontal length $n = 2m$, let the horizontal length of the leftmost compartment be $2k$. There are three ways to choose it when $k = 1$, and two ways when k is 2 or more. Thus, by classifying by k being 1, 2, ..., or m, we obtain

$$b_n = 2\ (b_0+b_2+\cdots+b_{n-4})\ +3\ b_{n-2}\ , \qquad (2)$$

when n is even. By subtracting this equation from equation

$$b_{n-2} = 2\ (b_0+b_2+\cdots+b_{n-6})\ +3\ b_{n-4} \qquad (3)$$

obtained by replacing n with n-2, we obtain the recurrence equation

$$b_n = 4\ b_{n-2}-b_{n-4}\ , \qquad (4)$$

for n 4 or more. Since we only need to consider the case where n is even, this is also a Fibonacci-type three-term recurrence formula, with a general term like Binet's formula. Calculating several values of b_n, we get the following table:

Table 1. Number of arranging tatami mats in a room of vertical length 3.

n	0	2	4	6	8	10
b_n	1	3	11	41	153	571

The author would also like to mention that the fourth question in the supplementary exam of 2021 University Admission Common Test in Japan is a guided-form question that shows another way to derive this recurrence formula.

Similar method is valid for rooms of vertical length four. Let c_n be the number of arranging tatami mats in a rectangular room of size 4 times n, and g_n that of compartments of horizontal length n. Figure 4 shows the complete set of compartments with horizontal length four and five.

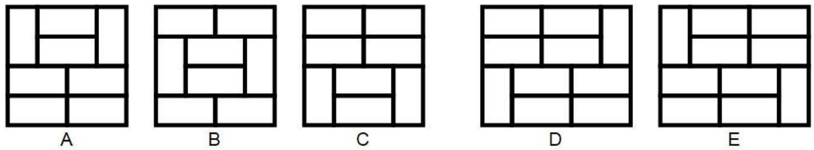

Fig. 4. Typical covering patterns of vertical length four which is vertically undividable.

As shown in Table 2, if the horizontal length n of the room increases as 1, 2, 3, 4, 5, 6 and so on, the number of compartments varies as 1, 4, 2, 3, 2, 3, with 2 and 3 repeated periodically from there on.

Table 2. Number of compartments in a room of vertical length 4.

n	1	2	3	4	5	6	7
g_n	1	4	2	3	2	3	2

This guess is correct, and just like the previous case with vertical length 3, if we focus on the leftmost compartment, we get a set of equations, such as

$$c_4 = c_3 + 4\,c_2 + 2\,c_1 + 3\,c_0, \quad (5)$$
$$c_6 = c_5 + 4\,c_4 + 2\,c_3 + 3\,c_2 + 2\,c_1 + 3\,c_0. \quad (6)$$

Now we get

$$c_n = c_{n\text{-}1} + 5\,c_{n\text{-}2} + c_{n\text{-}3} - c_{n\text{-}4} \quad (7)$$

by taking the difference between the equations for c_n and c_{n-2}. After calculating some values, we get Table 3.

Table 3. Number of arranging tatami mats in a room of vertical length 4.

n	0	1	2	3	4	5	6	7
c_n	1	1	5	11	36	95	281	781

The number c_5=95 is the number of arrangements of tatami mats in a room of size 4 times 5. In Japanese, a room of this size is called a 10-tatami room. The question of guessing this number was asked on a mathematics entertainment program "Takeshi's Komadai Mathematics Department" that aired on Fuji TV on Saturday, January 24, 2009.

In general, there is a famous formula for the number of ways to lay tatami mats in a rectangular room of size *m* times *n* that applies the dimer model of statistical mechanics, and it is explained in detail in Kenyon [12].

The method of covering a rectangular room of size 3 times *n*, with *n* odd, using only one half-tatami mat could also be taken up as an interesting theme of mathematical activities at the high school level. We will not discuss this problem here, but details are given in Ueno [26].

3 Conclusions

The general problem for a rectangular room of size *m* times *n* is called the dimer problem and is equivalent to the problem of counting the number of perfect matchings in a grid graph (see for instance Read [1]). Allowing half mats and imposing the auspicious restriction results in visually appealing structure, which is installed in combinatorial games by Alejandro Erickson in 2013 [2].

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Decision-Level Fusion on Healthcare

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Abstract. Patient care management is a crucial aspect that affects the quality of life of patients and the operational efficiency of hospitals. Patient care can be broadly categorized into two main categories: inpatient and outpatient. The objective of this study is to develop a machine learning model that can accurately predict whether a patient should be classified as inpatient or outpatient based on their laboratory test results. Five classification methods are applied in this study: Logistic Regression, Support Vector Machine (SVM), Random Forest, and Gradient Boosting. The feature variables in the dataset include laboratory test results and patient demographic information, such as Haematocrit, Haemoglobins, Erythrocyte, Leucocyte, Thrombocyte, MCH, MCHC, MCV, Age, and Gender. The target variable is the type of patient care, coded as 1 for inpatient and 0 for outpatient. This study also implements Decision Fusion to enhance prediction accuracy and stability. After preprocessing to detect and remove outliers, the data is split into training and testing sets. The models are then fitted and tested on the test data. Predictions from the four classification methods are combined using decision level fusion as majority voting, score level and weighted voting to obtain the final prediction. Thus, the fusion method can provide better performance compared to individual models by leveraging the collective strength of all the models used. In this study, we use two scenarios, a false negative ratio of 1% and a false negative ratio of 5% to show the performance of decision fusion.

1 Introduction

Patient care management is one of the crucial aspects that affect the quality of patient care and the operational efficiency of the hospital. In general, patient care can be divided into two main categories, namely inpatient and outpatient. Determining the right type of care is very important because it has an impact on the use of hospital resources, treatment costs, and patient comfort. One component that can help determine the type of care for patients is the results of laboratory tests. The use of technology is considered capable of supporting the identification process to be faster, more accurate, and more effective [1].

One way to classify patient treatment or disease is through machine learning. The patient classification process will be carried out automatically by a computer/machine with the help of certain algorithms. Previous study uses several algorithms to classify patient treatment

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status such as Logistic Regression [4-6], Decision/Classification Tree [7, 8], Support Vector Machine [7,8], Gradient Boosting [9,10], etc.

However, it cannot be denied that misclassification is one of the impacts of the use of automation technology. These misclassifications are dangerous because patients who actually need intensive care may not get it. These can worsen their health condition or even cause an emergency. For a non-emergency patient, misclassification can affect treatment choices, hospital risk management, medication error, and patient safety [14-17,26]. Several studies have been conducted to minimize misclassification in healthcare region [25, 26] using specified methods.

Recent study, such as [3] and [34], try to create and find the 'best' model to predict the patient care acurrately. In other words, those studies are focused on the number of True Positive and True Negative observations. However, in this study, we want to provide a different approach to minimize misclassification by considering several things such as risk, health, and urgency of patient care. When risk is a part of the consideration, the model's accuracy can't be the only metric that be noticed. A deeper explanation about the objectives of this study will be provided in Chapter 3. We're also use the same dataset as [3] and [34] so that readers can see the differences in the perspective used.

2 Theory

2.1 Logistic Regression

Logistic regression is a data analysis method that describes the relationship between a response variable and one or more predictor variables. The response of a binary logistic regression model consists of two categories (e.g. 0 and 1) so that the response variable will follow a Bernoulli distribution with a probability function $f(y_i) = \phi(x_i)^{y_i}\left(1-\phi(x_i)\right)^{1-y_i}$, $y_i = 0{,}1$ [27]. Based on this equation, obtained:

$$\phi(x) = \frac{\exp\left(\beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p\right)}{1 + \exp\left(\beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p\right)} \tag{1}$$

2.2 Random Forest

Random Forest is a type of classification algorithm that consists of more than one decision tree where each decision tree is formed depending on the values of a random vector sampled independently and identically distributed which is the same for all trees [11]. The prediction results from the model are obtained through the most results from each individual decision tree (voting for classification and average for regression). The model equation of random forest consisting of N trees is formulated as:

$$l(y) = argmax_c\left(\sum_{n=1}^{N} I_{h_N}(y) = c\right) \tag{2}$$

Where I is the indicator function and h_N is the n-th tree of the random forest [14].

2.3 Support Vector Machine

This model works by finding a line (hyperplane) to separate groups of data. The best Hyperplane is found by measuring the margin of the hyperplane and finding the maximum point. The equation used in the SVM model is as follows:

$$H_1: w_0 + w_1x_1 + w_2x_2 \geq 1, y_i = +1 \quad (3)$$
$$H_2: w_0 + w_1x_1 + w_2x_2 \leq -1, y_i = -1$$

For non-linearly separable data, the data will be mapped first to a higher dimension using non-linear mapping. This mapping uses some kind of function called kernel. The following presents several frequently used kernel functions [12].

2.3.1 Kernel Linear

$$K(x_i, x_j) = x_i^T x_j \quad (4)$$

2.3.2 Kernel Polynomial

$$K(x_i, x_j) = (\gamma x_i^T x_j + r)^d, \gamma > 0 \quad (5)$$

2.3.3 Kernel RBF (Radial Basis Function)

$$K(x_i, x_j) = \exp\left(-\gamma||x_i^T x_j||^2\right), \gamma > 0 \quad (6)$$

2.3.4 Kernel Sigmoid

$$K(x_i, x_j) = \tanh(\gamma x_i^T x_j + r) \quad (7)$$

2.4 Gradient Boosting

In simple terms, the GB algorithm works by combining the obtained model with a new candidate model. The purpose of this combination is to minimize the error obtained. Gradient Boosting Method starts by generating an initial classification tree and continuously adapting the new tree through minimizing the loss function [13]:

$$-logL = -\sum_{i=1}^{N} y_i \log(odds) + \log(1 + e^{\log(odds)}) \quad (8)$$

2.5 Decision Level Fusion

Decision-Level Fusion is a technique that combines decisions from several algorithms and then combines them to claim the final decision. Several techniques can be used in decision-level fusion, including majority voting, score level and weighted voting. These techniques have been used in various cases to improve the performance of the built model [18-24, 30-33].

2.5.1 Majority Voting

Majority voting is one of the approaches to voting based method. Voting based method is a voting-based method that operates only on labels. The majority voting process is carried out by voting to obtain the most votes. One algorithm that applies the concept of majority voting is the voting classifier algorithm [19]. Suppose the vector d is the prediction result defined as $d = [d_1, d_2, \dots, d_n]$ where $d \in \{c_1, c_2, \dots, c_m, r\}$ denotes the label of the i class and the r rejection. Given a binary characteristic function defined:

$$B_j(c_i) = \begin{cases} 1, & d_j = c_i \\ 0, & d_j \neq c_i \end{cases} \tag{9}$$

Hence, the general voting are defined by:

$$E(d) = \begin{cases} c_i, & \forall_{t \in \{1,\dots,m\}} \sum_{j=1}^{n} B_j(c_t) \leq \sum_{j=1}^{n} B_j(c_i) \geq \alpha m + k(d) \\ r, & otherwise \end{cases} \tag{10}$$

2.5.2 Score Level

Score-level fusion is a technique of combining several algorithms to improve system performance. Each classifier gives a different value related to the algorithm used. Combining values can provide more accurate decisions than those obtained from each classification method. Some algorithms commonly used in score-level fusion are average value fusion, maximum value fusion, minimum value fusion, with the average, maximum and minimum values being the final values [18].

2.6 Confusion Matrix

Certain measurements are needed to measure the performance of the system in data classification. In this study, confusion matrix is used. There are at least four metrics that can be used as representation of the results of the classification process, namely True Positive (TP), True Negative (TN), False Positive (FP) and False Negative (FN). Based on those metrics, the accuracy, precision and recall values of the model can be obtained using the following formula:

$$Accuracy = \frac{TP+TN}{TP+TN+FP+FN} \times 100\% \tag{11}$$

$$Precision = \frac{TP}{TP+FP} \times 100\% \tag{12}$$

$$Recall = \frac{TP}{TP+FN} \times 100\% \tag{13}$$

The probability of classifying an incorrect score as a correct score is called the False Positive Rate ($FPR(\Delta)$) with a threshold of Δ, while the probability of classifying a correct score as an incorrect score is called the False Negative Rate ($FNR(\Delta)$)). The complement of $FNR(\Delta)$ is called the True Positive Rate ($TPR(\Delta)$)), which is defined as the probability of accepting a genuine score as a genuine score. Since each genuine score will either be accepted or rejected by the system, we have $TPR(\Delta) = 1 - FNR(\Delta)$. The most common method used to view the system's performance is to plot the relationship between $FPR(\Delta)$ and $TPR(\Delta)$ for all $\Delta \in (-\infty, \infty)$, known as the Receiver Operating Characteristic (ROC) [35].

3 Methodology

3.1 Data

Data used in this research is "EHR Dataset for Patient Treatment Classification" collected from several private hospitals in Indonesia [35]. The response variable (namely `SOURCE` is categorical labeled by **inpatient** (0) and **outpatient** (1). A brief description about variables (columns) contained in the dataset is presented in **Table 1**.

Table 1. Description of Column in Dataset

Variable	Data Type	Description
HAEMATOCRIT	Numeric	Patient laboratory test result of haematocrit
HAEMOGOBLINS	Numeric	Patient laboratory test result of haemogoblins
ERYTHROCYTE	Numeric	Patient laboratory test result of erythrocyte
LEUCOCYTE	Numeric	Patient laboratory test result of leucocyte
THROMBOCYTE	Numeric	Laboratory test result of thrombocyte
MCH	Numeric	Laboratory test result of MCH (mean corpuscular hemoglobin). MCH value refers to average quantity of hemoglobin in a single red blood cell
MCHC	Numeric	Laboratory test result of MCHC (mean corpuscular hemoglobin concentrate). This value refers to amount of hemoglobin in a single red blood cell relative to cell's volume
MCV	Numeric	Laboratory test result of MCV (mean corpuscular volume). This valure refers to the average size of red blood cell in a blood sample
AGE	Numeric	Patient age
SEX	Categoric	Patient gender, labeled by Male or Female

Figure 1 give a correlation heatmap from the numeric variables in the dataset. It can be concluded that several variables have high dependencies, such as HAEMATOCRIT and HAEMOGLOBINS. But, it can be also seen that several variables is not correlated to the other. This information can be used as supplementary consideration to build the model later. Other information about the variables are given in the following section.

