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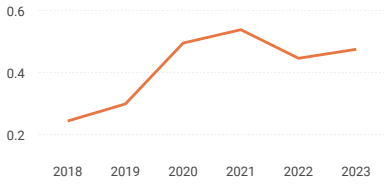

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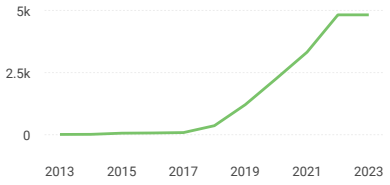

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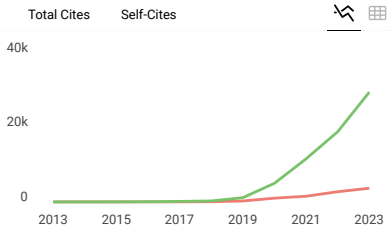

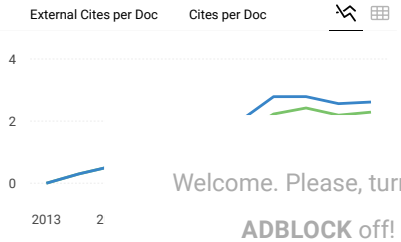

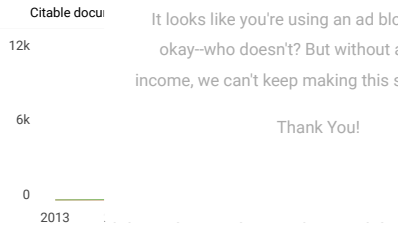

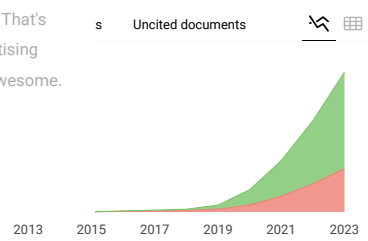

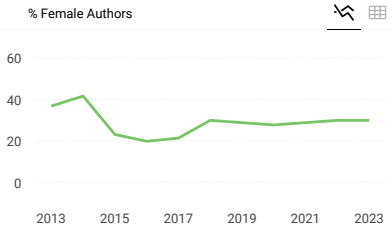

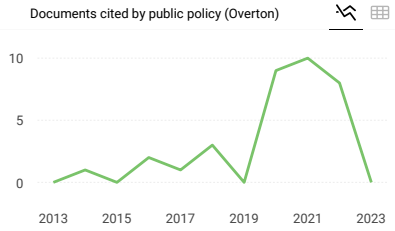

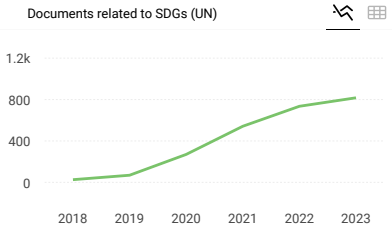

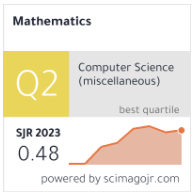

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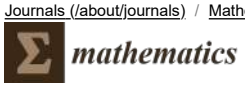

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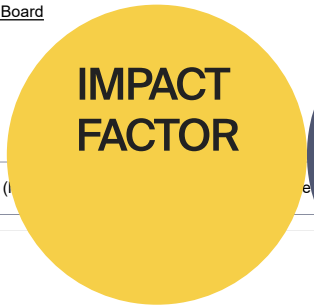

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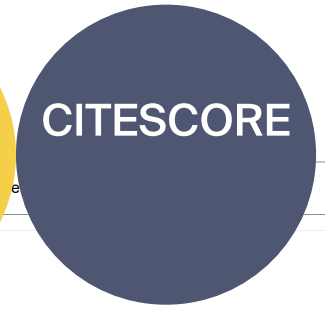

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Go to page **1** 2 (/journal/mathematics/editors?page_no=2) 3 (/journal/mathematics/editors?page_no=3) 4 (/journal/mathematics/editors?page_no=4) 5 (/journal/mathematics/editors?page_no=5) › (/journal/mathematics/editors?page_no=2) ›ı (/journal/mathematics/editors?page_no=35)

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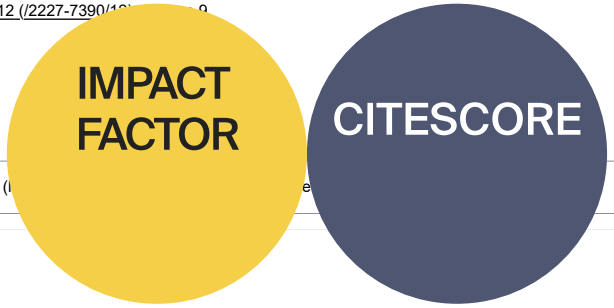

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Vol. 12 (2024) (/2227-7390/12)
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Vol. 6 (2018) (/2227-7390/6)
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Mathematics **2024**, *12*(9), 1335; https://doi.org/10.3390/math12091335 (registering DOI) - 27 Apr 2024

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Mathematics **2024**, *12*(9), 1334; https://doi.org/10.3390/math12091334 (registering DOI) - 27 Apr 2024

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Mathematics **2024**, *12*(9), 1329; https://doi.org/10.3390/math12091329 (registering DOI) - 26 Apr 2024

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Abstract The Sturm–Liouville boundary value problem (SLBVP) stands as a fundamental cornerstone in the realm of mathematical analysis and physical modeling. Also known as the Sturm–Liouville problem (SLP), this paper explores the intricacies of this classical problem, particularly the relationship between its canonical and **[...] Read more.**

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Mathematics **2024**, *12*(9), 1327; **https://doi.org/10.3390/math12091327 (https://doi.org/10.3390/math12091327)** - 26 Apr 2024

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Abstract Quaternions are a non-commutative associative number system that extends complex numbers, first described by Hamilton in 1843. We present algorithms for solving the eigenvalue problem for arrowhead and DPRk (diagonal-plus-rank-*k*) matrices of quaternions. The algorithms use the Rayleigh Quotient Iteration with **[...] Read more.**

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Essential Norm of *t*-Generalized Composition Operators from *F*(*p*, *q*, *s*) to Iterated Weighted-Type Banach Space (/2227-7390/12/9/1320)
by **Shams Alyusof** and **Nacir Hmidouch**
Mathematics **2024**, *12*(9), 1320; **https://doi.org/10.3390/math12091320 (https://doi.org/10.3390/math12091320)** - 26 Apr 2024
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Abstract In this work, we characterize the boundedness of *t*-generalized composition operators from $F(p,\ q,\ s)$ spaces to iterated weighted-type Banach space. We also provide estimates of the norm and the essential norm of *t*-generalized **[...] Read more.**

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Neural Network-Based Distributed Consensus Tracking Control for Nonlinear Multi-Agent Systems with Mismatched and Matched Disturbances (/2227-7390/12/9/1319)
by **Linxi Xu** and **Kaiyu Qin**
Mathematics **2024**, *12*(9), 1319; **https://doi.org/10.3390/math12091319 (https://doi.org/10.3390/math12091319)** - 26 Apr 2024
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Abstract In practice, disturbances, including model uncertainties and unknown external disturbances, are always widely present and have a significant impact on the cooperative control performance of a networked multi-agent system. In this work, the distributed consensus tracking control problem for a class of multi-agent **[...] Read more.**
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A Novel Method for Predicting the Behavior of a Sucker Rod Pumping Unit Based on the Polished Rod Velocity (/2227-7390/12/9/1318)

by **Jiaojian Yin** and **Hongzhang Ma**

Mathematics **2024**, *12*(9), 1318; **https://doi.org/10.3390/math12091318 (https://doi.org/10.3390/math12091318)** - 25 Apr 2024

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Abstract Fault dynamometer cards are the basis of the diagnosis technique for sucker rod pumping systems. Predicting fault cards with a pumping condition model is an economical and effective method. The usual model is described by a mixed function of the pump displacement and **[...] Read more.**

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The Blow-Up of the Local Energy Solution to the Wave Equation with a Nontrivial Boundary Condition (/2227-7390/12/9/1317)

by **Yulong Liu**

Mathematics **2024**, *12*(9), 1317; **https://doi.org/10.3390/math12091317 (https://doi.org/10.3390/math12091317)** - 25 Apr 2024

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Abstract In this study, we examine the wave equation with a nontrivial boundary condition. The main target of this study is to prove the local-in-time existence and the blow-up in finite time of the energy solution. Through the construction of an auxiliary function and **[...] Read more.**

