Paper Stochastic Epidemic E3S Herry 2024

by Ricky Aditya

E3S Web of Conferences **475**, 02016 (2024) https://doi.org/10.1051/e3sconf/202447502016
InCASST 2023

A study of stochastic epidemic model driven by liouville fractional brownian motion coupled with seasonal air pollution

Herry Pribawanto Suryawan[1*]

[1]Department of Mathematics, Faculty of Science and Technology, Sanata Dharma University, Yogyakarta, Indonesia

Abstract. Air pollution can cause and provoke respiratory diseases. It is an important topic to the public, particularly in developing countries. Since there are many uncertain factors in the environment, stochastic differential equation model is a powerful tool to study the changes of air pollution and the transmission of infectious diseases. The removal of air pollutants as well as the transmission of diseases can be influenced by random perturbations with memories. In this research, we develop a mathematical model in the form of a system of stochastic differential equations driven by fractional Brownian motion of Liouville-type, coupled with seasonal air pollution, to study the dynamics of infectious respiratory disease spread. As a result , by using stochastic calculus techniques, we derive the equation for the level of air pollution.

1 Introduction

Air pollution has aroused a great concern all over the world, because it can cause and aggravate respiratory diseases, see e.g. [1]. According to Alyousifi et al in [2] the level of air pollution reaches epidemic levels, which is caused by increasing use of fossil fuels, increasing vehicle exhaust emissions, the excessive industrialization and the destruction of vegetation. Numerous epidemiological studies have shown associations of particulate air pollution with risk for various adverse health outcomes, in which the most affected pathologies are chronic obstructive pulmonary disease, lung cancer, influenza and respiratory infectious diseases, see, for instances [3-5]. Hence, it is important to have a comprehensive understanding through mathematical means of the change of air pollution, and to evaluate the effects of air pollution on people's health. Since there are many random factors in the environment as mentioned in [6, 7], stochastic differential equation (SDE) model is a useful tool to study the changes of air pollution and the transmission of infectious diseases [8, 9].

The air quality index (AQI) is commonly used as a standard measurement of air quality in a quantitative description. Human activities are the main sources of air pollution and lead to atmospheric contamination with a wide variety of pollutants such as fine particulate matter, inhalable particles, carbon monoxide, ozone, nitrogen dioxide, and sulfur dioxide [10]. However, some pollutants may be cleared by vegetation or dispersed by wind. Some studies,

* Corresponding author: herrypribs@usd.ac.id

such as in [11, 12], show that high concentrations of air pollutants have a positive association with respiratory symptoms or hospitalization. Chung et al. in [13] showed that hospital admissions due to chronic obstructive pulmonary disease (COPD) and the increase in asthma prevalence were associated with the level of outdoor air pollutants. In particular, some epidemiological and experimental studies indicated an increased risk for respiratory viral infections, see e.g. [14]. To study the impact of seasonal changes in air pollution and random factors in the environment on infectious respiratory diseases, He et al in 2019 developed a stochastic susceptible-infective-susceptible (SIS) model driven by random change of air pollution, see [15]. This model has two features:

1. the equations of the value of AQI, denoted by $F(t)$, and the number of infectives, denoted by $I(t)$, are both time-inhomogeneous, i.e., the coefficients in the equations are constants.
2. the removal process of air pollution is disturbed by some random factors.

In reality, the periodic phenomenon in air pollution is widely observed in time series data under varying climatic and environment conditions as studied in [16, 17]. Therefore, it would be more realistic to assume the coefficients in the equation describing $F(t)$ to be periodic functions rather than constants. Deepa et al in [18] proved that there is a linear correlation between particulate matter and respiratory disease. The risk of respiratory diseases is related to the severity of air pollution. When we study the dynamics of respiratory diseases, the infection probability of infectious respiratory diseases is positively correlated with the value of AQI. In order to describe this, we assume that the transmission rate of the disease is a function dependent on the AQI.