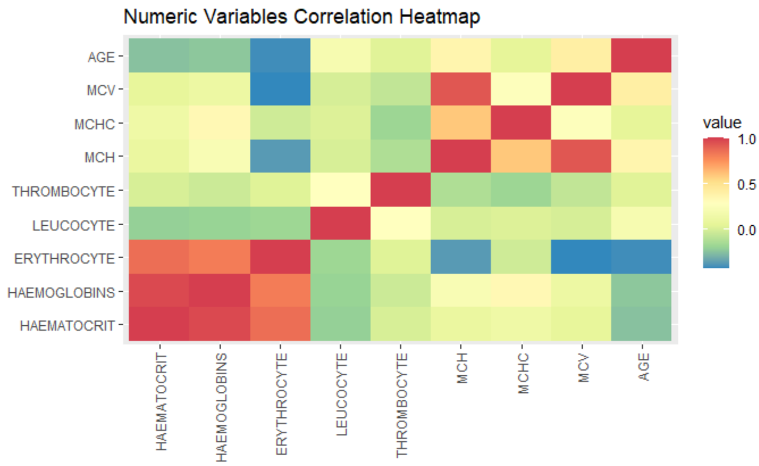

Fig. 1. Heatmap Correlation of Numerical Variables

Figure 2 gave the histogram representation of the data. With these histograms, the information about skewness of the data can be obtained. For example, most of the values from LEUCOCYTE variables are below 20. But from the histogram, it can be seen that there are some values more than 30. This can be a preliminary presumption that there are outliers from the numerical value. The same concept also applied for other variables.

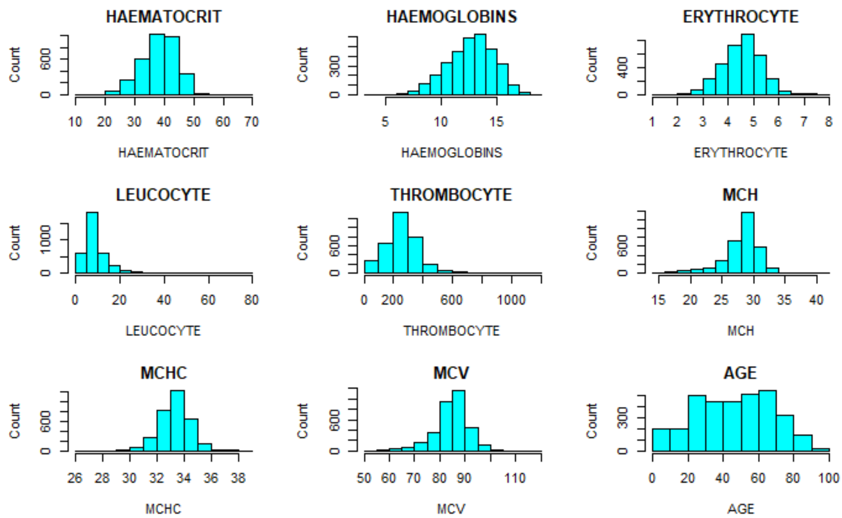

Fig. 2. Histogram of the Variables

Figure 3 gave information about the patient status and the number of patient belongs to each class if separated by their gender. Figure above shown that there are imbalance cases for patient status level (namely 0 and 1). The number of patient that labeled as 0 (outpatient) is approximately 2000, but the number of patient that labeled as 1 (inpatient) is approximately 1300. To make sure the constructed model can learn the pattern well enough, this imbalance case is considerable.

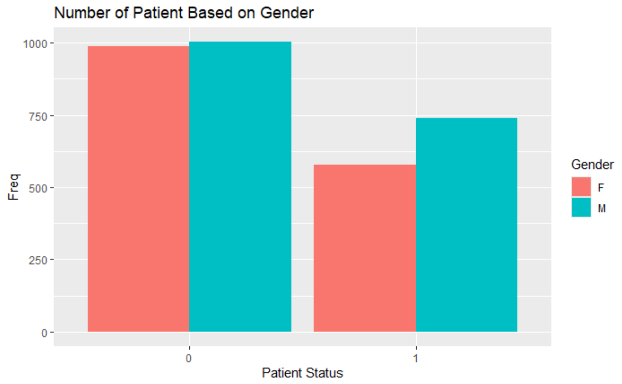

Fig. 3. Comparison of Patient Status Based on Gender

3.2 Objectives

This study was conducted to propose an optimal threshold inside statistical computing that minimize number of False Negatives. In this case, the False Negative Rate (FNR) represent the percentage of the inpatient that classified as the outpatient. This objective is supported by the fact that misclassification of inpatient care into outpatient care has a more dangerous risk to the health and life of the patient. In this study, two scenarios were carried out, namely by setting the FN percentage limit at 1% and 5% respectively.

3.3 Methods

The data was splited into 2 parts, namely *training* and *testing* with 3:1 ratio. Training data is used to build models with best threshold. Testing data is used to make sure that the fitted models can predict 'new' data according to the purpose.

There are at least two pre-processing steps before fitting the model to the data. The first pre-processing step is removing numerical outliers from the data. This step is done by using the concept of interquartile range (boxplot). For each numeric-value column, values that fall outside the closed interval $[Q_1 - 1.5IQR, Q_3 + 1.5IQR]$ will be considered as numeric outliers. Hence, the entire row will be dropped from the data.

The second pre-processing step is making the data more balance based on the response variable. It has been reported that the response variables are imbalanced. In other words, there are more information about the outpatient (class 0) compared to inpatient (class 1). This may affect the model's learning ability, specifically too good at understanding the outpatient class but poor at understanding the inpatient class. To avoid this scenario, in this study, the SMOTE resampling methods are used in the training data. By generate more sample of the inpatient class, it is hoped that the model will be able to understand the patterns in both classes more optimally.

Classification is carried out using four individual models, namely Logistic Regression (LR), Random Forest (RF), Support Vector Machine (SVM), and Gradient Boosting (GBM). After the results from each classifier appear, the results are processed using majority voting and the average of the probabilistic scores in the hope of obtaining more optimal results. The computational process is carried out with the help of RStudio and Google Colab using R language.

4 Results and Discussions

Each classifier is run with different parameters. Based on the result of `regsubset` function, the LR model was used by selecting four independent variables. RF model was used with the number of trees of 500. The SVM model is used by selecting the radial kernel with $\gamma = 0.1$ and $cost = 2$. GB model is used with 100 trees. The computational process is carried out with the aim of producing a model with a False Negative Rate of 1% or 5%. The results of the fitted model can be observed in the following table. In addition, the ROC curve is also presented to see the performance of the proposed method compared to individual classifiers.

4.1 False Negative Rate 1%

To achieve an expected FNR of 1%, a threshold equivalent to $8 - 15\%$ is required, where in general naive classification sets the threshold level at 50%. However, even with a very low threshold, the resulting precision is quite good for all classifiers. Furthermore, we applied two fusion methods: majority vote, which is based on the highest binary classification result, and score level, which is the average of the normalized scores of each classifier. The average score result is then examined for a threshold that produces 1% FNR. As for the summary of the results can be seen in **Table 2.**

Table 2. Model Comparison for False Negative Rate (FNR) 1%

Model	Treshold(Δ)	False Negative	Precision
Logistic	0.1198	3	0.7500
Random Forest	0.0801	3	0.8695
Support Vector Machine	0.0831	3	0.9117
Gradient Boosting	0.1532	3	0.9166
Fusion – Majority Voting	No threshold needed	2	0.8888
Fusion – Score Level	0.1215	3	0.6666

The majority voting method gives the best result of giving the smaller count of false negative than all classifiers. With this majority voting method, only 2 inpatients are classified as outpatient. In terms of precision metrics, the fusion is below individual classifiers such as Support Vector Machine and Gradient Boosting. However, compared to individual classifiers such as Logistic Regression and Random Forest, this model provides much better precision values.

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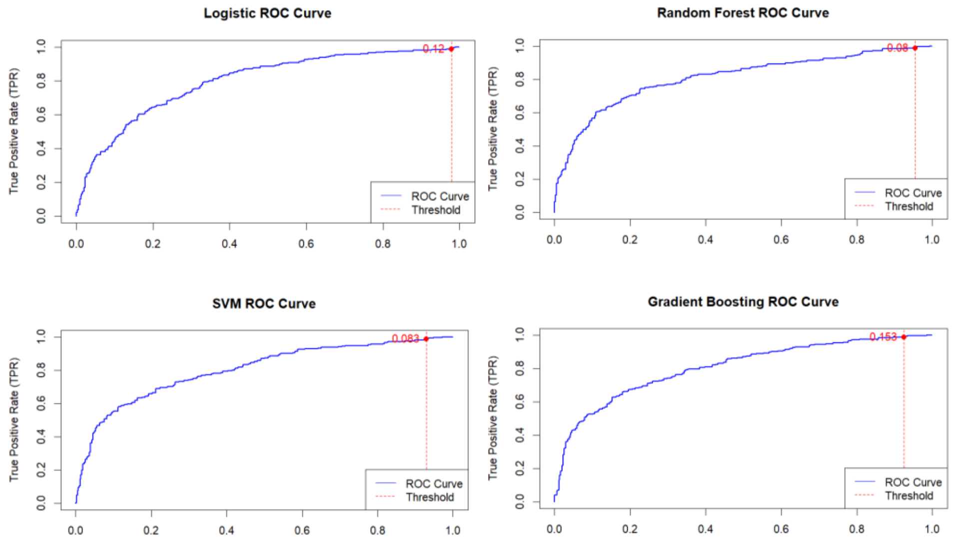

Fig. 4. ROC curves of individual classifier for FNR 1%

Based on Figure 4 above, it can be seen that the ROC curve with a threshold for 1% FNR tends to be to the right with an area of 1% from the right, or about 99% of the False Positive Rate (FPR). If we compare the ROC curve into one graph at once (Figure 5) comparing with the fusion score level, it can be seen that the black fusion line is more likely to be higher than the others. This means that the fusion score level method has a high TPR for each FPR.

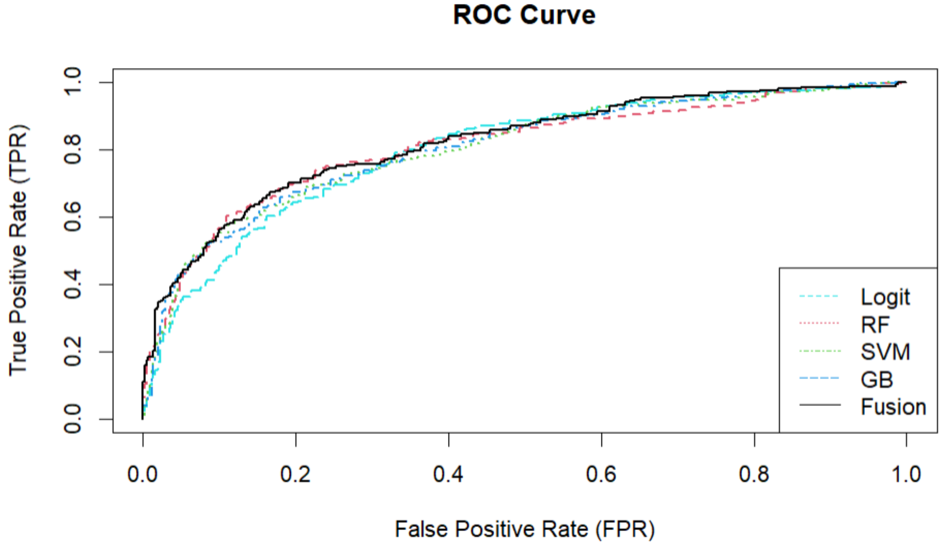

Fig. 5. ROC curves of different classification strategies for FNR 1%

4.2 False Negative Rate 5%

Compared to the previous 1% FNR, this 5% FNR scenario requires a threshold equivalent to 13-29% or more lenient in providing criteria. Although the threshold at 5% FNR is still far below the threshold of the naive classifier, the precision metric gives good results as shown in Table 3.

Table 3. Model Comparison for False Negative Rate (FNR) 5%

Model	Treshold(Δ)	False Negative	Precision
Logistic	0.2899	13	0.9166
Random Forest	0.1615	13	0.8785
Support Vector Machine	0.1300	13	0.8839
Gradient Boosting	0.2133	13	0.9064
Fusion – Majority Voting	No threshold needed	12	0.9172
Fusion – Score Level	0.2266	13	0.9166

It is obtained that the fusion by majority voting method gives the best result. From the count of false negative, the majority voting method gives better results with 12 people, while the fusion by score level method gives FN as many as 13 people. This shows that fusion can minimize the risk of misclassification. However, based on other metrics, the majority vote fusion outperformed any other methods with the precision of this model reaches 91.72%.