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Meshless Generalized Finite Difference Method Based on Nonlocal Differential Operators for Numerical Simulation of Elastostatics (/2227-7390/12/9/1316)

by **Yeying Zhou**, **Chunguang Li**, **Xinshan Zhuang** and **Zhifen Wang**

Mathematics **2024**, *12*(9), 1316; **https://doi.org/10.3390/math12091316 (https://doi.org/10.3390/math12091316)** - 25 Apr 2024

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Abstract This study proposes an innovative meshless approach that merges the peridynamic differential operator (PDDO) with the generalized finite difference method (GFDM). Based on the PDDO theory, this method introduces a new nonlocal differential operator that aims to reduce the pre-assumption required for the **[...] Read more.**

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Efficient Large-Scale IoT Botnet Detection through GraphSAINT-Based Subgraph Sampling and Graph Isomorphism Network (/2227-7390/12/9/1315)

by **Lihua Yin**, **Weizhe Chen**, **Xi Luo** and **Hongyu Yang**

Mathematics **2024**, *12*(9), 1315; **https://doi.org/10.3390/math12091315 (https://doi.org/10.3390/math12091315)** - 25 Apr 2024

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Abstract In recent years, with the rapid development of the Internet of Things, large-scale botnet attacks have occurred frequently and have become an important challenge to network security. As artificial intelligence technology continues to evolve, intelligent detection solutions for botnets are constantly emerging. Although **[...] Read more.**

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A Privacy Protection Scheme of Certificateless Aggregate Ring Signcryption Based on SM2 Algorithm in Smart Grid (/2227-7390/12/9/1314)

by **Hongna Song**, **Zhentao Liu**, **Teng Wang**, **Ling Zhao**, **Haonan Guo** and **Shuanggen Liu**

Mathematics **2024**, *12*(9), 1314; **https://doi.org/10.3390/math12091314 (https://doi.org/10.3390/math12091314)** - 25 Apr 2024

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Abstract With the rapid increase in smart grid users and the increasing cost of user data transmission, proposing an encryption method that does not increase the construction cost while increasing the user ceiling has become the focus of many scholars. At the same time, **[...] Read more.**

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Abstract This paper aimed to develop a method for generating sand–dust removal and dehazed images utilizing CycleGAN, facilitating object identification on roads under adverse weather conditions such as heavy dust or haze, which severely impair visibility. Initially, the study addressed the scarcity of paired **[...] Read more.**

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Deep Learning Based Fall Recognition and Forecasting for Reconfigurable Stair-Accessing Service Robots (/2227-7390/12/9/1312)

by **Jun Hua Ong**, **Abdullah Aamir Hayat**, **Braulio Felix Gomez**, **Mohan Rajesh Elara** and **Kristin Lee Wood**

Mathematics **2024**, *12*(9), 1312; **https://doi.org/10.3390/math12091312 (https://doi.org/10.3390/math12091312)** - 25 Apr 2024

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Abstract This paper presents a comprehensive study on fall recognition and forecasting for reconfigurable stair-accessing robots by leveraging deep learning techniques. The proposed framework integrates machine learning algorithms and recurrent neural networks (RNNs), specifically Long Short-Term Memory (LSTM) and Bidirectional LSTM (BiLSTM), for fall **[...] Read more.**

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A Few Similarity Measures on the Class of Trapezoidal-Valued Intuitionistic Fuzzy Numbers and Their Applications in Decision Analysis (/2227-7390/12/9/1311)

by **Jeevaraj Selvaraj** and **Melfi Alrasheedi**

Mathematics **2024**, *12*(9), 1311; **https://doi.org/10.3390/math12091311 (https://doi.org/10.3390/math12091311)** - 25 Apr 2024

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Abstract Similarity measures on trapezoidal-valued intuitionistic fuzzy numbers (TrVIFNs) are functions that measure the closeness between two TrVIFNs, which has a lot of applications in the area of pattern recognition, clustering, decision-making, etc. Researchers around the world are proposing various similarity measures on the **[...] Read more.**

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Robust Bias Compensation Method for Sparse Normalized Quasi-Newton Least-Mean with Variable Mixing-Norm Adaptive Filtering (/2227-7390/12/9/1310)

by **Ying-Ren Chien**, **Han-En Hsieh** and **Guobing Qian**

Mathematics **2024**, *12*(9), 1310; **https://doi.org/10.3390/math12091310 (https://doi.org/10.3390/math12091310)** - 25 Apr 2024
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Abstract Input noise causes inescapable bias to the weight vectors of the adaptive filters during the adaptation processes. Moreover, the impulse noise at the output of the unknown systems can prevent bias compensation from converging. This paper presents a robust bias compensation method for **[...] Read more.**
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Study of Random Walk Invariants for Spiro-Ring Network Based on Laplacian Matrices (/2227-7390/12/9/1309)

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Mathematics **2024**, *12*(9), 1309; **https://doi.org/10.3390/math12091309 (https://doi.org/10.3390/math12091309)** - 25 Apr 2024
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Abstract The use of the global mean first-passage time (GMFPT) in random walks on networks has been widely explored in the field of statistical physics, both in theory and practical applications. The GMFPT is the estimated interval of time needed to reach a state **[...] Read more.**
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Abstract Mathematical models and numerical simulations are necessary to understand the functions of biological rhythms, to comprehend the transition from simple to complex behavior and to delineate the conditions under which they arise. The aim of this work is to investigate the R **[...] Read more.**
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Monitoring the Wear Trends in Wind Turbines by Tracking Fourier Vibration Spectra and Density Based Support Vector Machines (/2227-7390/12/9/1307)

by **Claudiu Bisu**, **Adrian Olaru**, **Serban Olaru**, **Adrian Alexei**, **Niculae Mihai** and **Haleema Ushaq**
Mathematics **2024**, *12*(9), 1307; **https://doi.org/10.3390/math12091307 (https://doi.org/10.3390/math12091307)** - 25 Apr 2024
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Abstract To make wind power more competitive, it is necessary to reduce turbine downtime and reduce costs associated with wind turbine operation and maintenance (O&M). Incorporating machine learning in the development of condition-based predictive maintenance methodologies for wind turbines can enhance their efficiency and **[...] Read more.**
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A Reentry Trajectory Planning Algorithm via Pseudo-Spectral Convexification and Method of Multipliers (/2227-7390/12/9/1306)

by **Haizhao Liang**, **Yunhao Luo**, **Haohui Che**, **Jingxian Zhu** and **Jianying Wang**
Mathematics **2024**, *12*(9), 1306; **https://doi.org/10.3390/math12091306 (https://doi.org/10.3390/math12091306)** - 25 Apr 2024
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Abstract The reentry trajectory planning problem of hypersonic vehicles is generally a continuous and nonconvex optimization problem, and it constitutes a critical challenge within the field of aerospace engineering. In this paper, an improved sequential convexification algorithm is proposed to solve it and achieve **[...] Read more.**
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On α-Pseudo Spiralike Functions Associated with Exponential Pareto Distribution (EPD) and Libera Integral Operator (/2227-7390/12/9/1305)

by **Hamzat Jamiu Olusegun**, **Oluwayemi Matthew Olanrewaju**, **Oladipo Abiodun Tinuoye** and **Alb Lupas Alina**
Mathematics **2024**, *12*(9), 1305; **https://doi.org/10.3390/math12091305 (https://doi.org/10.3390/math12091305)** - 25 Apr 2024
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Abstract The present study aims at investigating some characterizations of a new subclass $G_\alpha(\mu,\tau)$ and obtaining the bounds on the first two Taylor–Maclaurin coefficients for functions belonging to the newly introduced subclass. In order to achieve this, a **[...] Read more.**
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Sparsity-Constrained Vector Autoregressive Moving Average Models for Anomaly Detection of Complex Systems with Multisensory Signals (/2227-7390/12/9/1304)

by **Meng Ma**, **Zhongyi Zhang**, **Zhi Zhai** and **Zhirong Zhong**
Mathematics **2024**, *12*(9), 1304; **https://doi.org/10.3390/math12091304 (https://doi.org/10.3390/math12091304)** - 25 Apr 2024
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Abstract Detecting anomalies in large, complex systems is a critical and challenging task, and this is especially true for high-dimensional anomaly detection due to the underlying dependency structures among sensors. To incorporate the interrelationships among various sensors, a novel sparsity-constrained vector autoregressive moving average **[...] Read more.**
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TPTM-HANN-GA: A Novel Hyperparameter Optimization Framework Integrating the Taguchi Method, an Artificial Neural Network, and a Genetic Algorithm for the Precise Prediction of Cardiovascular Disease Risk (/2227-7390/12/9/1303)

by **Chia-Ming Lin** and **Yu-Shiang Lin**
Mathematics **2024**, *12*(9), 1303; **https://doi.org/10.3390/math12091303 (https://doi.org/10.3390/math12091303)** - 25 Apr 2024
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Abstract The timely and precise prediction of cardiovascular disease (CVD) risk is essential for effective prevention and intervention. This study proposes a novel framework that integrates the two-phase Taguchi method (TPTM), the hyperparameter artificial neural network (HANN), and a genetic algorithm (GA) called TPTM-HANN-GA. **[...] Read more.**

