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# Developing LTBI for addition and multiplication

*by* Julie Hongki

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## Developing LTBI for addition and multiplication rules in probability theory with realistic mathematics education

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## <sup>1</sup>Developing LTBI for addition and multiplication rules in probability theory with realistic mathematics education

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**Abstract.** There were three components on a hypothetical learning trajectory (HLT) developed in this paper, namely: (1) the learning goal, (2) the learning activities, and (3) the way of students' thinking and learning. A learning trajectory based instruction (LTBI) was defined as a teaching and learning trajectory using HLT for instructional decisions. In this paper, the researcher will present a LTBI using Realistic Mathematics Education approach which helps mathematics education students following the Probabilistic Theory course to construct (1) the addition rule, and (2) the multiplication rule. The type of the research was the design research developed by Gravemeijer and Cobb. There were three phases in the research development, namely (1) the design preparation, (2) the design trial, and (3) the retrospective analysis. The researcher exposure in this paper was limited to the first stage of the design research developed by Gravemeijer and Cobb. As the products developed in this paper was limited to LTBI, so to construct a local instructional theory (LIT) for the sum and multiplication rules, the researcher needs to implement this LTBI in the classroom learning process.

### <sup>13</sup>Introduction

One of the mathematics courses that must be taken by students in Mathematics Education in the third semester is the probability course. The goal of this course is <sup>4</sup>help students to reinvent the basic probability concepts, and to apply these concepts to solve probabilistic problems. One of the importance of this subject was as it contained a provision for them to develop teaching and learning process about probability theory for senior high school students and/or vocational school students. From the lecture's experience of previous years, the most difficulties experienced by students were to perform a horizontal and vertical mathematizing process [1].

According to Sztajn et al. [2], Simon was the first researcher to use the term <sup>15</sup>hypothetical learning trajectory (HLT) to represent the student learning process [2]. Simon (1995) said three components on a HLT, namely: (1) the learning goal, (2) the learning activities, and (3) how students think and learn. Simon named a trajectory as a hypothetical learning trajectory because the student's learning trajectory was unknowable [3].

The learning and the teaching were often looked at as two sides of the same phenomenon, but often the studies carried out in these two areas were not connected to one another [2]. One attempt to link the research in both fields appeared in 2012 conducted by Sztajn <sup>1</sup>et al [2]. Their effort to combine study in both areas was to construct what they referred to as learning trajectory based instruction (LTBI). A LTBI was defined as a teaching and learning trajectory that HLT used for instructional decisions [2].



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The students' achievement taught by primary teachers were related directly and positively with mathematical and pedagogy abilities of their teachers [4,5]. There was a significantly connected between the knowledge achieved by the students and (1) the paradigm of teachers of mathematics teaching and learning process, and (2) teachers' care to the tendency of students' mathematics skills [4,5]. The mathematical knowledge of teachers related to the teachers' care of the tendency of students' mathematical skills [4,5]. The teacher mastery of the mathematics knowledge and the pedagogy would support the teachers' paradigm on the settlement of mathematical models and learning organization [4,5]. So, one of the determinants of student success in developing the probabilistic knowledge was the teacher's ability in managing the mathematics teaching and learning process and solving mathematical problems.

There were two types of understanding that students had in the process of learning mathematics, namely: (1) the instrumental understanding and (2) the relational understanding. The instrumental understanding means knowing how to use a rule or know how to use a formula to solve a problem, without understanding how the formula is derived, and why it can be used to solve the problem. Understanding in the relational understanding means knowing about (1) the relationship between concepts in mathematics (2) how to use a rule, (3) how to use a formula to solve a problem, (4) how the formula is derived, and (5) how to use a formula why the formula can be used to solve the problem [6].

From several studies reported that in the 21st century that humans not only required a content knowledge, but they also required skills that called as 21st-century skills. The 21st skills include the critical thinking and problem solving, creativity and innovation, communication and collaboration, flexibility and adaptability, initiative and self-direction, social and cross-cultural, productivity and accountability, leadership and responsibility, and information literacy [7,8]. In this LTBI, the developing of the critical thinking and problem solving were to become the focus of the researcher.

The philosophy of RME was mathematics as a human activity. It meant learning mathematics should be able to make the students thought that there was mathematics in human activities, and mathematics was used by them in real life [9,10,11]. There were five main characteristics in the RME [9,10,11], namely: (1) phenomenological exploration, (2) bridging by vertical instruments, (3) student contributions, (4) interactivity, and (5) intertwining.