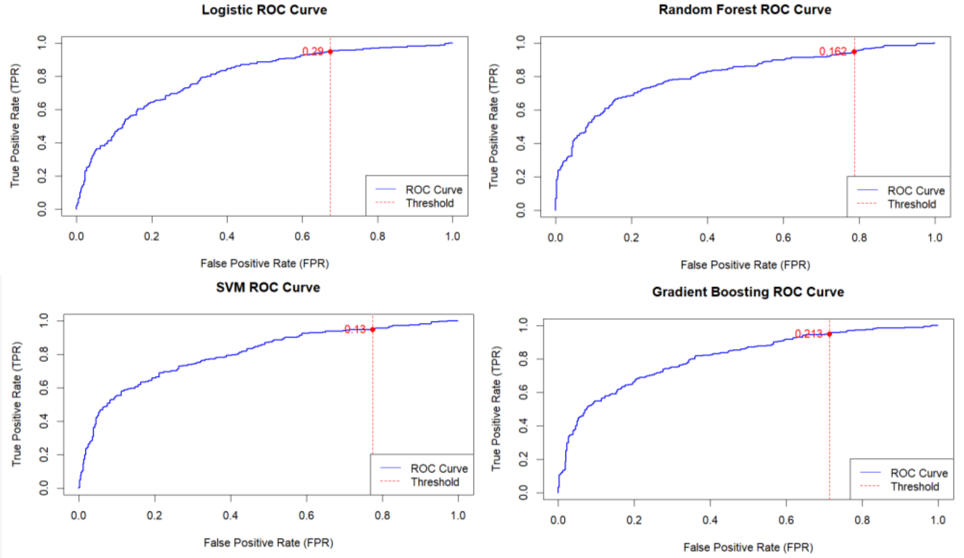

Fig. 6. ROC curves of different classification strategies for FNR 5%

When compared to the 1% FNR scenario as before, based on Figure 6 above, it can be seen that the ROC curves for each of the classifiers have a more overhanging threshold to the left away from the 100% FPR. This indicates that a larger threshold has the ability to separate the classes better.

5 Conclusion

Based on the study that has been conducted, it was found that for FNR scenarios of 1% and 5%, the proposed method provides good performance when viewed from the number of False Negatives and Precision. However, the model accuracy value becomes lower due to changes in the threshold given. Handling of data imbalance is also carried out in subsequent studies to overcome this problem. In addition, the model used in this study is very limited. Other studies can use more varied models and more diverse data so that the results provided are more optimal.

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Development of an Epidemiological Model with Transmission Matrix to Understand the Dynamics of Tuberculosis Spread

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Abstract. Tuberculosis remains a major challenge in the field of healthcare. The spread of tuberculosis depends on complex interactions between individuals within a population, involving factors such as mobility, physical contact, and age groups. Each age group has unique characteristics that influence how tuberculosis spreads among the population and how each group responds to the infection. To understand the dynamics of tuberculosis spread, an epidemiological model is required. This study aims to develop an epidemiological model based on a transmission matrix that can represent the pattern of tuberculosis spread within a population. The transmission matrix is used to describe the interactions between individuals and subpopulations, taking into account the transmission rate and incubation period. After building the model and transmission matrix, model calibration and validation are conducted. In this stage, model parameters are adjusted to ensure that the model can accurately replicate the observed epidemiological data. Subsequently, analysis is performed using the model and transmission matrix to understand the dynamics of disease spread, followed by interpretation of the results. The findings of this study indicate that the use of the transmission matrix provides valuable insights into the dynamics of tuberculosis spread and helps identify high-risk subpopulations.

1 Introduction

Tuberculosis is an infectious disease caused by the bacterium Mycobacterium tuberculosis [1]. According to the World Health Organization, tuberculosis remains one of the top ten causes of death, with millions of new cases diagnosed each year [2]. The rapid and widespread transmission of tuberculosis is caused by various factors, including human mobility, socioeconomic conditions, and population density. The dynamics of tuberculosis spread are also influenced by the complex interaction between epidemiological and biological factor, which make understanding the transmission patterns of this disease a major challenge [3]. Tuberculosis has a long and variable incubation period, usually several weeks to months after exposure [4].

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Tuberculosis is spread through the air when an infected person coughs, sneezes, or talks, releasing droplets containing the bacteria [5]. Many people infected with tuberculosis do not show symptoms immediately and are in a state of latent tuberculosis. The person cannot transmit tuberculosis but can develop active tuberculosis. The spread of tuberculosis is influenced by several factors, one of which is age groups. Each age group has unique characteristics that affect how tuberculosis spreads within that population and how each age group responds to the infection [6]. In order to, understand and control the spread of the disease, it is necessary to use mathematical models that are able to represent the complex interactions between individuals in a population. One important tool in epidemiology is the transmission matrix, which is used to model interactions between individuals in a population and describe how infectious disease can be transmitted from one individual to another.

Transmission matrices present a mathematical framework that makes it possible to investigate patterns of disease transmission, predict the development of outbreaks, and design effective control strategies. Transmission matrices in epidemiological modelling are used to look at the spread within sub-populations and between sub-populations Manna et al [1]. Mathematical models using transmission matrices help predict how different strategies affect tuberculosis disease in populations that are also affected by HIV [7]. This model identifies a dynamic tuberculosis case-finding policy in the context of an epidemic tuberculosis epidemic. [8] developed a mathematical model that use a transmission matrix to understand the spread of tuberculosis in highly endemic region in Asia-Pacific. The transmission matrix was used to capture the dynamics of social contact and disease spread across different age groups and socio-economic conditions. In addition, a transformation matrix contact data was developed to evaluate the risk of tuberculosis spread in Zambia and Western Cape, South Africa [9]. The results showed that focusing on young age groups can be effective in controlling the spread of tuberculosis.

In a transmission matrix, each entry represents the transmission rate from one individual to another, with various factors affecting this transmission rate including physical contact, population mobility, individual immunity levels and the effectiveness of control interventions. Transmission matrix can be used in epidemic models that are more expressive and able to capture behavioural heterogeneity Dunbar [2]. Using epidemiological data, behavioural science, and mathematical modelling techniques, transmission models can be built that can provide predictions about the number of new cases, the rate of spread, and the impact of various intervention scenarios. However, transmission matrices also have a number of challenges and limitations. Individual variability, data uncertainty and the complexity of human interactions are some of the many factors that need to be considered in the use of transmission matrix. The use of transmission matrices makes it possible to model the complexity of individual interactions within the population. Thus the use of transmission matrices can help in identifying hotspots where diseases may spread rapidly. Therefore, a deep understanding of the strengths and limitations of transmission matrix is essential in the development of infectious disease control strategies.

In this study, an epidemiological model with a transmission matrix is analysed to understand the dynamics of tuberculosis spread. Through this approach, it is expected that a deeper understanding of the factors affecting the transmission of tuberculosis can be obtained and the most effective controls can be identified to reduce the spread of this disease.

2 Research Methods

The methodology used in this research is a literature study, which involves searching for references in books and journals. The research begins by studying epidemiological modelling and how transmission matrices represent interactions between subpopulations, as well as how the spread of infectious diseases is represented using transmission matrices. The next step is to establish the research objective, which is to model the spread of tuberculosis among certain subpopulations and to understand the disease dynamics within those populations. Based on this objective, data collection is conducted, along with the formulation of several assumptions and the definition of variables and parameters, such as the number of infectious disease cases, the level of interaction/contact between subpopulations, the transmission rate of the infectious disease, and the incubation period.

Based on these assumptions and parameter identification, an epidemiological model that aligns with the research objectives and disease characteristics is developed. From the data obtained, a transmission matrix is created that includes the number of transmission cases between different subpopulations within the population. After building the model and the transmission matrix, model calibration and validation are conducted. In this stage, model parameters are adjusted to ensure that the model can accurately replicate the observed epidemiological data. Subsequently, analysis is performed using the model and transmission matrix to understand the dynamics of disease spread, and the results are interpreted.

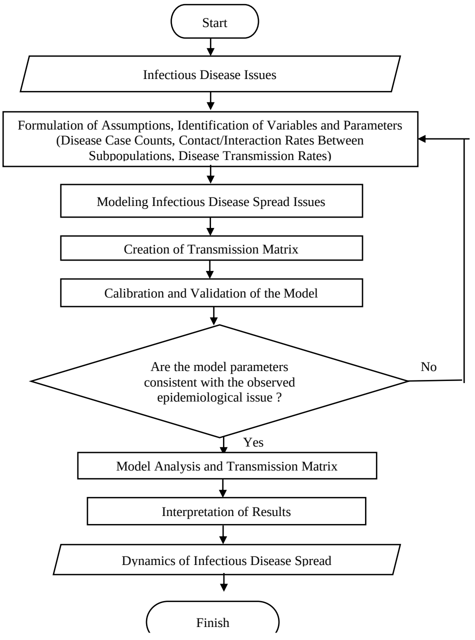

Fig. 1. Research method flowchart

3 Results and Discussions

Tuberculosis is an infectious disease caused by the bacteria Mycobacterium tuberculosis. Tuberculosis has a long and variable incubation period usually several weeks to months after exposure. Tuberculosis is spread through the air when an infected person coughs, sneezes, or talks, releasing droplets containing the bacteria. Many people infected with Tuberculosis do not show symptoms immediately and are in a state of Latent TB. The person cannot transmit tuberculosis but can develop active TB. The spread of tuberculosis is influenced by several factors, one of which is the age group. Each age group has unique characteristics that affect how tuberculosis spreads among the population and how each age group responds to the infection.

The teenagers age group (15-19 years old) begins to have wider social interactions such as school, playgrounds, which increases the risk of exposure to tuberculosis. Infected teenagers can transmit the disease and tend to realise symptoms sooner. However, sometimes the social stigma associated with tuberculosis can cause teenagers to not report their symptoms. The Adult age group (20 - 44 years) is at the peak of social and economic activity. This results in increased social interaction with many people and increased risk of spread. Adults infected with tuberculosis tend to be highly contagious. This is exacerbated by the presence of other chronic diseases such as diabetes or HIV. In the Elderly age group (45 years and above) the immune system declines. In addition, the presence of chronic diseases such as diabetes, heart disease, or malnutrition increases the susceptibility of the elderly to tuberculosis. In addition, elderly people who have been infected with latent TB may be at risk of reactivation of the infection when immunity declines. Tuberculosis in the elderly may be less contagious than in adults, but this age group can be a source of infection. In this age group, symptoms are difficult to diagnose as tuberculosis symptoms may mimic other chronic diseases common in the elderly. Mortality from Tuberculosis is higher in the elderly due to weakened immunity and complications from other chronic illnesses.

The mathematical model of the spread of the Tuberculosis virus is formed based on a population divided into 4 age groups, namely children (under 14 years), teenagers (15-19), adults (20-44) and the elderly (45 and above). The population is divided into four sub-populations, namely susceptible individuals, individuals who have been exposed to tuberculosis bacteria and are in the latent phase (exposed), individuals who have active tuberculosis and can transmit the disease (infected), and individuals who recover from tuberculosis (recovered). These sub-populations are denoted by S for susceptible, E for exposed, I for infected, and R for recovered, respectively.

The susceptible sub-population contains individuals who are not yet infected, but are susceptible to infection, and will become infected if they come into direct contact with infected individuals. The exposed sub-population contains individuals who have been exposed to tuberculosis bacteria but are not yet infected and cannot spread to other individuals. The infected sub-population contains individuals who have been infected and can transmit to other sub-populations. While the Recovered sub-population contains individuals who have successfully recovered, but still do not have permanent immunity, meaning that individuals who have recovered can still be infected again. Some assumptions that will be used to form the model are:

1. The number of individuals born equals the number of individuals who die (closed population)
2. Total population (N) is the total number of individuals in each sub-population. The number of individuals in the susceptible, exposed, infected, and recovered sub-populations are denoted by S, E, I, and R, respectively. The total population can be expressed as $N = S + E + I + R$ and is considered constant. The population is divided into age groups Children (0-14 years), teenagers (15-19 years), adults (20 - 44 years) and elderly (45 years and above).
3. Tuberculosis spreads through contact between susceptible and infected individuals.
4. Exposed individuals (E) are not immediately infectious until they turn into infected individuals.
5. Infected individuals may recover or die from tuberculosis.
6. Recovered individuals may lose immunity and become susceptible again.
7. There is natural mortality in each compartment. Death from tuberculosis occurs only in infected individuals.
8. Close contact with individuals infected with tuberculosis has a higher probability of transmission.
9. Contact between age groups (children, teenagers, adults and elderly) differs in terms of frequency and intensity.

Based on the above assumptions, the disease spread model is formed as follows:

$$\begin{cases} \frac{dS_i}{dt} = \Lambda N_i - \frac{\sum_{j=1}^{4} \beta_{ij} S_i I_j}{N_i} + \rho R_i - \mu_i S_i \\ \frac{d\mathrm{E}_i}{dt} = \frac{\sum_{j=1}^{4} \beta_{ij} S_i I_j}{N_i} - \sigma E_i - \mu_i E_i \\ \frac{dI_i}{dt} = \sigma E_i - (\mu_i + d_i) I_i - \tau I_i \\ \frac{dR_i}{dt} = \tau I_i - (\mu_i + \rho) R_i \end{cases} \tag{1}$$

With $N_i = S_i + E_i + I_i + R_i$, is the total population in age group i, with $i = 1$ children, $i = 2$ teenagers, $i = 3$ adults dan $i = 4$ elderly. The parameters of the model are presented in Table 1.