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Single and Multi-Valued Ordered-Theoretic Perov Fixed-Point Results for θ-Contraction with Application to Nonlinear System of Matrix Equations (/2227-7390/12/9/1302)

by **Fahim Ud Din**, **Salha Alshaikey**, **Umar Ishtiaq**, **Muhammad Din** and **Salvatore Sessa**

Mathematics **2024**, *12*(9), 1302; **https://doi.org/10.3390/math12091302 (https://doi.org/10.3390/math12091302)** - 25 Apr 2024

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Abstract This paper combines the concept of an arbitrary binary connection with the widely recognized principle of θ-contraction to investigate the innovative features of vector-valued metric spaces. This methodology demonstrates the existence of fixed points for both single- and multi-valued mappings within complete **[...] Read more.**

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Synergising an Advanced Optimisation Technique with Deep Learning: A Novel Method in Fault Warning Systems (/2227-7390/12/9/1301)

by **Jia Tian**, **Xingqin Zhang**, **Shuangqing Zheng**, **Zhiyong Liu** and **Changshu Zhan**

Mathematics **2024**, *12*(9), 1301; **https://doi.org/10.3390/math12091301 (https://doi.org/10.3390/math12091301)** - 25 Apr 2024

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Abstract In the realm of automated industry and smart production, the deployment of fault warning systems is crucial for ensuring equipment reliability and enhancing operational efficiency. Although there are a multitude of existing methodologies for fault warning, the proficiency of these systems in processing **[...] Read more.**

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Estimating the Complier Average Causal Effect with Non-Ignorable Missing Outcomes Using Likelihood Analysis (/2227-7390/12/9/1300)

by **Jierui Du**, **Gao Wen** and **Xin Liang**

Mathematics **2024**, *12*(9), 1300; **https://doi.org/10.3390/math12091300 (https://doi.org/10.3390/math12091300)** - 25 Apr 2024

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Abstract Missing data problems arise in randomized trials, which complicates the inference of causal effects if the missing mechanism is non-ignorable. We tackle the challenge of identifying and estimating the complier average causal effect parameters under non-ignorable missingness by increasing the covariates to mitigate **[...] Read more.**

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Robust Design Optimization of Electric Machines with Isogeometric Analysis (/2227-7390/12/9/1299)

by **Theodor Komann**, **Michael Wiesheu**, **Stefan Ulbrich** and **Sebastian Schöps**

Mathematics **2024**, *12*(9), 1299; **https://doi.org/10.3390/math12091299 (https://doi.org/10.3390/math12091299)** - 25 Apr 2024

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Abstract In electric machine design, efficient methods for the optimization of the geometry and associated parameters are essential. Nowadays, it is necessary to address the uncertainty caused by manufacturing or material tolerances. This work presents a robust optimization strategy to address uncertainty in the **[...] Read more.**

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Mining Trajectory Planning of Unmanned Excavator Based on Machine Learning (/2227-7390/12/9/1298)

by **Zhong Jin**, **Mingde Gong**, **Dingxuan Zhao**, **Shaomeng Luo**, **Guowang Li**, **Jiaheng Li**, **Yue Zhang** and **Wenbin Liu**

Mathematics **2024**, *12*(9), 1298; **https://doi.org/10.3390/math12091298 (https://doi.org/10.3390/math12091298)** - 25 Apr 2024

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Abstract Trajectory planning plays a crucial role in achieving unmanned excavator operations. The quality of trajectory planning results heavily relies on the level of rules extracted from objects such as scenes and optimization objectives, using traditional theoretical methods. To address this issue, this study **[...] Read more.**

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Open Access Article 17 pages, 6027 KiB **(/2227-7390/12/9/1297/pdf?version=1714034754)**

Exploring an Intelligent Classification Model for the Recognition of Automobile Sounds Based on EEG Physiological Signals (/2227-7390/12/9/1297)

by **Jingjing Guo**, **Tao Xu**, **Liping Xie** and **Zhien Liu**
Mathematics **2024**, *12*(9), 1297; **https://doi.org/10.3390/math12091297 (https://doi.org/10.3390/math12091297)** - 25 Apr 2024
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Abstract The advancement of an intelligent automobile sound switching system has the potential to elevate the market standing of automotive products, with the pivotal prerequisite being the selection of automobile sounds based on the driver's subjective perception. The subjective responses of diverse individuals to **[...] Read more.**

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Open Access Article 19 pages, 2006 KiB **(/2227-7390/12/9/1296/pdf?version=1714033082)**

Shearlet Transform Applied to a Prostate Cancer Radiomics Analysis on MR Images (/2227-7390/12/9/1296)

by **Rosario Corso**, **Alessandro Stefano**, **Giuseppe Salvaggio** and **Albert Comelli**
Mathematics **2024**, *12*(9), 1296; **https://doi.org/10.3390/math12091296 (https://doi.org/10.3390/math12091296)** - 25 Apr 2024
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Abstract For decades, wavelet theory has attracted interest in several fields in dealing with signals. Nowadays, it is acknowledged that it is not very suitable to face aspects of multidimensional data like singularities and this has led to the development of other mathematical tools. **[...] Read more.**
(This article belongs to the Special Issue **Artificial Intelligence for Biomedical Image Processing and Data Analysis (/journal/mathematics/special_issues/T635A6WNH6)**)

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Open Access Review 17 pages, 363 KiB **(/2227-7390/12/9/1295/pdf?version=1714031812)**

Feynman Diagrams beyond Physics: From Biology to Economy (/2227-7390/12/9/1295)

by **Nicolò Cangiotti**
Mathematics **2024**, *12*(9), 1295; **https://doi.org/10.3390/math12091295 (https://doi.org/10.3390/math12091295)** - 25 Apr 2024
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Abstract Feynman diagrams represent one of the most powerful and fascinating tools developed in theoretical physics in the last century. Introduced within the framework of quantum electrodynamics as a suitable method for computing the amplitude of a physical process, they rapidly became a fundamental **[...] Read more.**
(This article belongs to the Special Issue **Algebraic Structures and Graph Theory, 2nd Edition (/journal/mathematics/special_issues/1RKT503SSL)**)

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Open Access Article 14 pages, 5340 KiB **(/2227-7390/12/9/1294/pdf?version=1714008914)**

Improvement of Distributed Denial of Service Attack Detection through Machine Learning and Data Processing (/2227-7390/12/9/1294)

by **Fray L. Becerra-Suarez**, **Ismael Fernández-Roman** and **Manuel G. Forero**
Mathematics **2024**, *12*(9), 1294; **https://doi.org/10.3390/math12091294 (https://doi.org/10.3390/math12091294)** - 25 Apr 2024
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Abstract The early and accurate detection of Distributed Denial of Service (DDoS) attacks is a fundamental area of research to safeguard the integrity and functionality of organizations' digital ecosystems. Despite the growing importance of neural networks in recent years, the use of classical techniques **[...] Read more.**
(This article belongs to the Special Issue **Network Security in Artificial Intelligence Systems (/journal/mathematics/special_issues/S5N4U43HG4)**)

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Comparisons of Numerical and Solitary Wave Solutions for the Stochastic Reaction–Diffusion Biofilm Model including Quorum Sensing (/2227-7390/12/9/1293)

by **Muhammad Zafarullah Baber**, **Nauman Ahmed**, **Muhammad Waqas Yasin**, **Muhammad Sajid Iqbal**, **Ali Akgül**, **Alicia Cordero** and **Juan R. Torregrosa**
Mathematics **2024**, *12*(9), 1293; **https://doi.org/10.3390/math12091293 (https://doi.org/10.3390/math12091293)** - 24 Apr 2024
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Abstract This study deals with a stochastic reaction–diffusion biofilm model under quorum sensing. Quorum sensing is a process of communication between cells that permits bacterial communication about cell density and alterations in gene expression. This model produces two results: the bacterial concentration, which over **[...] Read more.**

Open Access Article 20 pages, 2900 KiB (/2227-7390/12/9/1292/pdf?version=1713972572)