According to Gravemeijer and Cobb [12], design research can be characterized as:

- Interventionist: the research leading to the design of an intervention in the real world.
- Iterative: the research incorporates a cyclic approach to the design, evaluation, and revision.
- Process-oriented: a model of research that avoids the measurement of inputs and outputs, focus on understanding and improving interventions.
- Oriented to usability: the benefits of design were measured by looking at the practicality of the design for the user in reality.
- Oriented to the theory: design (at least partially) made by theories that already exist, and field testing of the design contribute to the development of the theory.

According to Gravemeijer and Cobb [12], there were three phases in the design research, namely: (1) preparation of trial design, (2) trial design, and (3) a retrospective analysis [1]. In this paper, researchers will only discuss the results of research in stage 1.

The research question that would be answered by the researcher in this paper, namely: how to develop LTBI that could be used to build student knowledge about addition and multiplication rule using the RME approach?

## 8 Method

This research was classified as the design research. In this study, the researcher developed a LTBI to be used to build student knowledge concerning the addition and multiplication rule using RME approach. The research will only discuss the results at stage 1.

The subjects of this research were 38 students (7 male and 31 female) taking the Introduction Probability Theory course in one of the class at Sanata Dharma University. The research instrument used to obtain data about the students' reflection examined using the open questionnaire administered to students once the course completed. A teacher's journal was used to get data concerning teacher's reflection.

The study consisted of three cycles. Each cycle consisted of five main elements, namely context, experience, action, reflection, and evaluation. The learning process in the first cycle discussed about the counting principle, permutation and combination, experiment, sample space and events as well as the notion of the probability of the event and the definition of the axiomatic probability, the probability properties, independent and conditional events, and the conditional probability. The learning process in the second cycle discussed about random variables. The learning process in the third cycle concerned about the binomial, the Poisson, and the normal distribution. This paper is limited only at a LTBI that will help students to construct knowledge concerning addition and multiplication rules in the first cycle.

## Result and discussion

In this paper, the researcher will develop a LTBI that will help students to construct knowledge about addition and multiplication rules. The learning goals of this LTBI were (1) students could reinvent the addition rule, and (2) students could reinvent the multiplication rule. The teaching and learning trajectory which content the learning activities and the way of students thinking and learning in the research's LTBI as followed:

- Students are required to solve the first problem in a way that they understand as follows: there are four green balls, and five red balls in a bag. From inside the bag, two balls will be taken at once. Please, determine how many possible outcomes from the ball-taking experiment!

Problem 1 is intended to provide an opportunity for students to explore a phenomenon that will encourage students to construct knowledge of the rules of addition and multiplication. This activity is planned by the researcher to bring up a phenomenological exploration characteristic.

### The first possibility of a student answer for the problem 1 is:

Suppose, the green balls in the bag are  $H_1, H_2, H_3, H_4$ , and the red balls in the bag are  $M_1, M_2, M_3, M_4, M_5$ ; then

- Students will make the following list to state the number of events for two green balls as follows:  $H_1H_2, H_1H_3, H_1H_4, H_2H_3, H_2H_4, H_3H_4$ . So, the number of events for two green balls is six.
- Students make the following list to state the number of events of one green ball and one red ball as follows:  
 $H_1M_1, H_1M_2, H_1M_3, H_1M_4, H_1M_5, H_2M_1, H_2M_2, H_2M_3, H_2M_4, H_2M_5, H_3M_1, H_3M_2, H_3M_3, H_3M_4, H_3M_5, H_4M_1, H_4M_2, H_4M_3, H_4M_4, H_4M_5$ . So, the number of events for one green ball and one red ball is 20.
- Students make the following list to state the number of events for two red balls:  
 $M_1M_2, M_1M_3, M_1M_4, M_1M_5, M_2M_3, M_2M_4, M_2M_5, M_3M_4, M_3M_5, M_4M_5$ . So, the number of events for two red balls is 10.

Thus, the number of events in this experiment is  $6 + 20 + 10 = 36$ .