Table 1. Parameters of the model

Parameter	Description
Λ	Recruitment rate
β	Effective Tuberculosis contact rate
ρ	Rate at which recovered individuals become susceptible
σ	Rate at which exposed individuals become infected
τ	Rate at which infected individuals become recovered
μ	Natural death rate
d	Death rate because of Tuberculosis

An equilibrium point is a point that is constant and does not change with time. The equilibrium point can be determined if the following conditions are met:

$$\frac{dS_i}{dt} = 0; \frac{d\mathrm{E}_i}{dt} = 0; \frac{dI_i}{dt} = 0; \frac{dR_i}{dt} = 0 \tag{2}$$

Two kinds of equilibrium points for the Tuberculosis disease spread are obtained as follows:

1) Disease Free Equillibrum Point (E_0)

This equilibrium point occurs when there are no individuals infected with TB disease in a population, namely when $E = I = R = 0$. So that the equilibrium point is obtained as follows

$$E_0 = (S, E, I, R) = \left(\frac{\Lambda N_i}{\mu_i}, 0,0,0\right) \tag{3}$$

2) Endemic Disease Equillibrum Point (E_1)

This equilibrium point occurs if there is contact between individuals in the susceptible and infected TB subpopulations, which means that the exposed subpopulation, infected subpopulation and cured subpopulation remain. The following equilibrium point is obtained:

$$E_1 = (S^*, E^*, I^*, R^*) \tag{4}$$

$$S^* = \frac{N\big((d+\mu)(\mu+\sigma)+\tau(\mu+\sigma)\big)}{\beta\sigma}$$

$$E^* = \frac{(\Lambda\beta\sigma-\mu((d+\mu)(\mu+\sigma)+\tau(\mu+\sigma))N(d+\mu+\tau)(\mu+\rho)}{\beta\sigma\big((d+\mu)(\mu+\rho)(\mu+\sigma)+\mu\tau(\mu+\rho+\sigma)\big)}$$

$$I^* = \frac{(\Lambda\beta\sigma-\mu((d+\mu)(\mu+\sigma)+\tau(\mu+\sigma))N(\mu+\rho)}{\beta\big((d+\mu)(\mu+\rho)(\mu+\sigma)+\mu\tau(\mu+\rho+\sigma)\big)}$$

$$R^* = \frac{N\tau(\Lambda\beta\sigma-\mu((d+\mu)(\mu+\sigma)+\tau(\mu+\sigma))}{\beta\big((d+\mu)(\mu+\rho)(\mu+\sigma)+\mu\tau(\mu+\rho+\sigma)\big)}$$

After determining the equilibrium point, next determine the transmission matrix. To create a transmission matrix for the spread of disease, it is necessary to understand some basic concepts regarding the spread of Tuberculosis disease in the population. Transmission matrices are used in epidemiological models to describe how diseases spread between individuals or groups in the population. The transmission matrix includes elements that describe direct transmission from infected individuals to susceptible individuals (S) who become exposed (E). The Transmission matrix contains the value of β_{ij} which is the rate of transmission from age group j to age group i Since there are two infected classes (E and I) for 2 age groups of the population, the Transmission matrix is

$$T = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\beta_{11}}{N_1}S_1 & \frac{\beta_{12}}{N_1}S_1 & \frac{\beta_{13}}{N_1}S_1 & \frac{\beta_{14}}{N_1}S_1 \\ 0 & 0 & 0 & 0 & \frac{\beta_{21}}{N_2}S_2 & \frac{\beta_{22}}{N_2}S_2 & \frac{\beta_{23}}{N_2}S_2 & \frac{\beta_{24}}{N_2}S_2 \\ 0 & 0 & 0 & 0 & \frac{\beta_{31}}{N_3}S_3 & \frac{\beta_{32}}{N_3}S_3 & \frac{\beta_{33}}{N_3}S_3 & \frac{\beta_{34}}{N_3}S_3 \\ 0 & 0 & 0 & 0 & \frac{\beta_{41}}{N_4}S_4 & \frac{\beta_{42}}{N_4}S_4 & \frac{\beta_{43}}{N_4}S_4 & \frac{\beta_{44}}{N_4}S_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (5)$$

The disease spread transmission matrix provides an overview of how disease spreads between groups in the population based on contact rates and transmission probabilities. This matrix is very important in epidemiological models to understand the dynamics of disease spread and to design control strategies. Since it's at the disease-free equilibrium point (3) and the population is closed, so the transmission matrix is

$$T = \begin{pmatrix} 0 & 0 & 0 & 0 & \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ 0 & 0 & 0 & 0 & \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \\ 0 & 0 & 0 & 0 & \beta_{31} & \beta_{32} & \beta_{33} & \beta_{34} \\ 0 & 0 & 0 & 0 & \beta_{41} & \beta_{42} & \beta_{43} & \beta_{44} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (6)$$

After determining the transmission matrix we next determine the basic reproduction number. The basic reproduction number is the average of the number of new infections in the susceptible subpopulation produced by an infected person. The basic reproduction number (R_0) can be calculated using the transmission matrix and the infection transition matrix (V). The V matrix describes the rate at which individuals transition from susceptible to infected. The V matrix consists of the transition rate and the recovery rate from infection. Thus there are σ infection rate, death rate from tuberculosis (d_i), recovery rate (τ) and natural death (μ_i).

$$TV^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\beta_{11}}{\mu_1+d_1+\tau} & \frac{\beta_{12}}{\mu_2+d_2+\tau} & \frac{\beta_{13}}{\mu_3+d_3+\tau} & \frac{\beta_{14}}{\mu_4+d_4+\tau} \\ 0 & 0 & 0 & 0 & \frac{\beta_{21}}{\mu_1+d_1+\tau} & \frac{\beta_{22}}{\mu_2+d_2+\tau} & \frac{\beta_{23}}{\mu_3+d_3+\tau} & \frac{\beta_{24}}{\mu_4+d_4+\tau} \\ 0 & 0 & 0 & 0 & \frac{\beta_{31}}{\mu_1+d_1+\tau} & \frac{\beta_{32}}{\mu_2+d_2+\tau} & \frac{\beta_{33}}{\mu_3+d_3+\tau} & \frac{\beta_{34}}{\mu_4+d_4+\tau} \\ 0 & 0 & 0 & 0 & \frac{\beta_{41}}{\mu_1+d_1+\tau} & \frac{\beta_{42}}{\mu_2+d_2+\tau} & \frac{\beta_{43}}{\mu_3+d_3+\tau} & \frac{\beta_{44}}{\mu_4+d_4+\tau} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (7)$$

R_0 is the largest eigenvalue (spectral radius) of the matrix TV^{-1}. Mathematically

$$R_0 = \rho(TV^{-1}) \quad (8)$$

where ρ denotes the largest eigenvalue. If $R_0 > 1$, then the infection has the potential to spread within the population. If $R_0 < 1$, then the infection will wane and eventually disappear.

4 Numerical Solutions

Numerical simulation of the SEIRS model for tuberculosis spread provides insights into the dynamics of transmission across different age groups. The simulation was conducted using Runge-Kutta method by providing each parameter value can be seen in Table 2.

Table 2. The parameter values of the Mathematical model for the spread of tuberculosis

Parameter	value		Parameter	value
Λ	0,00904450447		β_{31}	0,001
μ_1	0,0003575985		β_{32}	0,01
μ_2	0,00120022		β_{33}	0,02
μ_3	0,005742038		β_{34}	0,005
μ_4	0,007114707		β_{41}	0,005
σ	0,1916113		β_{42}	0,005
β_{11}	0,02		β_{43}	0,005
β_{12}	0,01		β_{44}	0,01
β_{13}	0,005		ρ	0,01
β_{14}	0,005		τ	0.563878595
β_{21}	0,01		d_1	0,007150002
β_{22}	0,02		d_2	0,00239978
β_{23}	0,01		d_3	0.011480915
β_{24}	0,005		d_4	0,007114707

These parameter values were derived from Tuberculosis data in Kalimantan Barat. Based on Table 1, the transmission matrix is as follows:

$$T = \begin{pmatrix} 0{,}02 & 0{,}01 & 0{,}005 & 0{,}005 \\ 0{,}01 & 0{,}02 & 0{,}01 & 0{,}005 \\ 0{,}001 & 0{,}01 & 0{,}02 & ,0{,}005 \\ 0{,}005 & 0{,}005 & 0{,}005 & 0{,}01 \end{pmatrix} \tag{9}$$

Based on this matrix, children transmit tuberculosis to other children at a rate of 0,02 per year. Children transmit tuberculosis to teenagers at a rate of 0,01 per year. Children transmit tuberculosis to adults at a rate of 0,005 per year. Children transmit tuberculosis to the elderly at a rate of 0,005 per year.
The graph of the tuberculosis disease spread with the parameters from Table 1 can be seen in Fig 2.

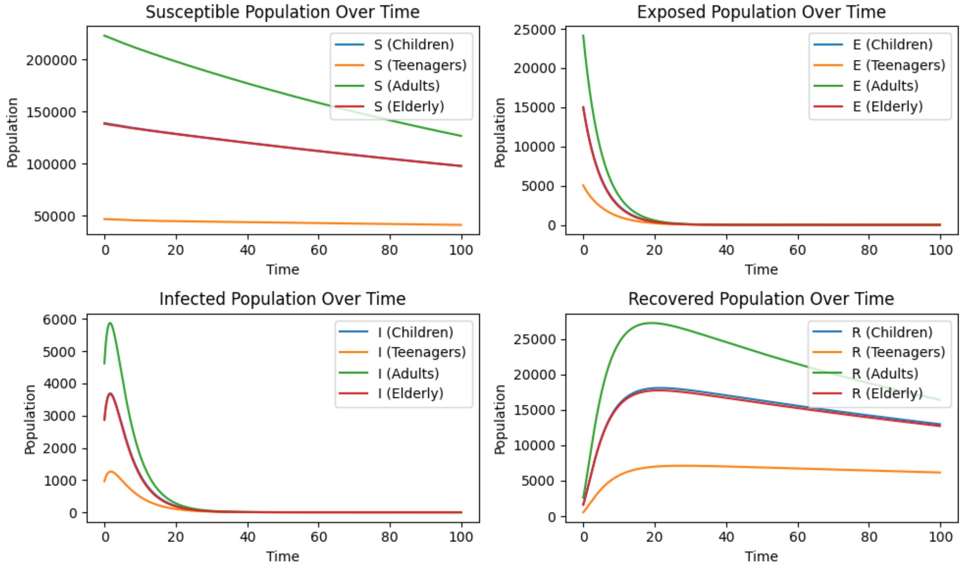

Fig 2. Graphs of Tuberculosis Spread by Age Group

Based on Fig 2, it can be observed that in children (0 -14 years), the number of susceptible individuals initially starts high, reflecting a large proportion of children who have not yet been exposed to tuberculosis. Over time, this number decreases as children become exposed or infected. The exposed subpopulation shows an initial increase as susceptible children come into contact with infected individuals and are exposed to tuberculosis. Subsequently, the infected subpopulation rises, following the trend of the exposed group, reaching a peak before eventually declining due to recovery or progression to other states. The recovered subpopulation gradually increases as children recover from tuberculosis. This increase reflects the role of immunity in reducing the susceptible population. In teenagers (15 – 19 years), the observed pattern is similar to that of children but differs in magnitude due to variations in transmission dynamics and contact rates. Teenagers tend to experience a faster decline in the susceptible population and a sharper peak in infections, likely caused by increased social interactions.

Among adults (20 – 44 years), the infection peak is higher, driven by more frequent interactions and a larger initial population base. The susceptible subpopulation declines more rapidly in this age group, while the exposed and infected populations remain significant for longer periods due to ongoing transmission dynamics. In the elderly (45 years and above), distinct dynamics are observed. The susceptible population declines more slowly, and infections tend to last longer, reflecting weaker immune responses and higher tuberculosis-induced mortality rates. Additionally, the number of recovered individuals increases more slowly, indicating a slower recovery process in this age group.

The model effectively captures the progression of tuberculosis epidemics with initial increases in exposed and infected populations, followed by recovery. The susceptible population steadily decreases, indicating the spread of the disease and a reduction in vulnerable individuals. Based on Figure 2, it is evident that the epidemic peaks at different times across age groups, emphasizing the need for control strategies tailored to specific age groups. Targeting particular groups would be more effective in managing the epidemic.

Factors influencing the dynamics of disease spread include varying transmission rates across age groups, influenced by their respective interaction patterns. Additionally, recovery and loss of immunity play crucial roles in shaping the long-term dynamics of the epidemic and determining the potential for future outbreaks. Variations in transmission rates between age groups reflect the impact of differing social interaction patterns on disease dynamics.

5 Conclusion

The use of transmission matrices in tuberculosis disease spread models allows for a deeper understanding of how the disease spreads among different age groups. The basic reproduction number is calculated using the transmission matrix and the infection transition matrix, which indicates the average number of new infections caused by an infected individual in a susceptible population. The transmission matrix depicts the rate at which tuberculosis is transmitted from one group to another. By understanding these interaction patterns, we can identify groups that are at higher risk of transmitting or contracting the disease.

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Application of the XGBoost Algorithm for Predicting the Target Effective Temperature in Closed Broiler Chicken Cage

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Abstract. Broiler chickens are a breed known for their rapid growth, typically reaching maturity in just 4-5 weeks. This growth is influenced by various factors, with cage management playing a significant role. One key factor in cage management is maintaining an optimal target temperature, which is determined by combining measurements of ambient temperature, humidity, and wind speed. This article examines how the XGBoost algorithm can be used to predict the target effective temperature in closed-house broiler chicken systems. The goal is to develop a predictive network model with high accuracy, enabling the regulation of cage conditions to ensure the chickens' comfort. The study findings demonstrate that the proposed algorithm effectively models target temperatures, aiding in the management of cage conditions.

1 Introduction

The livestock sector plays a vital role in meeting the nutritional needs of the Indonesian population. This sector provides high-protein food products such as meat, milk, and eggs commonly consumed. Specifically, the production and consumption of broiler chicken meat in Indonesia have shown significant growth year after year, according to a survey by the Indonesian Central Bureau of Statistics [1].

Broiler chickens are more widely cultivated due to their high productivity. In just 4 – 5 weeks, broiler chicks can grow into mature broiler chickens weighing around 2 kg and ready for harvest, offering promising profits for farmers. In general, there are two main broiler chicken farming systems: open housing and closed housing. The closed housing system (Figure 1) is considered by many farmers because it is less affected by weather conditions and environmental stress. Additionally, in terms of number of chickens per square meter, broiler chickens raised in closed housing can be higher [2].

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Fig. 1. Chicken farming in closed cages taken from [2].

1.1 Background

Several important factors influencing housing comfort must be controlled to ensure optimal broiler chicken growth. These factors include the actual enclosure temperature, air humidity, and wind speed. Housing with low temperatures causes chickens to huddle together and reduce their activity. Conversely, housing with relatively high temperatures makes chickens feel thirsty more easily, leading to higher water consumption compared to food consumption. This can ultimately affect the chickens' growth, resulting in lower weights at harvest.