Intelligent Low-Consumption Optimization Strategies: Economic Operation of Hydropower Stations Based on Improved LSTM and Random Forest Machine Learning Algorithm (/2227-7390/12/9/1292)

by **Hong Pan**, **Jie Yang**, **Yang Yu**, **Yuan Zheng**, **Xiaonan Zheng** and **Chenyang Hang**
Mathematics **2024**, *12*(9), 1292; **https://doi.org/10.3390/math12091292 (https://doi.org/10.3390/math12091292)** - 24 Apr 2024
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Abstract The economic operation of hydropower stations has the potential to increase water use efficiency. However, there are some challenges, such as the fixed and unchangeable flow characteristic curve of the hydraulic turbines, and the large number of variables in optimal load distribution, which **[...] Read more.**
(This article belongs to the Special Issue **Computational Methods and Applications for Numerical Analysis, 2nd Edition (/journal/mathematics/special_issues/0K108J6FA8)**)

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Multi-Objective Portfolio Optimization Using a Quantum Annealer (/2227-7390/12/9/1291)

by **Esteban Aguilera** , **Jins de Jong** , **Frank Phillipson**, **Skander Taamallah** and **Mischa Vos**
Mathematics **2024**, *12*(9), 1291; **https://doi.org/10.3390/math12091291 (https://doi.org/10.3390/math12091291)** - 24 Apr 2024
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Abstract In this study, the portfolio optimization problem is explored, using a combination of classical and quantum computing techniques. The portfolio optimization problem with specific objectives or constraints is often a quadratic optimization problem, due to the quadratic nature of, for example, risk measures. **[...] Read more.**
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Traffic Prediction with Self-Supervised Learning: A Heterogeneity-Aware Model for Urban Traffic Flow Prediction Based on Self-Supervised Learning (/2227-7390/12/9/1290)

by **Min Gao**, **Yingmei Wei**, **Yuxiang Xie** and **Yitong Zhang**
Mathematics **2024**, *12*(9), 1290; **https://doi.org/10.3390/math12091290 (https://doi.org/10.3390/math12091290)** - 24 Apr 2024
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Abstract Accurate traffic prediction is pivotal when constructing intelligent cities to enhance urban mobility and to efficiently manage traffic flows. Traditional deep learning-based traffic prediction models primarily focus on capturing spatial and temporal dependencies, thus overlooking the existence of spatial and temporal heterogeneities. Heterogeneity **[...] Read more.**

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Genetic Algorithms Application for Pricing Optimization in Commodity Markets (/2227-7390/12/9/1289)

by **Yiyu Li**, **Qingjie Xu**, **Ying Wang** and **Bin Liu**
Mathematics **2024**, *12*(9), 1289; **https://doi.org/10.3390/math12091289 (https://doi.org/10.3390/math12091289)** - 24 Apr 2024
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Abstract The perishable nature of vegetable commodities poses challenges for superstores, as reselling them is often unfeasible due to their short freshness period. Reliable market demand analysis is crucial for boosting revenue. This study simplifies the pricing and replenishment decision-making process by making reasonable **[...] Read more.**
(This article belongs to the Special Issue **Mathematical Modeling and Machine Learning with Application to Economics and Finance (/journal/mathematics/special_issues/mathmodel_economic_fina)**)

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Local $C^{0,1}$-Regularity for the Parabolic *p*-Laplacian Equation on the Group SU(3) (/2227-7390/12/9/1288)

by **Yongming He**, **Chengwei Yu** and **Hongqing Wang**
Mathematics **2024**, *12*(9), 1288; **https://doi.org/10.3390/math12091288 (https://doi.org/10.3390/math12091288)** - 24 Apr 2024
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Abstract In this article, when $2 \leq p \leq 4$, we establish the $C^{0,1}_{\mathrm{loc}}$-regularity of weak solutions to the degenerate parabolic *p*-Laplacian equation **[...] Read more.**
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A Weighted Skew-Logistic Distribution with Applications to Environmental Data (/2227-7390/12/9/1287)

by **Isaac Cortés**, **Jimmy Reyes** and **Yuri A. Iriarte**
Mathematics **2024**, *12*(9), 1287; **https://doi.org/10.3390/math12091287 (https://doi.org/10.3390/math12091287)** - 24 Apr 2024
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Abstract Skewness and bimodality properties are frequently observed when analyzing environmental data such as wind speeds, precipitation levels, and ambient temperatures. As an alternative to modeling data exhibiting these properties, we propose a flexible extension of the skew-logistic distribution. The proposal corresponds to a **[...] Read more.**
(This article belongs to the Section **Probability and Statistics (/journal/mathematics/sections/probability_and_statistics_theory)**)

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VTT-LLM: Advancing Vulnerability-to-Tactic-and-Technique Mapping through Fine-Tuning of Large Language Model (/2227-7390/12/9/1286)

by **Chenhui Zhang**, **Le Wang**, **Dunqiu Fan**, **Junyi Zhu**, **Tang Zhou**, **Liyi Zeng** and **Zhaohua Li**

Mathematics **2024**, *12*(9), 1286; **https://doi.org/10.3390/math12091286 (https://doi.org/10.3390/math12091286)** - 24 Apr 2024
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Abstract Vulnerabilities are often accompanied by cyberattacks. CVE is the largest repository of open vulnerabilities, which keeps expanding. ATT&CK models known multi-step attacks both tactically and technically and remains up to date. It is valuable to correlate the vulnerability in CVE with the corresponding **[...] Read more.**
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Open Access Article 25 pages, 8813 KiB **(/2227-7390/12/9/1285/pdf?version=1713956392)**

Spatial Network Analysis of Coupling Coordination between Digital Financial Inclusion and Common Prosperity in the Yangtze River Delta Urban Agglomeration (/2227-7390/12/9/1285)

by **Fanlong Zeng** and **Huaping Sun**
Mathematics **2024**, *12*(9), 1285; **https://doi.org/10.3390/math12091285 (https://doi.org/10.3390/math12091285)** - 24 Apr 2024
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Abstract Digital financial inclusion and common prosperity are pivotal elements in promoting the sustainable socioeconomic development of China. This study introduces a novel Multi-Criteria Decision Analysis (MCDA) method to evaluate the Common Prosperity Index (CPI). Using this index, alongside the Digital Financial Inclusion Index **[...] Read more.**
(This article belongs to the Special Issue **Mathematical Modelling of Economics and Regional Development (/journal/mathematics/special_issues/BIM75R3XZ9)**)

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Open Access Article 13 pages, 251 KiB **(/2227-7390/12/9/1284/pdf?version=1713953406)**

Asymptotic Behavior of Stochastic Reaction–Diffusion Equations (/2227-7390/12/9/1284)

by **Hao Wen**, **Yuanjing Wang**, **Guangyuan Liu** and **Dawei Liu**
Mathematics **2024**, *12*(9), 1284; **https://doi.org/10.3390/math12091284 (https://doi.org/10.3390/math12091284)** - 24 Apr 2024
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Abstract In this paper, we concentrate on the propagation dynamics of stochastic reaction–diffusion equations, including the existence of travelling wave solution and asymptotic wave speed. Based on the stochastic Feynman–Kac formula and comparison principle, the boundedness of the solution of stochastic reaction–diffusion equations can **[...] Read more.**
(This article belongs to the Special Issue **Dynamics of Predator-Prey and Infectious Disease Models (/journal/mathematics/special_issues/H617262S6K)**)

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Inter-Channel Correlation Modeling and Improved Skewed Histogram Shifting for Reversible Data Hiding in Color Images (/2227-7390/12/9/1283)

by **Dan He**, **Zhanchuan Cai**, **Dujuan Zhou** and **Zhihui Chen**
Mathematics **2024**, *12*(9), 1283; **https://doi.org/10.3390/math12091283 (https://doi.org/10.3390/math12091283)** - 24 Apr 2024
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Abstract Reversible data hiding (RDH) is an advanced data protection technology that allows the embedding of additional information into an original digital medium while maintaining its integrity. Color images are typical carriers for information because of their rich data content, making them suitable for **[...] Read more.**
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Numerical Investigation of Supersonic Flow over a Wedge by Solving 2D Euler Equations Utilizing the Steger–Warming Flux Vector Splitting (FVS) Scheme (/2227-7390/12/9/1282)

by **Mitch Wolff**, **Hashim H. Abada** and **Hussein Awad Kurdi Saad**
Mathematics **2024**, *12*(9), 1282; **https://doi.org/10.3390/math12091282 (https://doi.org/10.3390/math12091282)** - 24 Apr 2024
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Abstract Supersonic flow over a half-angle wedge (θ = 15°) with an upstream Mach number of 2.0 was investigated using 2D Euler equations where sea level conditions were considered. The investigation employed the Steger–Warming flux vector splitting (FVS) method executed in MATLAB 9.13.0 (R2022b) **[...] Read more.**
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MDER-Net: A Multi-Scale Detail-Enhanced Reverse Attention Network for Semantic Segmentation of Bladder Tumors in Cystoscopy Images (/2227-7390/12/9/1281)