In addition to seasonal changes, random disorder can also affect the disease dynamics, see, for instances, [19-21]. Therefore, it is necessary to consider randomness in the change of air pollution and respiratory disease transmission. He et al in [22] continued their study by assuming that both the clearance process of air pollution and the infection of disease are subject to random disturbance. Although there are many studies of periodic SDE and periodic stochastic epidemic models, such as in [23-26], there is a lack of the analysis for mathematical model that couples seasonal variations in air pollution with the dynamics of respiratory infections. The dynamical behavior of the air pollutant concentration is an inflow-clearance process and the equation of the change of AQI is independent of infection. However, the change of AQI affects the risk of infection of respiratory diseases. Thus, the periodicity of the whole system is caused by the periodicity of one equation. It is not trivial to analyze the full coupled system directly. Therefore, mathematical analysis of such stochastic periodic models needs to be investigated.

Motivated by the work of He et al in [22], in the present study we generalize the model by replacing the noise source from standard Brownian motion to Liouville fractional Brownian motion to capture the long memory property in the reality. The main section in the paper is organized as follows. First, we summarize a theoretical background on the Liouville fractional Brownian motion. Next, we build the epidemic model driven by Liouville fractional Brownian motion coupled with seasonal air pollution. Finally, since the level of air pollution in the model is independent of the number of infectious individuals, we start with a derivation of a solution of the stochastic equation on the air pollution level by using the stochastic calculus with respect to Liouville fractional Brownian motion developed via semimartingale approximation.

2 Model and analysis

2.1 Liouville rractional Brownian Motion

The fractional Brownian motion (fBm) of Hurst parameter $H \in (0,1)$ is a centered Gaussian process $B^H = (B_t^H)_{t\geq 0}$ defined on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with the covariance function

$$cov(t,s) = \mathbb{E}(B_t^H B_s^H) = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H})$$

This stochastic process is introduced by A.N. Kolmogorov in the 1940's and is the only Gaussian self-similar process with stationary increments. The fBm has attracted many attentions due to its wide range of applications such as in mathematical finance, hydrology, filtering theory, and queuing networks. In the case where $H = \frac{1}{2}$, the process B^H is a standard Brownian motion. If $H \neq \frac{1}{2}$, B^H is neither a semimartingale nor a Markov process and the classical Ito stochastic calculus cannot be applied. There are various approaches to stochastic calculus with respect to fBm by using some difficult tools such as: regularization approach [27], Malliavin calculus [28], theory of Wick product [29], pathwise approach [30], and white noise analysis [31]. However, it is not easy to find explicit solutions from these methods for many practical problems.

Mandelbrot and van Ness in [32] proved a stochastic integral representation of fBm in the form:

$$B_t^H = \frac{1}{\Gamma(H+\frac{1}{2})}(U_t + W_t^H),$$

where $(U_t)_{t\geq 0}$ is a stochastic process of absolutely continuous trajectories and $W^H = (W_t^H)_{t\geq 0}$ with

$$W_t^H := \int_0^t (t-s)^{H-\frac{1}{2}}\, dB_s$$

is called a Liouville fractional Brownian motion (LfBM). Here, $(B_t)_{t\geq 0}$ is a standard Brownian motion and Γ is the Euler gamma function. Note that for $H = \frac{1}{2}$, $W_t^H = B_t$. An LfBm shares many properties of a fBm, such as Holder continuity of trajectories and long-range dependence, except that it has non-stationary increments. For a more detailed discussion see e.g. [33]. Moreover, Comte and Renault in [34] gave a nice application of LfBm to finance. Because of these reasons and for simplicity we use W^H throughout this paper. It is known that W_t^H can be approximated in $L^2(\Omega)$ by semimartingale $(M_t^\varepsilon)_{t\geq 0}$, $\varepsilon > 0$ with

$$M_t^\varepsilon = \int_0^t (t-s+\varepsilon)^{H-\frac{1}{2}}\, dB_s,$$

and the convergence is uniform in $t \in [0,T]$. Stochastic calculus with respect to LfBm via semimartingale approximation was first introduced by Thao in [35] and then further investigated by Dung, see [36-38]. In particular, the SDE of the form

$$dX_t = b(t,X_t)dt + \sigma(t)X_t dW_t^H, \quad X_0 = x_0 \qquad (1)$$

is studied in [38]. In order to guarantee the existence and uniqueness of the solution of Eq. (1), the following standard assumptions on coefficients are made. The volatility $\sigma: [0,T] \to \mathbb{R}$ is a bounded deterministic function and the drift coefficient $b: [0,T] \times \mathbb{R} \to \mathbb{R}$ is a measurable function in all their arguments and satisfies the following conditions: there is a positive constant K such that

1) $b(t,x)$ is a continuously differentiable function in x and $|b(t,x) - b(t,y)| \leq K|x-y|$ for all $x,y \in \mathbb{R}$, $t \in [0,T]$, and

2) $|b(t,x)| \le K\,(1+|x|)$ for all $x \in \mathbb{R}$, $t \in [0,T]$.