Therefore, in managing poultry housing, it is crucial to regulate the enclosure temperature to ensure the chickens grow comfortably. Such a temperature is referred to as the target effective temperature, which varies depending on the chickens' age. Older chickens require a lower target effective temperature because their body temperature becomes higher as they age. Additionally, air humidity in the enclosure greatly influences the perceived temperature, as higher humidity makes the chickens feel warmer. To address this, closed housing is usually equipped with fans that can lower the actual enclosure temperature to achieve the target effective temperature (Figure 2).

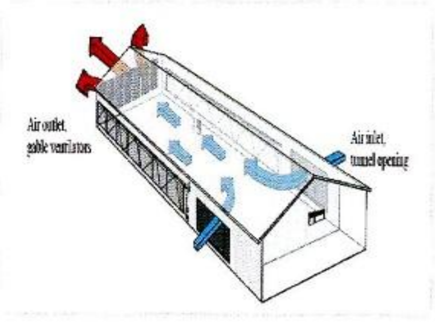

Fig. 2. Closed cages are equipped with fans to lower the temperature taken from [3].

Table 1 provides the target effective temperature in closed housing for chickens of various ages, ranging from 1 – 2-day-old chicks to mature chickens over 36 days old. The target effective temperature ranges from 21°C to 32°C. For 1 – 2-day-old chicks, the closed housing temperature must be set to around 32°C. This temperature is necessary for optimal growth. The table also shows a trend indicating that older chickens require lower housing temperatures for optimal growth.

Table 1. Target Effective Temperature in closed cage taken from [3].

Chicken age (in days)	Target Effective Temperature (in degrees Celcius)
1 – 2 days	32
3 – 4 days	31
5 – 7 days	30
8 – 14 days	29
15 – 21 days	27
22 – 28 days	25
29 – 35 days	22
More than 36 days	21

Additionally, data on predictor variables—such as actual temperature, air humidity, and wind speed (generated by fans)—and the response variable, the target effective temperature, are available for various humidity levels, as shown in Figure 3. In this table, the variables include: housing temperature in degrees Celsius (°C), humidity in the housing as relative humidity (%rH), and wind speed in the housing in feet per minute (fpm). These variables—temperature, humidity, and wind speed—are used as indicators in calculating the estimated target effective temperature such that the comfort of the chickens in the enclosure is maintained. The calculated target effective temperature is then used as an evaluation for adjusting the wind speed settings based on the chickens' needs and comfort during their care.

Actual Suhu (oC)	Air Velocity Feet Per Minute at Rh 70%										
	0	50	100	150	200	250	300	360	400	450	500
35.0	38	37	36	33	31	30	29	27	26	25	24
34.6	38	36	35	33	30	29	29	27	26	25	24
34.1	37	36	35	32	30	29	28	27	26	25	24
33.6	37	36	34	32	30	29	28	27	26	25	24
33.1	36	35	34	32	29	29	28	27	26	25	24
32.7	36	35	33	31	29	28	27	27	26	25	23
32.2	36	34	33	31	29	28	27	26	26	24	23
31.7	35	34	32	30	29	28	27	26	25	24	23
31.3	34	33	32	30	28	27	27	26	25	24	23
30.8	34	32	31	30	28	27	26	26	25	24	23
30.3	33	32	31	29	28	27	26	25	25	24	23
29.9	32	31	30	29	27	27	26	25	25	24	23
29.4	32	31	30	29	27	26	26	25	24	24	23
28.9	31	30	29	28	27	26	25	24	24	23	23
28.5	31	30	29	27	26	26	25	24	23	23	22
28.0	30	29	28	27	26	25	24	23	22	22	21
27.5	29	28	27	26	25	25	24	23	22	21	21
27.1	29	28	27	26	25	24	24	22	21	21	20
26.6	28	27	26	25	24	24	23	22	21	20	19
26.2	28	27	26	25	24	24	23	22	20	20	19
25.7	27	26	26	25	24	23	23	22	20	20	19
25.3	27	26	25	25	24	23	23	22	20	20	19
24.8	26	26	25	24	24	23	23	21	20	20	19
24.4	26	26	25	24	23	23	22	21	20	19	19
23.9	26	26	24	24	23	23	22	21	20	19	19
23.4	26	24	24	23	23	22	22	21	20	19	19
23.0	25	24	23	23	22	22	21	20	19	19	18
22.5	24	23	22	22	21	21	21	20	19	19	18
22.0	24	23	22	21	21	20	20	19	19	18	18
21.6	24	22	21	21	20	20	19	19	19	18	17
21.1	23	22	21	20	19	19	19	19	18	18	17

Target Efectif - Tempertur (TET)	
Umur	TET
1 - 2 hr.	32°C
3 - 4 hr.	31°C
5 - 7 hr.	30°C
8 - 14 hr.	29°C
15 - 21 hr.	28°C
22 - 28 hr.	26°C
29 - 35 hr.	23°C
36 - Lay.	22°C

70 %

Fig. 3. Relationship between actual temperature, wind speed and target effective temperature at 70% humidity taken from [3].

In practice, broiler chicken farmers using closed housing adjust the wind speed (with fans) and humidity (with an evaporator) to achieve the desired target effective temperature in the enclosure, doing so manually based on the data in Figure 2. For example, at a humidity level of 70%, for 1 – 2-day-old chicks in an enclosure with an actual temperature of 34.1°C, the fans must be set to a speed of 150 fpm to bring the enclosure temperature down to 32°C.

1.2 Problem Statement

However, the available tables are quite limited, covering only humidity levels of 50%, 70%, 80%, and 90% for a cage measuring 110 m x 10 m with a capacity of 20,000 chickens.

This study aims to design a mathematical model based on the XGBoost algorithm to formulate the relationship between humidity, wind speed, actual temperature, and the target effective temperature of the cage. As a result, standard tables for other humidity variations can be generated. If this is successfully implemented, the automation of closed-cage management can be achieved. This will further simplify the process for broiler chicken farmers in regulating the target effective temperature of their cages.

2 Method

In this part, we discuss the method outlining the approach and technique used to develop a mathematical model to achieve the objectives of this study. It provides a detailed description of the proposed model and the method of the hyperparameter tuning.

2.1 XGBoost

XGBoost, short for Extreme Gradient Boosting, is a fast, scalable, powerful and efficient algorithm which is used widely for supervised learning tasks, i.e. regression and classification. It is so popular because its superior performance in machine learning competitions and many real applications. The method proposed by Tianqi Chen in 2014 [4], belongs to gradient boosting algorithms which build many decision tree models sequentially and each new model corrects the errors resulted from the previous ones. Furthermore, the XGBoost outperforms the existing gradient boosting implementations in the speed and accuracy, handling missing values, reducing overfitting with regularization, and supporting parallel and distributed computing [5].

The objective function $\mathcal{L}$ of XGBoost algorithm has two key important components to be minimized, i.e. loss function ℓ and Ω regularization term.

$$\mathcal{L} = \sum_{i=1}^{n} \ell(y_i, \hat{y}_i) + \sum_{k=1}^{K} \Omega\,(f_k) \tag{1}$$

The loss function ℓ will measure how well the model's output $\hat{y}_i$ predicts the true label y_i, i.e. mean square error for regression and log loss for classification. The regularization term Ω will reduce the complexity of the model to avoid overfitting. For a tree f_k, the regularization is defined as

$$\Omega(f_k) = \gamma\, T + \frac{1}{2}\,\lambda \sum_{j=1}^{T} w_j^2 \tag{2}$$

where T the number leaves in the tree, w_j weight of the j-th leaf, γ regularization parameter for the number of leaves and λ regularization parameter for leaf weights. The prediction $\hat{y}_i$ for a datum x_i is the sum of the outputs from all K trees, i.e.

$$\hat{y}_i = \sum_{k=1}^{K} f_k(x_i) \tag{3}$$

where $f_k \in \mathcal{F}$ the space of decision trees. XGBoost will minimize $\mathcal{L}$ by adding trees sequentially, where each new tree f_k minimizes the following approximate objective

$$\mathcal{L}^{(t)} \approx \sum_{i=1}^{n} \left[g_i f_t(x_i) + \frac{1}{2} h_i f_t^2(x_i) \right] + \Omega\,(f_t) \tag{4}$$

where $g_i = \frac{\partial \ell\left(y_i, \hat{y}_i^{(t-1)}\right)}{\partial\, \hat{y}^{(t-1)}}$ is the gradient of the loss function and $h_i = \frac{\partial^2\, \ell\left(y_i, \hat{y}_i^{(t-1)}\right)}{\partial\, \hat{y}^{(t-1)}}$ is the Hessian of the loss function. Here t refers to the index of the current iteration or tree being optimized and the predicted value for an instance i at iteration t is

$$\hat{y}_i^{(t)} = \hat{y}_i^{(t-1)} + f_t(x_i) \tag{5}$$

where $\hat{y}_i^{(t-1)}$ is the prediction from the previous $(t-1)$ iterations and $f_t(x_i)$ is the contribution of the current tree t to correct the residual errors.

2.2 Hyperparameter Tuning

As it is known, XGBoost usually ensembles multiple weak different decision trees in a sequential and additive manner using gradient boosting. Each tree focuses on correcting the errors of the previous ones and the ensemble as a whole becomes a strong predictive model. Like a standard decision tree, the XGBoost model has also several hyperparameters that need to be tuned to optimize prediction accuracy [6,7,8]. The following table lists the most important hyperparameters of the XGBoost model along with their functions and effects.

Table 2. Several important hyperparameters of the XGBoost.

Name of hyparameters	Common abbreviations	Function	Effect
learning rate	eta	control the contribution of each tree to the final model	decreasing prevents overfitting
number of trees	n_estimators	specify the total number of boosting iterations (tree)	increasing may improve scores with large data
maximum tree depth	max_depth	determine the maximum depth of each tree	decreasing prevents overfitting
minimum child weight	min_child_weight	minimum sum of instance weights (hessian) needed to create a child node	increasing prevents overfitting
subsampling	subsample	fraction of training data used for growing each tree to reduce overfitting	decreasing prevents overfitting
column subsampling	colsample_bytree	fraction of features used to grow each tree to reduce overfitting	decreasing prevents overfitting

L2 regularization constant	lambda	penalizes large leaf weights to prevent overfitting by shrinking weights but retain all features	increasing prevents overfitting
L1 regularization constant	alpha	penalizes large leaf weights to prevent overfitting by introducing sparsity (zero weights for some features)	increasing prevents overfitting

To select the optimal set of the above hyperparameters, we used a method called randomized search [9] that randomly samples combinations of hyperparameter values from predefined ranges. Comparing to grid search method which evaluates all possible combinations of predefined distribution of hyperparameter values, it is more efficient due to its ability to save computational time and identify near-optimal hyperparameter settings.

3 Results and Discussion

This section begins by outlining the key results derived from the analyses, supported by relevant statistical and graphical representations. Emphasis is placed on finding patterns, relationships, or trends that align with or diverge from the initial hypotheses.

3.1 Data Analysis

The data used in this article consists of 1,227 entries, which represent the target effective temperature data for various actual temperature conditions in the cage, air humidity, and wind speed, as available in the literature [3]. Thus, there are three predictor variables: x_1 (air humidity), x_2 (wind speed), x_3 (actual cage temperature), and one response variable y (target effective temperature). Out of the total data points used, around 90% are used as training data to build the model, and the remaining 10% are used as test data.

Before building the model, preliminary data processing will be conducted to get the general description of the data and the relationships between the predictor variables and the response variable. Figure 3 shows the distribution of the data used in this analysis. Using a specific device, the humidity in cage can be adjusted to only a few values, namely 50%, 70%, 80%, and 90%. Wind speed can also be controlled by adjusting a number of fans, resulting in values of 0, 50, 100, ..., 500 feet per minute. The actual cage temperature, measured with a thermometer, ranges from 21.1°C to 35°C. The target effective temperature, as the response variable resulting from the interaction of the three predictor variables (humidity, wind speed, and actual cage temperature), is distributed as shown in the histogram in the bottom-right corner.

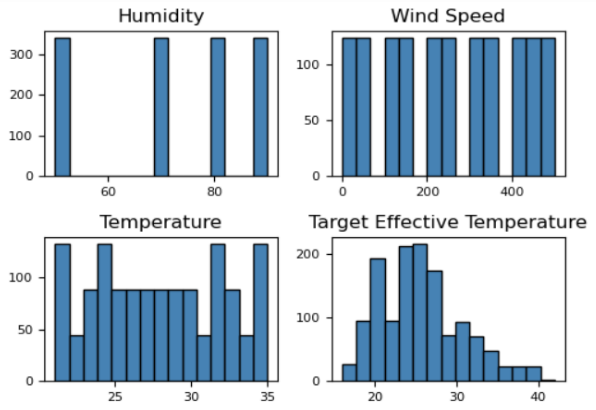

Fig. 4. Histogram of the data distribution.

Figure 5 shows the Pearson correlation coefficient between variable pairs. From the simple correlation between two variables, it can be concluded that the actual cage temperature variable is the most strongly (positively) correlated with the target effective temperature response variable, with a coefficient of 0.67. The wind speed variable has a negative correlation with the target effective temperature, while the humidity variable shows the weakest (positive) correlation. It means that the higher the humidity and temperature, the greater the target effective temperature. Conversely, the higher the wind speed, the lower the target effective temperature.

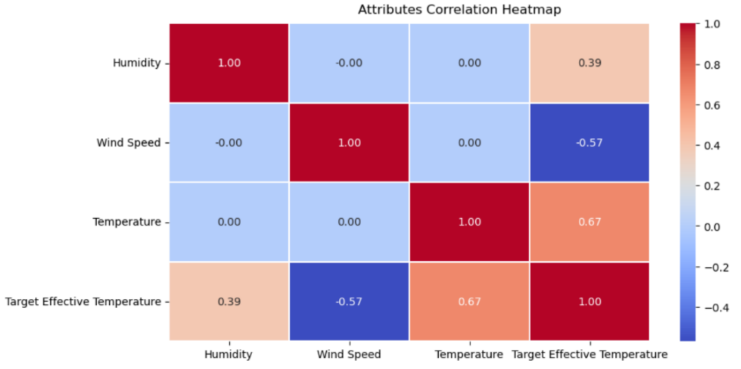

Fig. 5. Correlation coefficient between variables.