by **Chao Nie**, **Chao Xu** and **Zhengping Li**
Mathematics **2024**, *12*(9), 1281; **https://doi.org/10.3390/math12091281 (https://doi.org/10.3390/math12091281)** - 24 Apr 2024
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Abstract White light cystoscopy is the gold standard for the diagnosis of bladder cancer. Automatic and accurate tumor detection is essential to improve the surgical resection of bladder cancer and reduce tumor recurrence. At present, Transformer-based medical image segmentation algorithms face challenges in restoring **[...] Read more.**
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by **Leszek Gasiński** and **Nikolaos S. Papageorgiou**
Mathematics **2024**, *12*(9), 1280; **https://doi.org/10.3390/math12091280 (https://doi.org/10.3390/math12091280)** - 24 Apr 2024
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Abstract We consider a nonlinear Dirichlet problem driven by the $(p(z),q)$-Laplacian and with a logistic reaction of the equidiffusive type. Under a nonlinearity condition on a quotient map, we show existence and uniqueness of positive solutions **[...] Read more.**
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Statistical Solitonic Impact on Submanifolds of Kenmotsu Statistical Manifolds (/2227-7390/12/9/1279)

by **Abdullah Ali H. Ahmadini**, **Mohd. Danish Siddiqi** and **Aliya Naaz Siddiqui**
Mathematics **2024**, *12*(9), 1279; **https://doi.org/10.3390/math12091279 (https://doi.org/10.3390/math12091279)** - 24 Apr 2024
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Abstract In this article, we delve into the study of statistical solitons on submanifolds of Kenmotsu statistical manifolds, introducing the presence of concircular vector fields. This investigation is further extended to study the behavior of almost quasi-Yamabe solitons on submanifolds with both concircular and **[...] Read more.**
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by **Ramandeep Behl**, **Ioannis K. Argyros** and **Sattam Alharbi**
Mathematics **2024**, *12*(9), 1278; **https://doi.org/10.3390/math12091278 (https://doi.org/10.3390/math12091278)** - 23 Apr 2024
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Abstract The conventional approach of the local convergence analysis of an iterative method on $\mathbb{R}^m$, with m a natural number, depends on Taylor series expansion. This technique often requires the calculation of high-order derivatives. However, those derivatives may not be part of **[...] Read more.**
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by **Jesse Y. Rumbo-Morales**, **Jair Gómez-Radilla**, **Gerardo Ortiz-Torres**, **Felipe D. J. Sorcia-Vázquez**, **Hector M. Buenabad-Arias**, **Maria A. López-Osorio**, **Carlos A. Torres-Cantero**, **Moises Ramos-Martinez**, **Mario A. Juárez**, **Manuela Calixto-Rodriguez**, **Jorge A. Brizuela-Mendoza** and **Jesús E. Valdez-Resendiz**
Mathematics **2024**, *12*(9), 1277; **https://doi.org/10.3390/math12091277 (https://doi.org/10.3390/math12091277)** - 23 Apr 2024
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Abstract Currently, various light-emitting diode (LED) lighting systems are being developed because LEDs are one of the most used lighting sources for work environments, buildings, homes, and public roads in terms of some of their applications. Similarly, they have low energy consumption, quick responses, **[...] Read more.**
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Fixed/Preassigned-Time Synchronization of Fuzzy Memristive Fully Quaternion-Valued Neural Networks Based on Event-Triggered Control (/2227-7390/12/9/1276)

by **Shichao Jia**, **Cheng Hu** and **Haijun Jiang**

Mathematics **2024**, *12*(9), 1276; **https://doi.org/10.3390/math12091276 (https://doi.org/10.3390/math12091276)** - 23 Apr 2024

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Abstract In this paper, the fixed-time and preassigned-time synchronization issues of fully quaternion-valued fuzzy memristive neural networks are studied based on the dynamic event-triggered control mechanism. Initially, the fuzzy rules are defined within the quaternion domain and the relevant properties are established through rigorous **[...] Read more.**

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by **Salma Aljawi** and **Izhar Uddin**

Mathematics **2024**, *12*(9), 1275; **https://doi.org/10.3390/math12091275 (https://doi.org/10.3390/math12091275)** - 23 Apr 2024

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Abstract This article investigates certain fixed-point results enjoying nonlinear almost contraction conditions in the setup of relational metric space. Some examples are constructed in order to indicate the profitability of our results. As a practical use of our findings, we demonstrate the existence of **[...] Read more.**

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Joins, Secant Varieties and Their Associated Grassmannians (/2227-7390/12/9/1274)

by **Edoardo Ballico**

Mathematics **2024**, *12*(9), 1274; **https://doi.org/10.3390/math12091274 (https://doi.org/10.3390/math12091274)** - 23 Apr 2024

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Abstract We prove a strong theorem on the partial non-defectivity of secant varieties of embedded homogeneous varieties developing a general set-up for families of subvarieties of Grassmannians. We study this type of problem in the more general set-up of joins of embedded varieties. Joins **[...] Read more.**

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The Optimal Stopping Problem under a Random Horizon (/2227-7390/12/9/1273)

by **Tahir Choulli** and **Safa' Alsheyab**

Mathematics **2024**, *12*(9), 1273; **https://doi.org/10.3390/math12091273 (https://doi.org/10.3390/math12091273)** - 23 Apr 2024

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Abstract This paper considers a pair $(\mathbb{F}, \tau)$, where $\mathbb{F}$ is a filtration representing the "public" flow of information that is available to all agents over time, and τ is a random time that might not be an $\mathbb{F}$-stopping **[...] Read more.**

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Requirement Dependency Extraction Based on Improved Stacking Ensemble Machine Learning (/2227-7390/12/9/1272)

by **Hui Guan**, **Hang Xu** and **Lie Cai**

Mathematics **2024**, *12*(9), 1272; **https://doi.org/10.3390/math12091272 (https://doi.org/10.3390/math12091272)** - 23 Apr 2024

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Abstract To address the cost and efficiency issues of manually analysing requirement dependency in requirements engineering, a requirement dependency extraction method based on part-of-speech features and an improved stacking ensemble learning model (P-Stacking) is proposed. Firstly, to overcome the problem of singularity in the **[...] Read more.**

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Unveiling Fall Triggers in Older Adults: A Machine Learning Graphical Model Analysis (/2227-7390/12/9/1271)

by **Tho Nguyen**, **Ladda Thiamwong**, **Qian Lou** and **Rui Xie**

Mathematics **2024**, *12*(9), 1271; **https://doi.org/10.3390/math12091271 (https://doi.org/10.3390/math12091271)** - 23 Apr 2024

Abstract While existing research has identified diverse fall risk factors in adults aged 60 and older across various areas, comprehensively examining the interrelationships between all factors can enhance our knowledge of complex mechanisms and ultimately prevent falls. This study employs a novel approach—a *mixed* **[...] Read more.**

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Article

Green Measures for a Class of Non-Markov Processes

Herry P. Suryawan [1,†] and José L. da Silva [2,*,†]

[1] Department of Mathematics, Sanata Dharma University, Yogyakarta 55281, Indonesia; herrypribs@usd.ac.id
[2] CIMA, Faculty of Exact Sciences and Engineering, University of Madeira, Campus da Penteada, 9020-105 Funchal, Portugal
* Correspondence: joses@staff.uma.pt; Tel.: +351-291-705-185
† These authors contributed equally to this work.

Abstract: In this paper, we investigate the Green measure for a class of non-Gaussian processes in $\mathbb{R}^d$. These measures are associated with the family of generalized grey Brownian motions $B_{\beta,\alpha}$, $0 < \beta \leq 1$, $0 < \alpha \leq 2$. This family includes both fractional Brownian motion, Brownian motion, and other non-Gaussian processes. We show that the perpetual integral exists with probability 1 for $d\alpha > 2$ and $1 < \alpha \leq 2$. The Green measure then generalizes those measures of all these classes.

Keywords: fractional Brownian motion; generalized grey Brownian motion; green measure; subordination

MSC: 60G22; 65N80; 47A30

1. Introduction

In recent years, there has been a significant amount of research devoted to fractional dynamics related to fractional Brownian motion and related processes. These processes lack both the Markov and semimartingale properties from a mathematical standpoint. As a result, many traditional approaches in stochastic analysis do not apply, making their analysis more challenging. These processes are capable of modeling systems that exhibit long-range self-interaction and memory effects.