Theorem 2.1 ([38, Theorem 3.2])
SDE (1) has a solution which is given by

$$X_t = Y_t^{-1} Z_t,$$

where

$$Y_t = \exp\left(-\int_0^t \sigma(s)\, dW_s^H\right)$$

and Z_t is the unique solution of the random differential equation

$$dZ_t = Y_t b(t, Y_t^{-1} Z_t)\, dt, \;\; Z_0 = x_0.$$

Note that the stochastic integral in Y_t is well-defined as an $L^2(\Omega)$-limit of a sequence of stochastic integrals with respect to semimartingale.

2.2 The model

In the study of infectious respiratory disease dynamics affected by air pollution, an Susceptibe-Infectious System (SIS) model is used to describe the dynamic behavior of epidemics, where $S(t)$ and $I(t)$ denote the number of susceptible and infectious individuals at time t, respectively. The total population is assumed to be a constant N. The number of susceptible individuals can be represented by $N - I(t)$. Let $\beta(t)$ be the disease transmission rate function. The severity of air pollution affects the infectious respiratory disease transmission rate. Thus, we assume that the disease transmission rate $\beta(t)$ depends on the level of air pollution, denoted as $F(t)$. It is assumed to be a function of $F(t)$. This is the rate at which susceptibles come into $\beta(F(t))$ potentially infectious contact with infected individuals, which changes with the value of $F(t)$. In our model, the relationship is further assumed to be linear, i.e. $\beta\big(F(t)\big) = \beta F(t)$. To incorporate the seasonality in the variation of air pollutants, the clearance rate, denoted by $\mu(t)$, is assumed to change over time with a certain period. The inflow rate of pollutants is assumed to be a constant c. As mentioned in the introduction, the dynamics of the concentration of pollutants and infected individuals are stochastic. The diagram of the model is shown in Fig. 1.

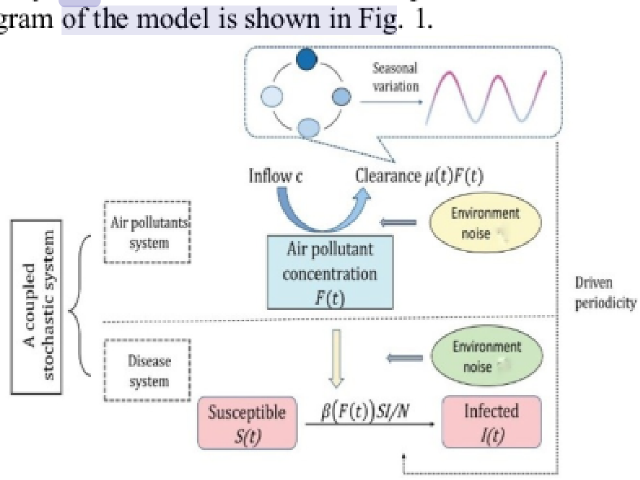

Fig. 1. Flow diagram of the coupled system [22].

In the model, we consider the situation in which both the disease transmission rate $\boldsymbol{\beta(t)}$ and the clearance rate of air pollutants $\boldsymbol{\mu(t)}$ are influenced by random perturbations. Thus, we replace $\boldsymbol{\beta(F(t))}$ by $\boldsymbol{\beta\big(F(t)\big) + \sigma_1\eta_1(t)}$ and $\boldsymbol{\mu(t)}$ by $\boldsymbol{\mu(t) + \sigma_2\eta_2(t)}$, where $\boldsymbol{\eta_1(t)}$ and