Using Python programming language, the XGBoost algorithm [10] is applied to the training data to identify the functional relationship between the predictor variables and the response variable. In this process, hyperparameter tuning is done to find the optimal hyparameters minimizing mean square error (mse). The optimal model obtained will then be tested using the test data, providing an estimate of the actual mse of the model.

3.2 XGBoost Model

The XGBoost regression model for this problem is built on the following hyperparameter setting. The first model using default values for hyperparameter (see third column of Table 3) gives mean square error around 0.1537. The second model using values resulting from randomized search (see fourth column of Table 3) produces a smaller mean square error, i.e. 0.146. Both models demonstrate strong predictive performance in estimating the target effective temperature.

Table 3. Default value hyperparameter in the XGBoost package.

name of hyparameters	common abbreviations	default values	optimal values
learning rate	eta	0.3	0.05
number of trees	n_estimators	100	1200
maximum tree depth	max_depth	6	3
minimum child weight	min_child_weight	1	6
subsampling	subsample	1	1
column subsampling	colsample_bytree	1	1
L2 regularization constant	lambda	1	0.05
L1 regularization constant	alpha	0	0.05

Furthermore, as shown in Figure 6, the feature importance analysis revealed that actual cage temperature contributed the most to the predictions, accounting for 42% of the total importance. Air humidity and wind speed also showed moderate contributions at 35% and 23%, respectively.

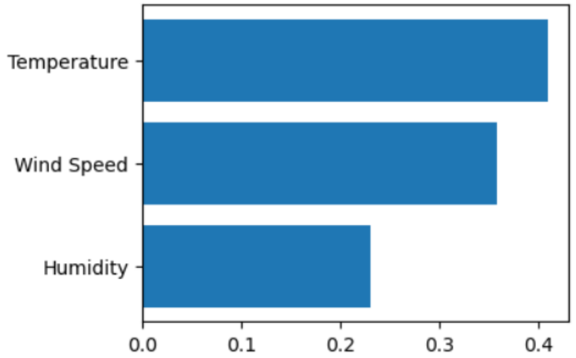

Fig. 6. Feature importance.

Overall, the results indicate that the XGBoost model can effectively predict the target effective temperature with minimal error, making it a robust tool for environmental management in broiler chicken farming.

4 Conclusion

This study demonstrates the effective application of the XGBoost algorithm for predicting the target effective temperature in closed broiler chicken cages. By utilizing key environmental variables such as actual cage temperature, wind speed, and humidity, the model achieved high accuracy and reliability in forecasting the target effective temperature. The results reveal that the actual cage temperature is the most influential predictor, showing a strong positive correlation with the target effective temperature, while wind speed and humidity contribute less significantly. The findings highlight the potential of XGBoost as a robust predictive tool for optimizing environmental control in closed broiler chicken cages, promoting animal welfare and operational efficiency. Future research could explore integrating additional variables and testing the model under diverse conditions to further enhance its adaptability and precision.

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Long Short-Term Memory and Bidirectional Long Short-Term Memory Algorithms for Sentiment Analysis of Skintific Product Reviews

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Abstract. In the era of ever-evolving digital technology, conducting customer sentiment analysis through product reviews has become crucial for businesses to improve their offerings and increase customer satisfaction. This research aims to analyze the sentiment of SKINTIFIC skincare products on the Shopee online store platform using advanced deep learning models: Long Short-Term Memory (LSTM) and Bidirectional Long Short-Term Memory (Bi-LSTM). These models were evaluated using learning rate, number of units, and dropout rate. The dataset consists of 9,184 product reviews extracted through the Shopee API. The reviews were pre-processed using stemming, normalization, and stopword removal techniques. The Bi-LSTM model showed superior performance, achieving an average accuracy of 95.91% and an average F1 score of 95.82%, compared to the standard LSTM model. The optimal configuration for Bi-LSTM included a learning rate 0.01, 64 units, and a dropout rate 0.2. These findings underscore the effectiveness of Bi-LSTM in understanding and classifying consumer sentiment toward specific products.

1 Introduction

In the digital age, online shopping through e-commerce has become one of the main shopping trends. Modern society encourages online shopping for several reasons. One reason people choose to shop online is the ability to see customer reviews posted on online stores, which can help them consider buying products based on customer reviews [1].

The Shopee online store platform is one of the e-commerce platforms that offer a wide variety of products and services, and it has become the leading destination for consumers looking for various kinds of things they need [2]. One of the trending products in e-commerce is skincare products. This research will analyze user sentiment towards SKINTIFIC products on Shopee by looking at these online shopping trends. SKINTIFIC is one of the popular brands in Indonesia, and its flagship product is ceramide moisturizer. This brand has become popular among teenagers and adults. SKINTIFIC originated in Canada and was founded in 1957 by Kristen Tveit and Ann Kristin Stokke. Its products use skincare industry technology that creates innovative products with pure active ingredients, brilliant formulations, and Trilogy Triangle Effect (TTE) Technology [3].

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Human interaction with computers has advanced significantly, thanks to Natural Language Processing (NLP). NLP plays a crucial role in many areas, including text generation, opinion mining, machine translation, named entity recognition (NER), speech recognition, and text summarization [4]. One of its most impactful applications is sentiment analysis, which predicts people's thoughts and emotions based on reviews, social media posts, and online forums. As businesses increasingly move online, unstructured data has become a treasure trove for insights, market research, and competitive analysis. Automated sentiment analysis is now vital for managing customer service, monitoring social media, and analyzing customer feedback. It simplifies processes by categorizing posts, survey responses, and scanned emails or documents [5].

In the real world, customers often share their opinions on product reviews. Many reviews are found with ratings that differ from their descriptions. For example, in the text of a review, a person gives a positive review while also giving a one-star rating, which means negative. A machine learning approach using NLP techniques is used to solve cases like this. The ability of NLP to perform product sentiment analysis has been studied in recent years. The use of several algorithms and comparing them with each other to see which algorithm can give optimal results in the study of product sentiment analysis.

Yadav, Vermar, and Katiyar [6] conducted a study by presenting a sentiment analysis method designed for Hindi e-commerce product reviews, utilizing a combination of the Long Short-Term Memory (LSTM) network and the Continuous Bag of Words (CBoW) model. Using five datasets, including Hindi SentiWordNet (HSWN), ABSA, and Twitter reviews, this research addresses the challenges posed by noisy data, word order dependency, and unbalanced datasets. Pre-processing steps such as tokenization, lemmatization, stopword removal, and max-pooling for dimensionality reduction significantly improve model performance. The proposed approach achieves an average accuracy of 87.71% across datasets, outperforming traditional methods such as SVM and state-of-the-art CNN-SVM models. In addition, the approach also exhibits superior precision, recall, and F1-score, with an F-score of 0.88. While the model demonstrates robustness in handling large data sets, it highlights the limitations of Hindi-specific text challenges, including polysemy and limited linguistic resources. The authors suggest further exploring solutions to these challenges in future work to refine the model's effectiveness in Hindi sentiment analysis.

Another study conducted by Yang, Li, Wang, and Sherratt [7] proposed a novel sentiment analysis model (SLCABG) for Chinese e-commerce product reviews, which combines sentiment lexicons with deep learning techniques, specifically CNN, BiGRU, and attention mechanisms. The model enhances sentiment features by using a lexicon to weight word vectors, followed by CNN to extract important features, BiGRU for contextual features, and attention mechanism to prioritize influential words. Experiments on a dataset of 100,000 reviews from the Chinese e-commerce site Dangdang show that the model outperforms traditional and deep learning models with higher classification accuracy, precision, recall, and F1-score. Key findings show that lexicon weighting improves sentiment feature extraction, optimal performance is achieved with a specific thesaurus size and number of iterations, and dropout regularization improves generalization. The paper concludes that the SLCABG model effectively classifies sentiment as positive or negative but lacks granularity for subtle sentiment differences, thus suggesting future research to refine sentiment categorization.

Another study that explored the application of a Bidirectional Long Short-Term Memory (Bi-LSTM) network for sentiment analysis on product reviews was conducted by Mahadevaswami and Swathi [8]. They used the Amazon Product Reviews dataset, explicitly focusing on the Mobile Electronics category, which contained 104,975 reviews. The main objective was to classify user reviews into positive or negative sentiments, utilizing deep learning techniques. The study employed several pre-processing steps, such as tokenization and case-folding, to clean and standardize the text data. A text encoder transformed the text into word vectors, and padding ensured uniform input length for the Bi-LSTM model. The proposed architecture included embedding layers, two Bi-LSTM layers with 128 units each, and dense layers with ReLU activation to improve classification performance. Dropout layers were integrated to mitigate overfitting, and binary cross-entropy was used as the loss function. The model achieved significant accuracy improvements over baseline methods. During training, the model reached a final accuracy of 90.14% with a validation accuracy of 88.08% after five epochs. The test set achieved an overall accuracy of 91.4%, outperforming other models such as CNN and standard LSTM, which attained 87.62% and 86.56%, respectively. The study concluded that the Bi-LSTM model is highly effective for sentiment analysis due to its ability to process long-term dependencies in sequential data. Future work suggested incorporating more nuanced sentiment classifications (e.g., multi-tier emotions) and employing ensemble methods to enhance performance further.

Other research on NLP has also been done using customer reviews on women's clothing. Several deep learning algorithms were applied and compared to obtain optimal results. There are various neural network (NN) models, including CNN, RNN, Bi-LSTM, and ensemble models, and word embedding techniques such as Word2Vec, FastText, and BERT variants (RoBERTa and ALBERT). Data augmentation enhanced the dataset, and experiments were conducted with two sentiment ranking settings: 5-class (detailed) and 3-class (compressed) classification. Findings show that Bi-LSTM with Word2Vec embedding consistently outperforms other NN models, especially when using the augmented dataset and 3-class setting. RoBERTa performed best among the BERT variants, although ensemble NN models, specifically CNN-RNN-Bi-LSTM, achieved the highest accuracy (96%) and F1-score (91.1%). The deep learning models significantly outperformed traditional machine learning models such as SVM, Naïve Bayes, and Random Forest, which showed at least 20% lower accuracy. This study concludes that deep learning is more effective for sentiment rating prediction, especially with enhanced classes and augmented datasets. However, limitations include the potential impact of spam or fake reviews in the dataset and the exclusive focus on English reviews. Thus, future research should combine multilingual datasets and lexicon-based approaches to improve sentiment analysis further [9].

In our research, we will use review description data because the description can provide more context and depth about the product's user experience. Shopee e-commerce customer review data about SKINTIFIC cosmetic products is used to understand, analyze, and describe the opinions and feelings contained in product reviews given by consumers, both positive and negative. By categorizing customer reviews of SKINTIFIC products into two groups, namely positive and negative reviews, it is hoped that it can provide consideration to people who want to buy SKINTIFIC products. To get optimal results, two deep learning algorithms, namely LSTM and bi-LSTM, are used with various supporting parameters so that, in the end, it can be seen which algorithm with what parameters will be the most optimal.

2 Research Methodologies

The research phase begins with data collection; then, pre-processing is carried out to clean and prepare it for further analysis. Then, the process of labelling, tokenizing, and dividing training and testing data is carried out. This is followed by creating a testing scenario that aims to test the model with different situations to measure the extent to which the model can function in various contexts. Then, the model will be created according to the designed scenario, and the model will be trained using the data that has been prepared previously. In the last stage, the model will be evaluated to measure how much it performs according to the research objectives.

In general, the research is carried out in several stages, as seen in Fig. 1.

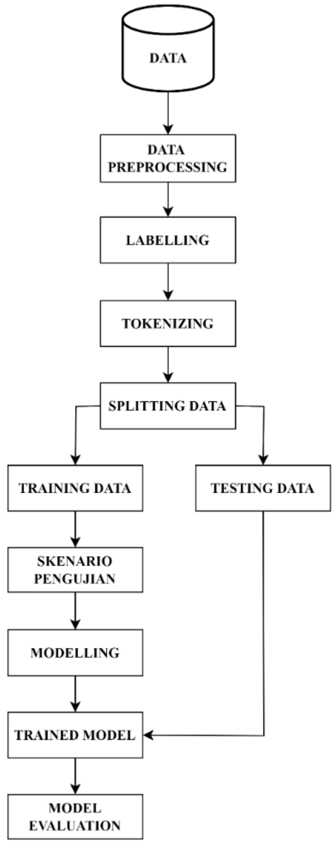

Fig. 1 Research workflow

2.1 Data

The data used are SKINTIFIC skin care product review data. The data was taken from the Shopee online store platform API on October 8, 2023. The data features are item ID, product name, comment, and rating. 20 SKINTIFIC skin care products were randomly selected and analyzed for reviews. The API used is the product API, which can be accessed via the following URL:

https://shopee.co.id/api/v2/item/get_ratings?filter=0&flag=0&itemid={item_id}&offset={offset}&shopid={shop_id}&type=0

Where item_id and shop_id was obtained from each product link, offset and limit were defined at the beginning.

The first process in data collection is to collect product links in a list and iterate over each product link [10]. In the iteration, it will extract the item_id and shop_id using regular expressions and compile the product's API URL. After that, it will request the Shopee API using the URL. After getting a response in the form of data, the data will be processed to be more structured, and the data retrieved includes item ID, product name, link, comment, and rating.

The total data obtained is 9,192, and the data that will be used for training is data whose word length is less than or equal to 100, so the remaining data is 9,184. Furthermore, the data will be stored in the form of data frames.