In 1992, Schneider introduced the grey Brownian motion [1], a class of non-Gaussian processes, to solve the time-fractional diffusion equation with a Caputo–Djrbashian derivative of fractional order. During the 1990s, Mainardi and their co-authors conducted a systematic investigation into fractional differential equations; see [2] and the references therein. They introduced the notion of generalized grey Brownian motion (ggBm for short), and the corresponding time-fractional differential equations governing its densities. This family of processes is denoted by $B_{\beta,\alpha}$ with parameters $0 < \beta \leq 1$ and $0 < \alpha \leq 2$. If $\beta \neq 1$, the process $B_{\beta,\alpha}$ is non-Gaussian with stationary increments and $\alpha/2$-self-similar; see Section 2 for details. The process $B_{\beta,\alpha}$ admits different representations (cf. (12) and (13) below) in terms of other known processes, which are useful for simulation and to derive other properties. In a recent work, Grothaus et al. [3] elaborated an infinite dimensional analysis for (non-Gaussian) measures of the Mittag-Leffler type. They used ggBm to solve the time-fractional heat equation, extending the fractional Feynman–Kac formula of Schneider [1].

The goal of this paper (see Theorem 1 and Corollary 1 below) is to prove the existence of the Green measure for the class of non-Gaussian processes ggBm in $\mathbb{R}^d$. This result will extend the results of Kondratiev et al. [4]. More precisely, for a Borel function $f : \mathbb{R}^d \longrightarrow \mathbb{R}$, the potential of f (see [5,6] for details) is defined as

$$V_{\beta,\alpha}(f,x) = \int_0^\infty \mathbb{E}\left[f(x + B_{\beta,\alpha}(t))\right] \mathrm{d}t, \quad x \in \mathbb{R}^d. \tag{1}$$

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We would like to investigate the class of functions f for which the potential of f has the representation

$$V_{\beta,\alpha}(f,x) = \int_{\mathbb{R}^d} f(y)\mathcal{G}_{\beta,\alpha}(x,\mathrm{d}y), \tag{2}$$

where $\mathcal{G}(x,\cdot) := \mathcal{G}_{\beta,\alpha}(x,\cdot)$ is a Radon measure on $\mathbb{R}^d$ called the Green measure corresponding to the ggBm $B_{\beta,\alpha}$; see Definition 2 below. If $B_{\beta,\alpha}$ admits a generator $L_{\beta,\alpha}$, then the potential $V(x,f)$ can be obtained from the equation

$$-LV = f.$$

The Green measure can be seen as the fundamental solution for the generator $L_{\beta,\alpha}$ of the process $B_{\beta,\alpha}$. First, we establish the existence of the perpetual integral (cf. Theorem 1):

$$\int_0^\infty f(x + B_{\beta,\alpha}(t))\,\mathrm{d}t \tag{3}$$

with probability one. This leads to an explicit representation of the Green measure for ggBm, namely (cf. Corollary 1)

$$\mathcal{G}_{\beta,\alpha}(x,\mathrm{d}y) = \frac{D}{|x-y|^{d-2/\alpha}}\,\mathrm{d}y, \quad d\alpha > 2, \quad 1 < \alpha \leq 2,$$

where D is a constant that depends on β, α, and the dimension d; see (17) for the explicit expression. Note that as $d\alpha > 2$ and $1 < \alpha \leq 2$, the Green measure $\mathcal{G}_{\beta,\alpha}(x,\cdot)$ exists for $d \geq 2$, since $d > 2/\alpha \in [1,2)$. The Brownian case ($\alpha = 1$) is covered only for $d \geq 3$. We emphasize that the existence of the Green measure for a given process X is not always guaranteed. In addition, finding a proper space of functions $f : \mathbb{R}^d \longrightarrow \mathbb{R}$ that guarantees the existence of (1) is crucial. As an example, the d-dimensional Bm starting at $x \in \mathbb{R}^d$ has a density given by $p_t(x,y) = (2\pi t)^{-d/2}\exp\big(-|x-y|^2/(2t)\big)$, $y \in \mathbb{R}^d$. It is not difficult to see that $\int_0^\infty p_t(x,y)\,\mathrm{d}t$ does not exist for $d = 1,2$. Hence, the Green measure of Bm for $d = 1,2$ does not exist. On the other hand, for $d \geq 3$, the Green measure of Bm on $\mathbb{R}^d$ exists and is given by $\mathcal{G}(x,\mathrm{d}y) = C(d)|x-y|^{2-d}\,\mathrm{d}y$, where $C(d)$ is a constant depending on the dimension d; see [4] and the references therein for more details. In a two-dimensional space, the Green measure of ggBm is determined by the parameter α that is related to the roughness of the path. The Green measure of ggBm for $d = 1$ requires further analysis (for Bm, see [7], Ch. 4), which we will postpone for a future paper.

This paper is organized as follows. In Section 2, we recall the definition and main properties of ggBm that will be needed later. In Section 3, we show the existence of the perpetual integral with probability one, which leads to the explicit formula for the Green measure for ggBm. In Section 4, we discuss the obtained results, connect them with other topics, and draw conclusions.

2. Generalized Grey Brownian Motion

We recall the class of non-Gaussian processes, called generalized grey Brownian motion, which we study below. This class of processes was first introduced by Schneider [8,9], and was generalized by Mura et al. (see [10,11]) as a stochastic model for slow/fast anomalous diffusion described by the time-fractional diffusion equation.

2.1. Definition and Properties

For $0 < \beta \leq 1$, the (entire) Mittag-Leffler function E_β is defined by the Taylor series

$$E_\beta(z) := \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\beta n + 1)}, \quad z \in \mathbb{C}, \tag{4}$$

where

$$\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}\,dt, \quad z \in \mathbb{C},\ \mathrm{Re}(z) \geq 0$$

is the Euler gamma function.

The M-Wright function is a special case of the class of Wright functions $W_{\lambda,\mu}$, $\lambda > -1$, $\mu \in \mathbb{C}$, via

$$M_\beta(z) := W_{-\beta,1-\beta}(-z) = \sum_{n=0}^{\infty} \frac{(-z)^n}{n!\Gamma(-\beta n + 1 - \beta)}.$$

The special choice $\beta = 1/2$ yields the Gaussian density on $[0,\infty)$:

$$M_{1/2}(z) = \frac{1}{\sqrt{\pi}}\exp\left(-\frac{z^2}{4}\right). \tag{5}$$

The Mittag-Leffler function E_β is the Laplace transform of the M-Wright function, that is,

$$E_\beta(-s) = \int_0^\infty e^{-s\tau}M_\beta(\tau)\,d\tau. \tag{6}$$

The generalized moments of the density M_β of order $\delta > -1$ are finite and are given (see [10]) by

$$\int_0^\infty \tau^\delta M_\beta(\tau)\,d\tau = \frac{\Gamma(\delta+1)}{\Gamma(\beta\delta+1)}. \tag{7}$$

Definition 1. *Let $0 < \beta \leq 1$ and $0 < \alpha \leq 2$ be given. A d-dimensional continuous stochastic process $B_{\beta,\alpha} = \{B_{\beta,\alpha}(t),\ t \geq 0\}$, starting at $0 \in \mathbb{R}^d$ and defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$, is a ggBm in $\mathbb{R}^d$ (see [11] for $d = 1$) if the following is satisfied:*

1. *$\mathbb{P}(B_{\beta,\alpha}(0) = 0) = 1$, that is, $B_{\beta,\alpha}$ starts at zero $\mathbb{P}$-almost surely ($\mathbb{P}$-a.s.).*
2. *Any collection $\{B_{\beta,\alpha}(t_1), \ldots, B_{\beta,\alpha}(t_n)\}$ with $0 \leq t_1 < t_2 < \cdots < t_n < \infty$ has a characteristic function given, for any $\theta = (\theta_1, \ldots, \theta_n) \in (\mathbb{R}^d)^n$ with $\theta_k = (\theta_{k,1}, \ldots \theta_{k,d})$, $k = 1, \ldots, n$, by*

$$\mathbb{E}\left[\exp\left(\mathrm{i}\sum_{k=1}^{n}(\theta_k, B_{\beta,\alpha}(t_k))_{\mathbb{R}^d}\right)\right] = E_\beta\left[-\frac{1}{2}\sum_{j=1}^{d}(\theta_{\cdot,j}, \gamma_\alpha\theta_{\cdot,j})_{\mathbb{R}^n}\right], \tag{8}$$

   where $\mathbb{E}$ denotes the expectation with regard to $\mathbb{P}$ and

$$\gamma_\alpha := \gamma_{\alpha,n} := \left(t_k^\alpha + t_j^\alpha - |t_k - t_j|^\alpha\right)_{k,j=1}^{n}.$$