### The second possibility of a student answer for the problem 1 is:

Suppose, the green balls in the bag are  $H_1, H_2, H_3, H_4$ , and the red balls in the bag are  $M_1, M_2, M_3, M_4, M_5$ ; then

- The number of events for two green balls can be described with the diagram as follows (look at figure 1):

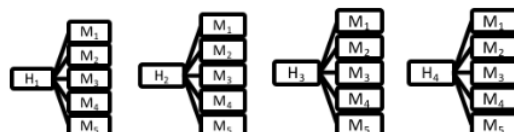




**Figure 1.** The possibility for two green balls.

So, the number of events for two green balls is  $3 + 2 + 1 = 6$

- The number of events for one green ball and one red ball can be drawn with the diagram as follows (look at figure 2):



**Figure 2.** The possibility for one green ball and one red ball.

So, the number of events for one green ball and one red ball is  $(1 \times 5) + (1 \times 5) + (1 \times 5) + (1 \times 5) = 4 \times (1 \times 5) = 4 \times 5 = 20$ .

- Figure 3 describes the number of events for two red balls.



**Figure 3.** The possibility for two red balls.

So, the number of events for two green balls is  $4 + 3 + 2 + 1 = 10$ .

So, the number of possibilities in this experiment is  $6 + 20 + 10 = 36$ .

**The third possibility of a student answer for the problem 1 is:**

Suppose, the green balls in the bag are  $H_1, H_2, H_3, H_4$ , and the red balls in the bag are  $M_1, M_2, M_3, M_4, M_5$ ; then

- Students will create the following chart to record the number of events for two green balls (look at table 1):

**Table 1.** The possibility for two green balls.

	$H_1$	$H_2$	$H_3$	$H_4$
$H_1$	-	$H_1 H_2$	$H_1 H_3$	$H_1 H_4$
$H_2$	-	-	$H_2 H_3$	$H_2 H_4$
$H_3$	-	-	-	$H_3 H_4$
$H_4$	-	-	-	-

So, the number of events for two green balls is  $1 + 2 + 3 = 6$ .

- Students will create the following table to record the number of events for one green ball and one red ball (look at table 2):

**Table 2.** The possibility for one green ball and one red ball.

	$M_1$	$M_2$	$M_3$	$M_4$	$M_5$
$H_1$	$H_1 M_1$	$H_1 M_2$	$H_1 M_3$	$H_1 M_4$	$H_1 M_5$
$H_2$	$H_2 M_1$	$H_2 M_2$	$H_2 M_3$	$H_2 M_4$	$H_2 M_5$
$H_3$	$H_3 M_1$	$H_3 M_2$	$H_3 M_3$	$H_3 M_4$	$H_3 M_5$
$H_4$	$H_4 M_1$	$H_4 M_2$	$H_4 M_3$	$H_4 M_4$	$H_4 M_5$

So, the number of events for one green ball and one red ball is  $4 \times 5 = 20$ .

- Students will create the following table to record the number of events for two red balls (look at table 3):

**Table 3.** The possibility for two red balls.

	$M_1$	$M_2$	$M_3$	$M_4$	$M_5$
$M_1$	-	$M_1M_2$	$M_1M_3$	$M_1M_4$	$M_1M_5$
$M_2$	-	-	$M_2M_3$	$M_2M_4$	$M_2M_5$
$M_3$	-	-	-	$M_3M_4$	$M_3M_5$
$M_4$	-	-	-	-	$M_4M_5$
$M_5$	-	-	-	-	-

So, the number of events for two red balls is  $4 + 3 + 2 + 1 = 10$ .

So, the number of possibilities in this experiment is  $6 + 20 + 10 = 36$ .

The researcher provides the opportunity for students to solve the problem individually so that researchers can bring up a student contribution characteristic in the LTBI design.

- Once the student completed this problem, they are asked to discuss their solution with a friend's seat. The researcher gave the opportunity for students to present their solution to their colleague so that the researcher brings up an interactivity characteristic in the LTBI design.
- The lecturer asks the student who answers like the first, second, and third possibility for the problem 1 to write his/her answer on the board. This step is designed so that the researcher can generate a bridging by vertical instrument characteristics in the LTBI design.
- Other students are asked to observe and criticize their friends' answers. The researcher can create a student contribution characteristic in the LTBI design with this activity.
- Students are required to solve the second problem in a way that they understand individually: in SMP Maju Jaya, there is an election of OSIS management. There are four candidates. From the four people selected three people to occupy the position of chairman, secretary, and treasurer. The rule of the election is a person should not obtain multiple positions. How many forms of organizational structure can be made?