2.2 Pre-processing

Pre-processing steps include lowercasing, removing unnecessary characters, removing non-alphanumeric characters, normalizing slang words, stemming, and stopping word removal.
Lowercasing: Changing all letters into lowercase so that the text becomes insensitive to the difference in uppercase and lowercase letters. Lowercasing can improve classification performance without having to worry about inconsistencies in the text [11].
Remove Unnecessary Characters: Removing unnecessary characters is the process of cleaning the text from irrelevant elements such as punctuation marks, special symbols, and excessive spaces. This process improves the quality of text data, making it easier to process and analyze.
Remove Non-Alphanumeric: The remove non-alphanumeric process removes characters or symbols that are not letters or numbers in the tweet. This is done because these symbols have no information or meaning in the text. In the remove non-alphanumeric process, the program will separate words by connecting characters such as punctuation marks and remove all characters that are not letters or numbers. Non-alphanumeric characters can also represent regular expression patterns (Regex Pattern) but must not contain spaces [12].
Slang Word Normalization: Slang words refer to highly informal uses of language and expressions, which tend to be more figurative, playful, brief, lively, and short-lived than everyday language. In addition, slang words are not an official language listed in dictionaries. They are only a language style certain groups use [13].
Stemming: Stemming is a step where words are mapped and separated into their basic form [14]. The purpose is to facilitate the analysis and understanding of the language and to group words that have the same meaning into basic forms so that they can be considered a single unit in the text. For example, stemming can convert varied words such as "play," "game," and "play" into their simpler base form, namely "play."
Stop word Removal: Stop word removal is the process of removing words that have no meaning in a particular context, such as removing links, hash marks, and words that do not play an important role in sentiment. These stop words are words that do not affect sentiment. For example, words like "at", "by", "of", "a", and the like [15].

2.3 Labelling

The data collected do not yet have a label, so they need to be labelled. Labelling is the process of labelling a text based on the polarity of the sentiment contained in it. This research will do labelling with the output of 2 positive and negative labels. This research does not use neutral labels because neutral labels are between positive and negative, so there is a possibility that data in neutral labels can be identified as positive or negative [16].

Labelling is done using the Indonesian RoBERTa Base Sentiment Classifier. RoBERTa (Robustly optimized BERT approach) is a model that thoroughly understands text context and is used to handle various natural language processing tasks [17]. The Indonesian RoBERTa Base Sentiment Classifier is an optimized version of the RoBERTa model for classifying sentiment in text. The model is initially based on the Indonesian RoBERTa Base, which has gone through a previous training process, and then adapted to the indole SmSA dataset, consisting of Indonesian comments and reviews [18].

Tokenizing is the process of dividing text into smaller units, or "tokens," so that it is easy to process further.

2.4 Data splitting.

Data splitting will be done using the k-fold cross-validation method. The data obtained will be divided into training and test data using the k-fold approach. The dataset will be divided into k equal subsets. Each subset will be used alternately as test data, while the other subset will be used as training data. This process will be repeated k times, where each subset will be used as test data exactly once.

2.5 Modelling.

Modelling will be done using the Keras framework in Python. The model uses Sequential objects, a simple and linear model type. The models that will be created are LSTM and Bi-LSTM models. Then, create a parameter_search function to run the model using the specified parameter combination. First, it divides the data into five folds. After that, iterate each combination of bidirectional parameters, learning rate, num units, and dropout. For each parameter combination, it will be entered into the create_model function. Then, the model will be trained using training data from each fold with five epochs and a batch size of 64. Next, we will examine the model's performance by calculating Loss, accuracy, and f1-score.

2.6 Experiments

The experiment will compare the use of LSTM and Bi-LSTM layers, learning rate, num_units, and dropout parameters. Table 1 shows the parameter combinations to be tested, and Table 2 shows the model architecture.

Table 1. Parameter of experiments

Parameters	**Values**
bidirectional	[True, False]
learning_rate	[0,001, 0,01]
num_units	[64, 128, 256]
dropout	[0,2, 0,5]

Table 2. Model Architectures

Model	***Layer***
LSTM	Embedding (100) LSTM (num_units/ activation) GlobalMaxPooling1D () Dense(num_units/activation) Dropout () Dense(2/softmax)
Bi-LSTM	Embedding (100) Bi-LSTM (num_units/ activation) GlobalMaxPooling1D () Dense(num_units/activation) Dropout () Dense(2/softmax)

The dataset will be divided into five folds, each of which will take the performance results of the model evaluation in the form of Loss, accuracy, and f1-score. The model is trained using several combinations of parameters determined above. The model evaluation results from each fold will then be averaged to obtain the average Loss, accuracy, and f1-score. The average results of the three evaluations metrics will be analyzed.

3 Results and discussions

The results are analyzed based on the five highest average accuracy values because it wants to know how precise the model is in classifying. The results of the five highest values can be seen in Table 3. The table highlights the performance of different configurations of LSTM and Bi-LSTM models based on average Loss, accuracy, and F1-score. These metrics are crucial for evaluating the models' ability to generalize and perform well on a given task.

Table 3. The best five results of the implementation of LSTM and Bi-LSTM

No.	Layer	Learning rate	Num Units	Dropout	Average Loss	Average accuracy	Average F1-score
1	Bi-LSTM	0,01	64	0,2	14,17%	95,91%	95,82%
2	Bi-LSTM	0,01	64	0,5	14,34%	95,79%	95,74%
3	LSTM	0,01	128	0,2	13,67%	95,78%	95,82%
4	Bi-LSTM	0,01	128	0,2	13,79%	95,57%	95,60%
5	Bi-LSTM	0,01	128	0,5	17,34%	95,54%	95,52%

A detailed analysis of the results reveals important trends and trade-offs between model complexity, dropout rates, and performance. Average Loss is a key metric for understanding how well the model fits the training data. The lowest Loss, 13.67%, is achieved by the LSTM configuration with 128 units and a dropout rate of 0.2 (Row 3). In contrast, the highest Loss, 17.34%, occurs in the Bi-LSTM configuration with 128 units and a higher dropout rate of 0.5 (Row 5). This trend suggests that increasing dropout, while helpful for reducing overfitting, can lead to worse model performance if over-applied.

When evaluating Average Accuracy, the Bi-LSTM configuration with 64 units and a 0.2 dropout rate (Row 1) stands out with the highest accuracy of 95.91%. Accuracy slightly decreases when dropout is increased, as seen in the Bi-LSTM configurations in Rows 1 and 2 (0.2 vs. 0.5 dropout) or Rows 4 and 5. This indicates that a moderate dropout value (e.g., 0.2) is generally better at balancing overfitting and performance.

In terms of the Average F1 Score, which balances precision and recall, two configurations achieve the highest value of 95.82%: Bi-LSTM with 64 units and 0.2 dropouts (Row 1) and LSTM with 128 units and 0.2 dropouts (Row 3). However, Row 1 also has a slightly higher accuracy than Row 3, making it the more reliable configuration overall.

Comparing the configurations, the best overall model is Row 1 (Bi-LSTM, 64 units, 0.2 dropout). This configuration offers the highest accuracy, a very high F1 Score, and a reasonably low average loss. While Row 3 (LSTM, 128 units, 0.2 dropouts) achieves the lowest Loss, it slightly lags in accuracy compared to Row 1. Since accuracy and F1 Score are often prioritized in classification tasks, Row 1 provides the best trade-off between performance metrics.

Of the five highest results, more models use the Bi-LSTM layer than the LSTM layer. According to the existing theory, the Bi-LSTM layer has a better understanding because it can receive information from both directions. Based on average accuracy, the combination of a learning rate of 0.01 dominates the top five results; in other words, the use of a learning rate of 0.01 gives better results than the learning rate of 0.001.

The results of comparing the Num-unit parameter can be seen in the following table.

Table 4. The Num-unit comparison results where other parameters are equal

No.	***Layer***	***Learning rate***	***Num Units***	***Dropout***	***Average Loss***	***Average accuracy***	***Average F1-score***
1	Bi-LSTM	0,01	64	0,2	14,17%	95,91%	95,82%
4	Bi-LSTM	0,01	128	0,2	13,79%	95,57%	95,60%

From the two comparisons of the Num-unit 64 and 128 with the same Bi-LSTM layer, learning rate, and dropout, it can be seen that the model with the Num-unit 64 is superior in terms of average accuracy and average f1-score. In terms of average Loss, the model with num-units 128 is superior. In terms of model performance and accuracy, the model with the Num-unit 64 is better because it has superior accuracy.

Next, the Bi-LSTM model will be compared if the dropout parameter is changed. The following table shows the results of the bi-LSTM model if the dropout parameter is changed. The results of comparing the dropout parameters can be seen in Table 5.

Table 5. Dropout comparison results where other parameters are equal

No.	***Layer***	***Learning rate***	***Num Units***	***Dropout***	***Average Loss***	***Average accuracy***	***Average F1-score***
4	Bi-LSTM	0,01	128	0,2	13,79%	95,57%	95,60%
5	Bi-LSTM	0,01	128	0,5	17,34%	95,54%	95,52%

The table above shows that using dropout with a value of 0.2 gives better performance for all metrics. This shows that a 0.2 dropout performs better in suppressing Loss and improving average accuracy and f1-score than a 0.5 dropout for the same parameters. When the dropout is increased to 0.5, the number of disabled neurons becomes more significant, making it harder for the model to learn patterns from the data [13]. When dropout is applied, the neurons in the neural network are deactivated randomly, which can affect the calculation of the weight and bias of the neural network in performing classification.

In conclusion, the Bi-LSTM model with 64 units and a 0.2 dropout rate (Row 1) is the optimal configuration. Its high accuracy and F1 score make it particularly well-suited for tasks requiring reliable classification. Increasing dropout or model complexity (e.g., the number of units) can degrade performance, highlighting the importance of tuning these parameters for optimal results.

4 Conclusions

Based on the results of this study, it can be concluded that the Bi-LSTM model with a combination of learning rate parameters 0.01, num units 64, and dropout 0.2 provides the best results with a relatively small average loss (14.17%), high average accuracy (95.91%), and high average f1-score (95.82%). The average Loss of 14.17% indicates that the model has a relatively low prediction error rate. The average accuracy of 95.91% indicates that the model performs accurately and is 95.91% correct in predicting sentiment in a sentence. An average f1-score of 95.82% indicates that the model has an excellent balance between precision (the model's ability to avoid false positives) and recall (the model's ability to find all true positives). The best model has been proven to use the optimal parameters according to the parameter comparison that has been done.

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On Independent [1, 2]-sets in Hypercubes

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Abstract. Given a simple graph G, a subset $S \subseteq V(G)$ is an independent [1, 2]-set if no two vertices in S are adjacent and for every vertex $v \in V(G)\backslash S$, $1 \leq |N(v) \cap S| \leq 2$, that is, every vertex $v \in V(G)\backslash S$ is adjacent to at least one but not more than two vertices in S. This paper investigates the existence of independent [1, 2]-sets of hypercubes. We show that for some positive integer k, if $n = 2^k - 1$, then hypercubes Q_n and Q_{n+1} have an independent [1, 2]-set. Furthermore, for $1 \leq n \leq 4$, we find bounds for the minimum and maximum cardinality of an independent [1, 2]-set of hypercube Q_n, while for $n = 5, 6$, we get the maximum of cardinality of an independent [1, 2]-set of hypercube Q_n.

1 Introduction

Let G be a simple graph, that is, it is an undirected graph, has no loop, and has no multiple edges. The *open neighborhood* of a vertex $v \in V(G)$ is the set $N(v) = \{u|uv \in E(G)\}$ of vertices adjacent to v. Each vertex in $u \in N(v)$ is called a *neighbor* of v and the degree of v is $d(v) = |N(v)|$. For a set S and a vertex v, we denote the number of neighbors of v in S as $d_S(v)$, that is, $d\_S\ (v) = |N(v) \cap S\ |$. A set S is *independent* if no two vertices in S are adjacent and *dominating* if every vertex not in S is adjacent to some vertices in S.

Chellali et al., in [1], define a subset $S \subseteq V(G)$ to be a $[j, k]$-set if for every vertex $v \in V(G)\backslash S, j \leq dS\ (v) \leq k$, that is, every vertex in $V(G)\backslash S$ is adjacent to at least j vertices, but not more than k vertices in S. For $j = 1$, a $[1, k]$-set S is a dominating set, since every vertex in $V(G)\backslash S$ has at least one neighbor in S (is dominated by S). The major focus in this study is finding bounds on the minimum cardinality of a [1, 2]-set [1]–[4].

In [5], Chellali et al. continue the study of $[j, k]$-sets and add the requirement that the sets be independent. A dominating set S is an independent $[1, k]$-set of G if S is independent and $1 \leq d_S(v) \leq k$ for all $v \in V(G)\backslash S$. In this paper, we will exclusively focus on independent $[1, 2]$-set. Given a graph G, we denote by $i_{[1,2]}(G)$ the minimum cardinality of an independent $[1, 2]$-set of G and by $\alpha_{[1,2]}(G)$ the maximum cardinality of an independent $[1, 2]$-set of G. Unfortunately, not every graph has an independent $[1, 2]$-set. Thus, beside finding the lower and upper bounds cardinality of an independent $[1, 2]$-set of a graph, investigating the existence of an independent $[1, 2]$-set for some graphs is another focus in this study [6], [7].

In this work, we investigate the existence of independent $[1, 2]$-sets of hypercube Q_n. Moreover, we find bounds for the minimum and maximum cardinality of an independent

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$[1,2]$-set of hypercube Q_n, for $n = 1,2,3$, and 4. For $n = 5,6$, we get bounds for the maximum cardinality of an independent $[1,2]$-set of hypercube Q_n.

The n-cube or n-dimensional hypercube Q_n is defined recursively in terms of the Cartesian product of two graphs as follows [8]:

$$Q_1 = K_2 \text{ (a complete graph of order 2)}$$
$$Q_n = K_2 \square Q_{n-1}$$

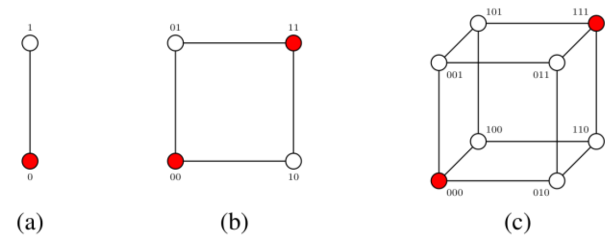

Fig. 1. Hypercubes (a) Q_1, (b) Q_2, and (c) Q_3 with an independent $[1,2]$-set.