$\boldsymbol{\eta_2(t)}$ are Liouville fractional white noises (the time derivative of LfBm in the sense of generalized function), $\boldsymbol{\sigma_1}$ and $\boldsymbol{\sigma_2}$ are two nonnegative real numbers, which represent the intensities of noises $\boldsymbol{\eta_1(t)}$ and $\boldsymbol{\eta_2(t)}$, respectively. Modifying the model in the [22], the seasonally forced stochastic model driven by LfBm can be written as

$$\begin{cases} dI(t) = \left(\beta F(t)\dfrac{(N - I(t))I(t)}{N} - \gamma I(t)\right)dt \\ \qquad + \sigma_1 \dfrac{(N - I(t))I(t)}{N}\, dW_1^H(t) \\ dF(t) = (c - \mu(t)F(t))\, dt - \sigma_2\, F(t)\, dW_2^H(t) \end{cases} . \quad (2)$$

Here, W_1^H dan W_2^H are independent LfBms, $\beta F(t)$ is the infection rate, and $\gamma > 0$ is the recovery rate for infected individuals. In this paper, we choose the pollution clearance rate $\mu(t)$ to be

$$\mu(t) = \mu_0 + \mu_1 \sin(\omega t + \phi), \qquad (3)$$

where $\omega = \frac{2\pi}{365}$ is a periodic parameter assuming that there are 365 days in a year, ϕ and μ_0 are the phase parameter and offset parameter, respectively. The amplitude is controlled by μ_1. It is clear that $\mu_0 - \mu_1 \le \mu(t) \le \mu_0 + \mu_1$.

2.3 Analysis of the model

In model (2), the second equation of $F(t)$ is independent of the first equation of $I(t)$. The explicit solution of $F(t)$ can be obtained using Theorem 2.1 as follows. Let $Y_t = \exp\left(-\int_0^t \sigma_2\, dW_s^H\right) = \exp(-\sigma_2 W_t^H)$. The solution $F(t)$ is given by $F(t) = Y_t^{-1} Z_t$, where Z_t is the unique solution of the random differential equation

$$dZ_t = Y_t(c - \mu(t) Y_t^{-1} Z_t),$$

where $\mu(t)$ is given by (3). Hence,

$$dZ_t = \exp(-\sigma_2 W_t^H)\,(c - \mu(t)\exp(\sigma_2 W_t^H) Z_t)$$
$$= c\exp(-\sigma_2 W_t^H) - \mu(t) Z_t.$$

Rewriting the last expression, we obtain a nonhomogeneous linear first order random differential equation

$$\frac{dZ_t}{dt} + \mu(t) Z_t = c\exp(-\sigma_2 W_t^H),\, Z_0 = F(0) = F_0 \quad (4)$$

Let us use the integrating factor

$$\exp\left(\int \mu(t)dt\right)$$
$$= \exp\left(\int (\mu_0 + \mu_1 \sin(\omega t + \phi))\, dt\right)$$
$$= \exp\left(\mu_0 t - \frac{\mu_1}{\omega}\cos(\omega t + \phi)\right).$$

Then, multiply both sides of (4) by the integrating factor to obtain

$$\exp\left(\mu_0 t - \frac{\mu_1}{\omega}\cos(\omega t + \phi)\right)\left(\frac{dZ_t}{dt} + \mu(t) Z_t\right)$$
$$= c\exp\left(\mu_0 t - \frac{\mu_1}{\omega}\cos(\omega t + \phi)\right)\exp(-\sigma_2 W_t^H).$$

The last equation is equivalent with

$$\frac{d}{dt}\left(\exp\left(\mu_0 t - \frac{\mu_1}{\omega}\cos(\omega t + \phi)\right) Z_t\right)$$

$$= c\exp\left(\mu_0 t - \frac{\mu_1}{\omega}\cos(\omega t + \phi) - \sigma_2 W_t^H\right).$$

Integrating from 0 to t gives

$$\int_0^t \frac{d}{ds}\exp\left(\mu_0 s - \frac{\mu_1}{\omega}\cos(\omega s + \phi)\right) Z_s\, ds$$
$$= \int_0^t c\exp\left(\mu_0 s - \frac{\mu_1}{\omega}\cos(\omega s + \phi) - \sigma_2 W_s^H\right)\, ds.$$