3. *The joint probability density function of $(B_{\beta,\alpha}(t_1), \ldots, B_{\beta,\alpha}(t_n))$ is equal to*

$$\rho_\beta(\theta, \gamma_\alpha) = \frac{(2\pi)^{-\frac{nd}{2}}}{(\det\gamma_\alpha)^{d/2}}\int_0^\infty \tau^{-\frac{nd}{2}}\mathrm{e}^{-\frac{1}{2\tau}\sum_{j=1}^{d}(\theta_{\cdot,j}, \gamma_\alpha^{-1}\theta_{\cdot,j})_{\mathbb{R}^n}}M_\beta(\tau)\,d\tau. \tag{9}$$

The following are the most important key properties of ggBm:

(P1). For each $t \geq 0$, the moments of any order of $B_{\beta,\alpha}(t)$ are given by

$$\begin{cases} \mathbb{E}\left[|B_{\beta,\alpha}(t)|^{2n+1}\right] & = 0, \\ \mathbb{E}\left[|B_{\beta,\alpha}(t)|^{2n}\right] & = \frac{(2n)!}{2^n\Gamma(\beta n+1)}t^{\alpha n}. \end{cases}$$

(P2). The covariance function has the form

$$\mathbb{E}\left[\left(B_{\beta,\alpha}(t), B_{\beta,\alpha}(s)\right)\right] = \frac{d}{2\Gamma(\beta+1)}\left(t^\alpha + s^\alpha - |t-s|^\alpha\right), \quad t, s \geq 0. \tag{10}$$

(P3). For each $t, s \geq 0$, the characteristic function of the increments is

$$\mathbb{E}\left[e^{i(k,B_{\beta,\alpha}(t)-B_{\beta,\alpha}(s))}\right] = E_\beta\left(-\frac{|k|^2}{2}|t-s|^\alpha\right), \quad k \in \mathbb{R}^d. \tag{11}$$

(P4). The process $B_{\beta,\alpha}$ is non-Gaussian and $\alpha/2$-self-similar with stationary increments.

(P5). The ggBm is not a semimartingale. Furthermore, $B_{\alpha,\beta}$ cannot be of finite variation in $[0,1]$ and, by the scaling and stationarity of the increment, on any interval in $\mathbb{R}^+$.

(P5). For $n = 1$, the density $\rho_\beta(x,t)$, $x \in \mathbb{R}^d$, $t > 0$ is the fundamental solution of the following fractional differential equation (see [12]):

$$\mathbb{D}_t^{2\beta}\rho_\beta(x,t) = \Delta_x \rho_\beta(x,t),$$

where Δ_x is the d-dimensional Laplacian in x and $\mathbb{D}_t^{2\beta}$ is the Caputo–Dzherbashian fractional derivative; see [13] for the definition and properties.

2.2. Representations of Generalized Grey Brownian Motion

The ggBm admits different representations in terms of well-known processes. It follows from (8) that ggBm has an elliptical distribution; see Section 3 in [3]. On the other hand, ggBm is also given as a product (see [10] for $d = 1$) of two processes, as follows:

$$\left\{B_{\beta,\alpha}(t),\, t \geq 0\right\} \stackrel{\mathcal{L}}{=} \left\{\sqrt{Y_\beta}B^{\alpha/2}(t),\, t \geq 0\right\}. \tag{12}$$

Here, $\stackrel{\mathcal{L}}{=}$ means equality in law, the non-negative random variable Y_β has density M_β, and $B^{\alpha/2}$ is a d-dimensional fBm with Hurst parameter $\alpha/2$ and is independent of Y_β.

We give another representation of ggBm $B_{\beta,\alpha}$ as a subordination of fBm (see Section 2.14 in [14] for $d = 1$) which is used below. For completeness, we give a short proof.

Proposition 1. *The ggBm has the following representation:*

$$\left\{B_{\beta,\alpha}(t),\, t \geq 0\right\} \stackrel{\mathcal{L}}{=} \left\{B^{\alpha/2}(tY_\beta^{1/\alpha}),\, t \geq 0\right\}. \tag{13}$$

Proof. We must show that both representations (12) and (13) have the same finite-dimensional distribution. For every $\theta = (\theta_1, \ldots, \theta_n) \in (\mathbb{R}^d)^n$, we have

$$\begin{aligned}
\mathbb{E}\left[\exp\left(\mathrm{i}\sum_{k=1}^n (\theta_k, B^{\alpha/2}(t_k Y_\beta^{1/\alpha}))\right)\right] &= \int_0^\infty \mathbb{E}\left[\exp\left(\mathrm{i}\sum_{k=1}^n (\theta_k, B^{\alpha/2}(t_k y^{1/\alpha}))\right)\right] M_\beta(y)\,\mathrm{d}y \\
&= \int_0^\infty \mathbb{E}\left[\exp\left(\mathrm{i}\sum_{k=1}^n (\theta_k, y^{1/2}B^{\alpha/2}(t_k))\right)\right] M_\beta(y)\,\mathrm{d}y \\
&= \mathbb{E}\left[\exp\left(\mathrm{i}\sum_{k=1}^n (\theta_k, Y_\beta^{1/2}B^{\alpha/2}(t_k))\right)\right].
\end{aligned}$$

In the second equality, we use the $\alpha/2$-self-similarity of fBm. This completes the proof. □

3. The Green Measure for Generalized Grey Brownian Motion

In this section, we show the existence of the Green measure for ggBm; see (1) and (2). Let us begin by discussing the existence of the Green measure for a general stochastic process X.

Let $X = \{X(t),\ t \geq 0\}$ be a stochastic process in $\mathbb{R}^d$ starting from $x \in \mathbb{R}^d$. If $X(t)$, $t \geq 0$, has a probability distribution $\rho_{X(t)}(x,\cdot)$, then Equation (1) becomes

$$V_X(x,f) = \int_0^\infty \int_{\mathbb{R}^d} f(y)\,\rho_{X(t)}(x,\mathrm{d}y)\,\mathrm{d}t. \tag{14}$$

Then, applying the Fubini theorem, the Green measure $\mathcal{G}_X(x,\cdot)$ of X is given by

$$\mathcal{G}_X(x,\mathrm{d}y) = \int_0^\infty \rho_{X(t)}(x,\mathrm{d}y)\,\mathrm{d}t,$$

assuming the existence of $\mathcal{G}_X(x,\cdot)$ as a Radon measure on $\mathbb{R}^d$. That is, for every bounded Borel set $B \in \mathcal{B}_b(\mathbb{R}^d)$ we have

$$\mathcal{G}_X(x,B) = \int_0^\infty \rho_{X(t)}(x,B)\,\mathrm{d}t < \infty.$$

If the probability distribution $\rho_{X(t)}(x,\cdot)$ is also absolutely continuous with respect to the Lebesgue measure, say $\rho_{X(t)}(x,\mathrm{d}y) = \rho_t(x,y)\,\mathrm{d}y$, then the function

$$g_X(x,y) := \int_0^\infty \rho_t(x,y)\,\mathrm{d}t, \quad \forall y \in \mathbb{R}^d, \tag{15}$$

is called the Green function of the stochastic process X. Moreover, the Green measure in this case is given by $\mathcal{G}_X(x,\mathrm{d}y) = g_X(x,y)\,\mathrm{d}y$.

This leads us to the following definition of the Green measure of a stochastic process X.

Definition 2. *Let $X = \{X(t),\ t \geq 0\}$ be a stochastic process on $\mathbb{R}^d$ starting from $x \in \mathbb{R}^d$ and $\rho_{X(t)}(x,\cdot)$ be the probability distribution of $X(t)$, $t \geq 0$. The Green measure of X is defined as a Radon measure on $\mathbb{R}^d$ by*

$$\mathcal{G}_X(x,B) := \int_0^\infty \rho_{X(t)}(x,B)\,\mathrm{d}t, \quad B \in \mathcal{B}_b(\mathbb{R}^d),$$

or

$$\int_{\mathbb{R}^d} f(y)\mathcal{G}_X(x,\mathrm{d}y) = \int_{\mathbb{R}^d} f(y) \int_0^\infty \rho_{X(t)}(x,dy)\,\mathrm{d}t, \quad f \in C_0(\mathbb{R}^d)$$

whenever these integrals exist.

In other words, $\mathcal{G}_X(x,B)$ is the expected length of time the process remains in B.

To state the main theorem that establishes the existence of the Green measure for ggBm, first, we introduce a proper Banach space of functions $f : \mathbb{R}^d \longrightarrow \mathbb{R}$ such that the perpetual integral (3) is finite $\mathbb{P}$-a.s. Without a loss of generality, we can assume that $f \geq 0$ above. We define the space $CL(\mathbb{R}^d)$, of continuous real valued, on $\mathbb{R}^d$ by

$$CL(\mathbb{R}^d) := \{f : \mathbb{R}^d \to \mathbb{R} \mid f \text{ is continuous, bounded and } f \in L^1(\mathbb{R}^d)\}.$$

The space $CL(\mathbb{R}^d)$ becomes a Banach space with the norm

$$\|f\|_{CL} := \|f\|_\infty + \|f\|_1, \quad \forall f \in CL(\mathbb{R}^d),$$

where $\|\cdot\|_\infty$ denotes the sup-norm and $\|\cdot\|_1$ is the norm in $L^1(\mathbb{R}^d)$. The choice of $CL(\mathbb{R}^d)$ allows us to show that the family of random variables (3) with $f \in CL(\mathbb{R}^d)$ have finite expectations $\mathbb{P}$-a.s.