The hypercube of dimension n may also be defined as a graph with vertex set $V(Q_n)$ the set of all binary n-tuples of zeros and ones and set $E(Q_n)$ the set of pairs of vertices $u = u_1u_2 \ldots u_n$ and $v = v_1v_2 \ldots v_n$, where $\sum_{i=1}^{n} |u_i - v_i| = 1$. In this representation, two vertices of Q_n are adjacent if and only if their binary n-tuples differ in exactly one place. **Fig. 1** illustrates hypercubes Q_1, Q_2, and Q_3.

Observation 1 *The sets* $S_1 = \{0\}$ *and* $S_2 = \{00,11\}$ *are independent* $[1,2]$*-sets of* Q_1 *and* Q_2*, respectively. Furthermore,* $i_{[1,2]}(Q_1) = \alpha_{[1,2]}(Q_1) = 1$, $i_{[1,2]}(Q_2) = \alpha_{[1,2]}(Q_2) = 2$*, and* S_1 *is an efficient dominating set of* Q_1.

Observation 2 *The set* $S_3 = \{000,111\}$ *is an independent* $[1,2]$*-set of* Q_3*. No singleton subset of* $V(Q_3)$ *can be a dominating set and any independent set of cardinality three is not a* $[1,2]$*-set; hence* $i_{[1,2]}(Q_3) = \alpha_{[1,2]}(Q_3) = 2$*. Furthermore,* S_3 *is an efficient dominating set of* Q_3*. The independent* $[1,2]$*-set* S_3 *is not unique.*

In Observations 1 and 2 we noted that hypercubes Q_1 and Q_3 have efficient dominating sets. The study of the existence of efficient dominating sets in Q_n has been done in the context of single error-correcting codes [9].

Theorem 1 (Livingston [10]). *An* n*-dimensional hypercube* Q_n *has an efficient dominating set if and only if* $n = 2^k - 1$*, for some positive integer* k.

2 Main Result

In the following discussion, we show that hypercubes Q_n, for $n = 4,5$, and 6, have an independent $[1,2]$-set.

Proposition 1. Q_4 *has an independent* $[1,2]$*-set. Furthermore,* $i_{[1,2]}(Q_4) = \alpha_{[1,2]}(Q_4) = 4$.

Proof. We prove the proposition by construction. We will construct an independent $[1,2]$-set S_4 using S_3 in Observation 2.

Since by definition, $Q_4 = K_2 \square Q_3$, as illustrated in **Fig. 2**, we may consider Q_4 as formed from two copies of Q_3, say A and B, respectively. We label the vertices of Q_4 using vertex labeling of Q_3 with prefix 0 added in the vertices in A, and 1 for the vertices in B. Thus if $v_1v_2v_3 \in V(Q_3)$, then its corresponding vertices in Q_4 will be $0v_1v_2v_3$ and $1v_1v_2v_3$.

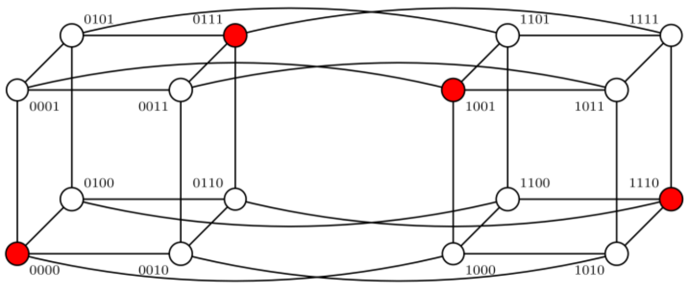

Fig. 2. The hypercubes Q_4 with an independent $[1,2]$-set.

We form S_4 by taking a copy of S_3 in A, and another copy in B. For example, $S_4 = \{0000,0111,1001,1110\}$. It follows that $i_{[1,2]}(Q_4) = 4$ since a vertex subset with at most 3 elements can only dominate 12 vertices at most, while $|V(Q_4)| = 16$. Finally, $\alpha_{[1,2]}(Q_4) = 4$ since an independent $[1,2]$-set with 5 elements will have at least 3 elements in A or B, contradicting $\alpha_{[1,2]}(Q_3) = 2$. □

We note that the independent $[1,2]$-set of Q_4 is not unique. **Fig. 3** shows another independent $[1,2]$-set of Q_4. The advantage of the construction in the proof of Proposition 1 is that we use the independent $[1,2]$-set of Q_3 to construct an independent $[1,2]$-set of Q_4. Using a similar technique, we construct an independent $[1,2]$-set of Q_5 and Q_6. Also, we observe in **Fig. 2**, that S_4 is not an independent $[1,1]$-set. Furthermore, some vertices in Q_4, namely vertices $0001, 0110, 1000, 1111$, are adjacent to two elements of S_4. We need to consider such vertices in the construction of S_5.

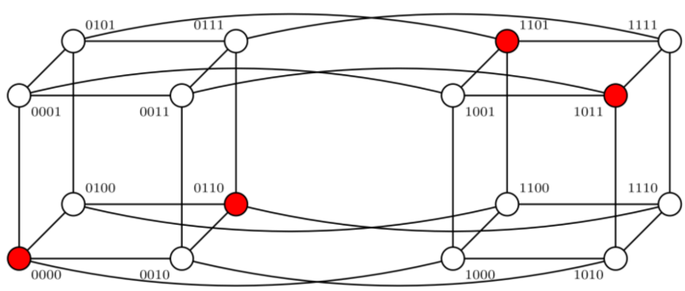

Fig. 3. Another independent $[1,2]$-set for Q_4.

Proposition 2. *Q_5 has an independent $[1,2]$-set and* $\alpha_{[1,2]}(Q_5) = 8$.

Proof. We prove the proposition by construction. We will construct an independent $[1,2]$-set S_5 using S_4 in Proposition 1. We consider Q_5 as formed from two copies of Q_4, say A and B, respectively. We label the vertices of Q_5 using vertex labeling of Q_4 with prefix 0 added in the vertices in A, and 1 for the vertices in B.

As illustrated in **Fig. 4**, we form S_5 by taking a copy of S_4 in Proposition 1 for A. We consider another independent [1,2]-set in B such that the union with S_4 is an independent of Q_5. For example, $S_5 = \{00000,00111,01001,01110,10101,10010,11100,11011\}$. It follows that $\alpha_{[1,2]}(Q_5) = 8$ since an independent $[1,2]$-set with 9 elements will have at least 5 elements in A or B, contradicting $\alpha_{[1,2]}(Q_4) = 4$. □

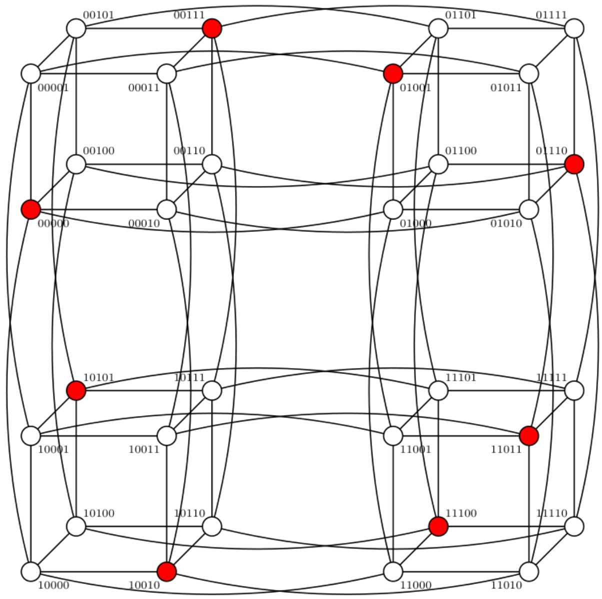

Fig. 4. The hypercube Q_5 with an independent $[1, 2]$-set.

Proposition 3. *Q_6 has an independent $[1,2]$-set and $\alpha_{[1,2]}(Q_6) = 16$.*

Proof. Using similar technique as in Propositions 1 and 2, we construct an independent $[1,2]$-set S_6 using S_5. We consider Q_6 as formed from two copies of Q_5, say A and B, respectively. We use S_5 in Proposition 2 as an independent [1,2]-set for A. As illustrated in **Fig. 5**, we construct an independent [1,2]-set for B such that the union with S_5 is an independent [1,2]-set of Q_6.

$$S_6 = \{000000, 000111, 001001, 001110, 010101, 010010, 011100, 011011, 100100, 100011, 101101, 101010, 110001, 110110, 111000, 111111\}$$

is an independent $[1,2]$-set of Q_6. It follows that $\alpha_{[1,2]}(Q_6) = 16$ since an independent $[1,2]$-set with 17 elements will have at least 9 elements in A or B, contradicting $\alpha_{[1,2]}(Q_5) = 8$. □

Theorem 2 If $n = 2^k - 1$, for some positive integer k, then Q_n and Q_{n+1} have an independent $[1,2]$-set.

Proof. By proposition 1, Q_n with $n = 2^k - 1$ has an efficient dominating set, that is, an independent [1, 1]-set, say S_n. Then S_n is also an independent $[1,2]$-set. If $A = \{a_1a_2a_3 \dots a_na_{n+1} \in V(Q_{n+1}) | a_1 = 0\}$, then $\mathrm{T} = \{0\mathrm{s}_1\mathrm{s}_2\mathrm{s}_3 \dots \mathrm{s}_\mathrm{n} | \mathrm{s}_1\mathrm{s}_2\mathrm{s}_3 \dots \mathrm{s}_\mathrm{n} \in \mathrm{S}_\mathrm{n}\}$ is an independent [1, 1]-set of Q_{n+1}[A]. We observe that

$$W = \{u_1u_2u_3 \dots u_n \in V(Q_n) | u_k = s_k, 1 \le k \le n-1, \text{and } |u_n - s_n| = 1, \text{for all } s_1s_2s_3 \dots s_n \in S_n\}$$

is also an independent $[1,1]$-set of Q_n. Let $B = \{b_1b_2b_3 \dots b_nb_{n+1} \in V(Qn+1) | b1 = 1\}$. Then a set $U = \{1u_1u_2u_3 \dots u_n | u_1u_2u_3 \dots u_n \in W\}$ is an independent [1, 1]-set of Q_{n+1}[B].

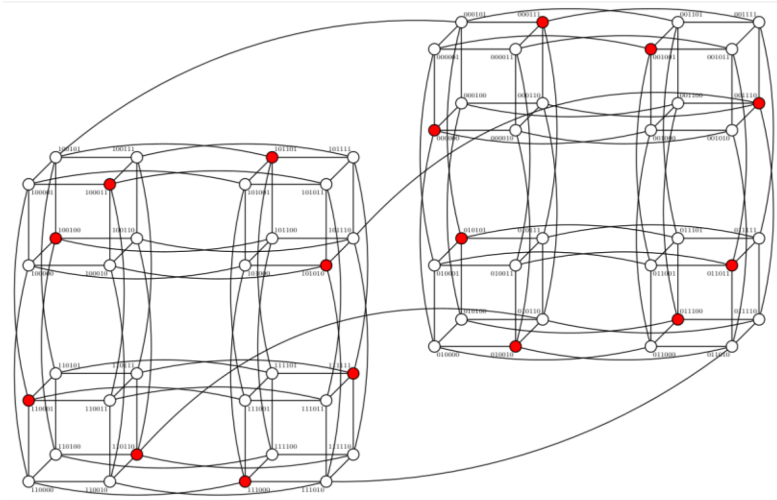

Fig. 5. The hypercube Q_6 with an independent $[1,2]$-set. To avoid a confusion, we only represent some edges from vertices $0v_2v_3v_4v_5v_6$ to $1v_2v_3v_4v_5v_6$.

Now we consider $\mathrm{S_{n+1}} = \mathrm{T} \cup \mathrm{U}$. First we note that S_{n+1} is a dominating set of Q_{n+1} since sets T and U are independent $[1,1]$-sets of $Q_{n+1}[A]$ and $Q_{n+1}[B]$, respectively, where sets A and B are disjoint sets with union $V(Q_{n+1})$. Let $v = v_1v_2v_3...v_{n+1}$ and $w = w_1w_2w_3...w_{n+1}$ be elements of S_{n+1}. If $v_1 = w_1$ then they are not adjacent since either both of them are elements of T or U.

Suppose $v_1 \neq w_1$. Then one of them is an element of T and the other one is an element of U. We recall the construction of T and U. If $0v_2v_3...v_nv_{n+1}$ is an element of T then vertex $1v_2v_3...v_nk$ with $k = |v_{n+1} - 1|$ is an element of U. Conversely if $1w_2w_3...w_nw_{n+1}$ is an element of U, then vertex $0w_2w_3...w_nk$ with $k = |w_{n+1} - 1|$ is an element of T. Thus, if $v_1v_2v_3...v_{n+1}$ and $w_1w_2w_3...w_{n+1}$ are elements of S_{n+1} with $v_1 \neq w_1$, then $\sum_{i=1}^{n+1}|v_i - w_i| \geq 2$. Hence they are not adjacent.

We have shown that S_{n+1} is an independent dominating set. We need to show that it is a $[1,2]$-set. Let $t \in A$ or $t \in B$. In any case, t is adjacent to at most one element in T and at most one element in U. So, t is adjacent to at most two elements of $S\_\{n + 1\}$. By similar argument, if $t\_1 = 1$, then t is adjacent to at most two elements of S_{n+1}, and it follows that S_{n+1} is an independent $[1,2]$-set of Q_{n+1}. □

3 Conclusion and Remarks

This paper has shown the existence of an independent [1, 2]-set of hypercube Q_n for $1 \leq n \leq 6$ and $n = 2k$, for some positive integer k. For further study, one may investigate the existence of an independent $[1,2]$-set of hypercube Q_n for all positive integer n together with the bounds of minimum and maximum cardinality of an independent $[1,2]$-set.

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