Applying the fundamental theorem of calculus we get

$$\exp\left(\mu_0 s - \frac{\mu_1}{\omega}\cos(\omega s + \phi)\right) Z_s\Big]_0^t$$
$$= c\int_0^t \exp\left(\mu_0 s - \frac{\mu_1}{\omega}\cos(\omega s + \phi) - \sigma_2 W_s^H\right)\, ds$$

and hence

$$\exp\left(\mu_0 t - \frac{\mu_1}{\omega}\cos(\omega t + \phi)\right) Z_t$$
$$= \exp\left(-\frac{\mu_1}{\omega}\cos\phi\right) F_0$$
$$+ c\int_0^t \exp\left(\mu_0 s - \frac{\mu_1}{\omega}\cos(\omega s + \phi) - \sigma_2 W_s^H\right)\, ds.$$

By dividing both sides with $\exp\left(\mu_0 t - \frac{\mu_1}{\omega}\cos(\omega t + \phi)\right)$ and simplifying the result gives the solution of (4):

$$Z_t = F_0\exp\left(-\mu_0 t + \frac{\mu_1}{\omega}\cos(\omega t + \phi) - \frac{\mu_1}{\omega}\cos\phi\right)$$
$$+ c\int_0^t \exp(-\mu_0(t - s) - \sigma_2 W_s^H)$$
$$\exp\left(\frac{\mu_1}{\omega}\cos(\omega t + \phi) - \cos(\omega s + \phi)\right)\, ds.$$

Now,

$$F(t)$$
$$= Y_t^{-1} Z_t$$
$$= \exp(\sigma_2 W_t^H) Z_t$$
$$= F_0\exp\left(-\mu_0 t + \frac{\mu_1}{\omega}(\cos(\omega t + \phi) - \cos\phi) + \sigma_2 W_t^H\right) + c\int_0^t \exp\left(-\mu_0(t - s) + \sigma_2(W_t^H - W_s^H) + \frac{\mu_1}{\omega}(\cos(\omega t + \phi) - \cos(\omega s + \phi))\right)\, ds. \qquad (5)$$

Observe that the solution (5) is complicated and hence, cannot be directly substituted into the first equation of model (2) for a further theoretical analysis. This makes it difficult to obtain the properties of the equation of $I(t)$. In view of this obstacle, one should first study some properties of $F(t)$ in model (2).

3 Conclusion

This preliminary study introduces a coupled periodic SDE model driven by LfBm to study the dynamics of infectious respiratory disease in an environment with air pollution. We assume that the clearance process of air pollutant has seasonal variation and random disturbance. In addition, the transmission rate of the disease is assumed to depend on the concentration of air pollutants and also has random disturbance. The connection between the air pollutants and the number of infected individuals is that the change of concentration of air pollutants is independent of the disease infection of human but affects the the susceptibility to infection. It is mathematically challenging to analyze this coupled stochastic system by exact methods. So far, we are able to give analytical solution for $F(t)$. Unfortunately, this analytical solution of $F(t)$ is intricate and cannot be directly used in the equation of $I(t)$ for further analysis. For future research we plan to do a more deep study on $F(t)$ such as investigation of its boundedness and the existence of periodic solutions. It is also planned to verify the theoretical results by numerical experiments. To see that our model whether suitable for studying infectious respiratory diseases whose transmission can be investigated by an SIS epidemiological model we should validate the model and estimate parameter values by data fitting. This will be also considered in a forthcoming work.