Theorem 1. *Let $f \in CL(\mathbb{R}^d)$ and $x \in \mathbb{R}^d$ be given and consider ggBm $B_{\beta,\alpha}$ with $d\alpha > 2$ and $1 < \alpha \leq 2$. Then, the perpetual integral functional $\int_0^\infty f(x + B(t))\,\mathrm{d}t$ is finite $\mathbb{P}$-a.s. and its expectation equals*

$$\mathbb{E}\left[\int_0^\infty f(x + B_{\beta,\alpha}(t))\,\mathrm{d}t\right] = D\int_{\mathbb{R}^d} \frac{f(x+y)}{|y|^{d-2/\alpha}}\,\mathrm{d}y, \tag{16}$$

where

$$D = D(\beta,\alpha,d) = \frac{1}{\alpha}2^{-1/\alpha}\pi^{-\frac{d}{2}}\Gamma\left(\frac{d}{2} - \frac{1}{\alpha}\right)\frac{\Gamma(1-\frac{1}{\alpha})}{\Gamma(1-\frac{\beta}{\alpha})}. \tag{17}$$

Proof. Given that $x \in \mathbb{R}^d$ and $f \in CL(\mathbb{R}^d)$ are non-negative, let $\rho_\beta(\cdot, t^\alpha)$ denote the density of $B_{\beta,\alpha}(t)$, $t \geq 0$, which is given by (see (9) with $n = 1$)

$$\rho_\beta(y, t^\alpha) = \frac{1}{(2\pi t^\alpha)^{d/2}} \int_0^\infty \tau^{-d/2} \mathrm{e}^{-\frac{|y|^2}{2t^\alpha \tau}} M_\beta(\tau)\, \mathrm{d}\tau, \quad y \in \mathbb{R}^d.$$

First, we show equality (16). It follows from the above considerations that

$$\begin{aligned}\mathbb{E}\left[\int_0^\infty f(x + B_{\beta,\alpha}(t))\, \mathrm{d}t\right] &= \int_0^\infty \int_{\mathbb{R}^d} f(x+y)\rho_\beta(y, t^\alpha)\, \mathrm{d}y\, \mathrm{d}t. \\ &= \int_0^\infty \int_{\mathbb{R}^d} f(x+y) \frac{1}{(2\pi t^\alpha)^{d/2}} \int_0^\infty \tau^{-d/2} M_\beta(\tau) e^{-\frac{|y|^2}{2t^\alpha \tau}}\, \mathrm{d}\tau\, \mathrm{d}y\, \mathrm{d}t.\end{aligned}$$

Using Fubini's Theorem, we first compute the t-integral and use the assumption $d\alpha > 2$. We obtain

$$\int_0^\infty \frac{1}{(2\pi t^\alpha \tau)^{d/2}} e^{-\frac{|y|^2}{2t^\alpha \tau}}\, \mathrm{d}t = C(\alpha, d) \frac{\tau^{-\frac{1}{\alpha}}}{|y|^{d-2/\alpha}},$$

where

$$C(\alpha, d) := \frac{1}{\alpha} 2^{-1/\alpha} \pi^{-\frac{d}{2}} \Gamma\left(\frac{d}{2} - \frac{1}{\alpha}\right).$$

Next, we compute the τ-integral using (7) so that

$$\int_0^\infty \tau^{-1/\alpha} M_\beta(\tau)\, \mathrm{d}\tau = \frac{\Gamma(1 - \frac{1}{\alpha})}{\Gamma(1 - \frac{\beta}{\alpha})}, \quad \alpha > 1.$$

Combining them gives the equality (16) where $D = D(\beta, \alpha, d) = C(\alpha, d) \frac{\Gamma(1-\frac{1}{\alpha})}{\Gamma(1-\frac{\beta}{\alpha})}$.

Now, we show that the right-hand side of (16) is finite for every non-negative $f \in CL(\mathbb{R}^d)$. To see this, we can use the local integrability of $|y|^{d-2/\alpha}$ in y and obtain

$$\begin{aligned}\int_{\mathbb{R}^d} \frac{f(x+y)}{|y|^{d-2/\alpha}}\, \mathrm{d}y &= \int_{\{|y|\leq 1\}} \frac{f(x+y)}{|y|^{d-2/\alpha}}\, \mathrm{d}y + \int_{\{|y|>1\}} \frac{f(x+y)}{|y|^{d-2/\alpha}}\, \mathrm{d}y \\ &\leq C_1 \|f\|_\infty + C_2 \|f\|_1 \leq C \|f\|_{CL}.\end{aligned}$$

Therefore, the integral in (16) is, in fact, well defined. In other words, the integral $\int_0^\infty f\big(x + B_{\beta,\alpha}(t)\big)\, \mathrm{d}t$ exists with probability one. This completes the proof. □

As a consequence of the above theorem, we immediately obtain the Green measure of ggBm $B_{\beta,\alpha}$, that is, comparing (2) and (16).

Corollary 1. *The Green measure of ggBm $B_{\beta,\alpha}$ for $d\alpha > 2$ is given by*

$$\mathcal{G}_{\beta,\alpha}(x, \mathrm{d}y) = \frac{D}{|x-y|^{d-2/\alpha}}\, \mathrm{d}y,$$

where D is given by (17).

Remark 1.

1. *It is possible to show that, given $f \neq 0$, the perpetual integral (3) is a non-constant random variable. As a consequence, for $f \geq 0$ the variance of the random variable (3) is strictly positive. The proof uses the notion of conditional full support of ggBm. We do not provide a detailed explanation of this result that closely follows the ideas of Theorem 2.2 in [4], to which we address interested readers.*

2. *Note also that the functional in* (1),

$$V_{\beta,\alpha}(\cdot, x) : CL(\mathbb{R}^d) \longrightarrow \mathbb{R}$$

is continuous. In fact, from the proof of Theorem 1, any $f \in CL(\mathbb{R}^d)$ *yields*

$$|V_{\beta,\alpha}(f, x)| \leq K\|f\|_{CL},$$

where K is a constant depending on the parameters β, α, *and* d.

4. Discussion and Conclusions

We derived the Green measure for the class of stochastic processes called generalized grey Brownian motion in Euclidean space $\mathbb{R}^d$ for $d \geq 2$. This class includes, in particular, fractional Brownian motion and other non-Gaussian processes. To address the case where $d = 1$, a renormalization process is needed. However, this will be postponed to future work. For $\beta = \alpha = 1$ ggBm, $B_{1,1}$ is nothing but a Brownian motion. In this case, the Green measure exists for $d \geq 3$. Green measures and Green functions are well known to be intrinsically connected and applied to (stochastic partial) differential equations. In this context, the Green measures discussed in this paper play the same role for space-time-fractional derivatives. The presented method can be applied to other processes with sufficient information on the density and existence of the integrals. If we consider a Markov process X that admits a Green measure and T, a random time change given by an inverse subordinator, then the Green measure of the subordinated process $X(T(t))$, $t \geq 0$ exists only after renormalization. Mixing different types of processes, e.g., fBm and scaled Bm, as described in [15], or Markovian and non-Markovian, as in [16], may lead us to a renormalization procedure to guarantee the existence of the Green measure.

The relationship between the Green measure and the local time of the ggBm can be described as follows. For any $T > 0$ and a continuous function $f : \mathbb{R}^d \longrightarrow \mathbb{R}$, the integral functional

$$\int_0^T f(B_{\beta,\alpha}(t))\,\mathrm{d}t \tag{18}$$

is well defined. For $d = 1$, the integral (18) with $f \in L^1(\mathbb{R})$ is represented as

$$\int_0^T f(B_{\beta,\alpha}(t))\,\mathrm{d}t = \int_{\mathbb{R}} f(x) L_{\beta,\alpha}(T, x)\,\mathrm{d}x,$$

where $L_{\beta,\alpha}(T, x)$ is the local time of ggBm up to time T at the point x (see [3]). The Green measure corresponds to the asymptotic behaviour in T of the expectation of local time $L_{\beta,\alpha}(T, x)$. The existence of this asymptotic depends on the dimension d and the transient or recurrent properties in the process.

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