References

1. M. Fernandes, C. Carletti, L. Sierra de Araujo, R. Santo, J. Reis, Rev. Neurol **175**, 10 (2019)
2. Y. Alyousifi, N. Masseran, K. Ibrahim, Stoch. Env. Res. Risk. **32**, (2018)
3. C. A. Pope, R.T. Burnett, M. J. Thun, E. E. Calle, D. Krewski, K. Ito, G. D. Thurston, JAMA **287**, 9 (2002).
4. P. M. Manucci, Europ. Heart J. **34**, 17 (2013)
5. G. P. Bala, R. M. Rajnoveanu, E. Tudorache, R. Motisan, C. Oancea, Environ. Sci. Pollut. Res. Int. **28**, 16 (2021)
6. F. Benth, J. Saltyte-Benth, Appl. Math. Financ **12**, 1 (2005)
7. X. Dong, X. Zhao, F. Peng, D. Wang, Sci. Rep. **10**, 1 (2020)
8. S. Jha, C. Langmead, *Exploring behaviors of SDE models of biological systems using change of measures*, in IEEE International Conference on Computational Advances in Bio and Medical Sciences (2011)
9. S. Makhno, S. Melnik, J. Math. Sci. **231**, 1 (2018)
10. S. Nagendra, V, K. Shiva, S. L. Jones, Transp. Res. D. Transp. Environ. **12**, 3 (2007)
11. A. Petroeschevsky, R. W. Simpson, L. Thalib, S. Rutherford, Arch. Environ. Health **56**, 1 (2001)
12. J. Sun, N. Zhang, M. Wang, J. Wang, Stoch. Environ. Res. Risk. **34**, (2020)
13. K. F. Chung, J. Zhang, N. Zhong, Respirology **16**, 7 (2011)
14. F. J. Kelly, EMJ Respir. **2**, (2014)
15. S. He, S. Tang, W. Wang, Phys. A **532**, (2019)
16. A. Augustaitis, I. Augustaitiene, A. Kliucius, G. Pivoras, D. Sopauskiene, R. Girgzdiene, Eur. J. Forest Res. **129**, 3 (2010)
17. G. Touloumi, E. Samoli, A. Le-Tertre, R. Atkinson, K. Katsouyanni, Stat. Med. **25**, 24 (2010)
18. M. Deepa, M. Rajalakshmi, R. Nedunchezhian, Data-Enabled Discov. Appl. **1**, 1 (2017)
19. I. Cattadori, A. Pathak, M. Ferrari, Ecol. Evol. **9**, 23 (2019)
20. W. Ma, G. Lin, J. Liang, Ecol. Model. **419**, 1 (2020)
21. W. Wang, C. Ji, J. Bi, S. Liu, Appl. Math. Lett. **104**, (2020)
22. S. He, S. Tang, Y. Cai, W. Wang, L. Rong, Stoch. Environ. Res. Risk. **34**, (2020)
23. Y. Cai, Y. Kang, M. Banerjee, W. Wang, J. Differ. Equ. **259**, 12 (2015)
24. S. He, S. Banerjee, Phys. A **501**, (2018)

25. W. Zhao, J. Liu, M. Chi, F. Bian, Adv. Differ. Equ. **2019**, 1 (2019)
26. B. Yang, Y. Cai, K. Wang, W. Wang, Phys. A **526**, (2019)
27. F. Russo, P. Vallois, Probab. Th. Rel. Fields **97**, 3 (1993)
28. E. Alòs, O. Mazet, D. Nualart, Annals Prob. **29**, 2 (2001)
29. T. Duncan, B. Pasic-Duncan, Y. Hu, *Stochastic calculus for fractional Brownian motion I. Theory*, in Proceedings of the IEEE Conference on Decision and Control 38 (2), (2000)
30. S.J. Lin, SIAM Review **10**, (1995)
31. C. Bender, Stochastic Process. Appl. **104**, (2003)
32. B. Mandelbrot, J. W. van Ness, SIAM Review **10**, (1968)
33. S. C. Lim, J. Phys. A: Math. Gen. **34**, (2001)
34. F. Comte, E. Renault, Math. Finance **8**, 4 (1998)
35. T. H. Thao, Nonlinear Anal. **7**, (2006)
36. N. T. Dung, Vietnam J. Math. **36**, 3(2008)
37. N. T. Dung, Comput. Math. with Appl. **61**, 7 (2011)
38. N. T. Dung, J. Math. Anal. Appl. **397**, (2013)

Paper Stochastic Epidemic E3S Herry 2024

ORIGINALITY REPORT

19%	10%	14%	%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	Petrus Iwan, I. Gusti Ketut Puja. "Effect of curing temperature on the mechanical properties of coconut shell nano carbon reinforced composites with epoxy matrix", E3S Web of Conferences, 2024 Publication	3%
2	citeseerx.ist.psu.edu Internet Source	2%
3	Francesca Biagini, Yaozhong Hu, Bernt Øksendal, Tusheng Zhang. "Stochastic Calculus for Fractional Brownian Motion and Applications", Springer Science and Business Media LLC, 2008 Publication	2%
4	pubag.nal.usda.gov Internet Source	1%
5	www.aimspress.com Internet Source	1%
6	downloads.hindawi.com Internet Source	1%
7	Paulina Dwi Nawanti, Dionisius Brian Deva Erwandha, Budi Setyahandana, I.M.W. Ekaputra. "Investigation of Eichhornia crassipes as a natural fibre in PMC for noise controller", E3S Web of Conferences, 2024 Publication	1%
8	Kong, Lingcai, Jinfeng Wang, Weiguo Han, and Zhidong Cao. "Modeling Heterogeneity in	1%

Direct Infectious Disease Transmission in a Compartmental Model", International Journal of Environmental Research and Public Health, 2016.
Publication

9 R. Ruiz-Páez, J. Díaz, J.A. López-Bueno, C. Asensio, M.S. Ascaso, M. Saez, M.Y. Luna, M.A. Barceló, M.A. Navas, C. Linares. "Short-term effects of air pollution and noise on emergency hospital admissions in Madrid and economic assessment", Environmental Research, 2023
Publication
1%

10 Nguyen Tien Dung. "Semimartingale approximation of fractional Brownian motion and its applications", Computers & Mathematics with Applications, 2011
Publication
<1%

11 www.cambridge.org
Internet Source
<1%

12 Djoko Untoro Suwarno. "Using a stepper motor as a low-power, low-rotation DC generator for renewable energy harvesting", E3S Web of Conferences, 2024
Publication
<1%

13 Issoglio, E., and M. Riedle. "Cylindrical fractional Brownian motion in Banach spaces", Stochastic Processes and their Applications, 2014.
Publication
<1%

14 tailieu.vn
Internet Source
<1%

15 Goldberg, M.S.. "Counterpoint: Time-series studies of acute health events and environmental conditions are not confounded
<1%

by personal risk factors", Regulatory Toxicology and Pharmacology, 200807
Publication

16	Lisen Lin, Tianyu Li, Mengqi Sun, Qingqing Liang, Yuexiao Ma, Fenghong Wang, Junchao Duan, Zhiwei Sun. "Effect of particulate matter exposure on the prevalence of allergic rhinitis in children: A systematic review and meta-analysis", Chemosphere, 2020 Publication	<1 %
17	M. Erraoui, E. H. Essaky. "Chapter 13 Canonical Representation for Gaussian Processes", Springer Science and Business Media LLC, 2009 Publication	<1 %
18	McClendon-Peralta, Joyce. "Modeling of Air Pollution and Related Health Issues in Baton Rouge, Louisiana", Southern University and Agricultural and Mechanical College, 2024 Publication	<1 %
19	Ramziya Rifhat, Lei Wang, Zhidong Teng. "Dynamics for a class of stochastic SIS epidemic models with nonlinear incidence and periodic coefficients", Physica A: Statistical Mechanics and its Applications, 2017 Publication	<1 %
20	kipdf.com Internet Source	<1 %
21	www.hindawi.com Internet Source	<1 %
22	Hariharan, Dharmarajan. "Essays on estimation of non-linear state-space models", Proquest, 20111004 Publication	<1 %

23	Wang, Yong. "Integrated measurement technique to measure curing process-dependent mechanical and thermal properties of polymeric materials using fiber bragg grating sensors", Proquest, 20111108 Publication	<1 %
24	lmba.math.univ-brest.fr Internet Source	<1 %
25	sumitkumarjha.com Internet Source	<1 %
26	Qi Zhou, Huaimin Yuan, Qimin Zhang. "Dynamics and approximation of positive solution of the stochastic SIS model affected by air pollutants", Mathematical Biosciences and Engineering, 2022 Publication	<1 %
27	João Guerra, David Nualart. "Stochastic Differential Equations Driven by Fractional Brownian Motion and Standard Brownian Motion", Stochastic Analysis and Applications, 2008 Publication	<1 %
28	Čoupek, P., and B. Maslowski. "Stochastic evolution equations with Volterra noise", Stochastic Processes and their Applications, 2016. Publication	<1 %

Exclude quotes	On	Exclude matches	< 5 words
Exclude bibliography	On		