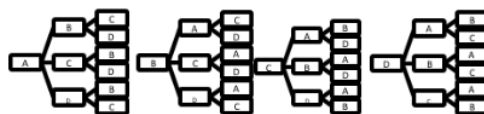
The problem 2 is intended to provide an opportunity for students to explore a phenomenon that will encourage students to construct knowledge of the rules of multiplication. This activity is planned by researchers to create a phenomenological exploration characteristic.

#### The first possibility of a student answer for the second problem is:

Suppose the 4 candidate are A, B, C, and D. Students write the following list as possible of OSIS stewardship arrangement: ABC, ABD, ACB, ACD, ADB, ADC, BAC, BAD, BCA, BCD, BDA, BDC, CAB, CAD, CBA, CBD, CDA, CDB, DAB, DAC, DBA, DBC, DCA, and DCB. Thus, there are 24 possible arrangements of OSIS management in Maju Jaya Junior High School.

#### The second possibility of a student answer for the second problem is:

Suppose the four candidates are A, B, C, and D. Students describe the following diagram to illustrate the possibility of OSIS stewardship (look at figure 4):



**Figure 4.** The possibility of OSIS stewardship.

Thus, the number of possible arrangements of OSIS management in SMP Maju Jaya is  $(3 \times 2) + (3 \times 2) + (3 \times 2) + (3 \times 2) = 4 \times (3 \times 2) = 4 \times 3 \times 2 = 24$ .



**The third possibility of a student answer for the second problem is:**

Suppose the four candidates are A, B, C, and D. Students describe the following table to illustrate the possibility of OSIS stewardship (refertoTable 4):

**Table 4.** The possibility of OSIS stewardship.

Chairman	Secretary	Treasurer
Four possibility	Three possibility	Two possibility

So, the possibilities of OSIS stewardship in Maju Jaya Junior High School are  $4 \times 3 \times 2 = 24$ . The researcher provides the opportunity for students to solve the problem individually so that researchers can bring up a student contribution characteristic in the LTBI design.

- Once the student completed this problem, they are asked to discuss their solution with a friend's seat. The researcher gave the opportunity for students to present their solution to their colleague so that the researcher brings up an interactivity characteristic in the LTBI design.
- The lecturer asks the student who answers like the first, second, and third possibility for the problem 2 to write his/her answer on the board. This step is designed so that the researcher can generate a bridging by vertical instrument characteristics in the LTBI design.
- Other students are asked to observe and criticize their friends' answers. The researcher can create a student contribution characteristic in the LTBI design with this activity.
- Students are invited to discuss about (1) the connecting between the concepts that have been built by them when studying set theory and number theory and problem 1 and 2 and (2) concluding about the addition and multiplication rules. The expected conclusions are as follows:
  - The principle of addition: if the object  $A_1$  can be selected by  $n_1$  ways, the object  $A_2$  can be selected by  $n_2$  ways, until to object  $A_k$  can be selected by  $n_k$  ways, then the number of ways to choose objects  $A_1, A_2, \dots$ , or  $A_k$  is  $n_1 + n_2 + \dots + n_k$ .
  - The Multiplication Principle: if a process can be formed from  $n_1$  different ways followed by the next process inside  $n_2$  different ways, and followed by the next process inside  $n_3$  different ways, up to the k-th procedure formed from  $n_k$  different ways, then the number of ways to form the procedure is  $n_1 \times n_2 \times \dots \times n_k$ .
- The lecturer gives the exercise to be done by the students to strengthen the student' understanding about the rule of addition and multiplication. Here are two practical exercises that students can use to reinforce their understanding of the set and multiplication rules:
  - Two judges for the mathematics Olympiad were chosen from four people. Three judges for the Olympiad physics contest were selected from six people. "How many ways can you choose to choose a jury of a mathematics Olympiad or a physics Olympiad jury?" and "How many ways can you choose to choose a jury of a mathematics Olympiad and a physics Olympiad jury?"
  - The teacher will form a learning group consisting of four students from eight male students and four female students. How many study groups may be created by the teacher if in each group there are at least two male students?
- Lecturers hold class discussions to discuss student completion outcomes.

#### 4. Conclusion

Three conclusions can be drawn from the above explanation, namely:

- To develop a LTBI to help students constructing the addition and multiplication rules using the RME approach, the researcher needs to conduct the several steps. First, providing the phenomenon to be explored by the student. Second, providing the opportunity for students to discuss with their friends. Third, asking the students to present the answers according to the

formalization degree of the answers. Fourth, inviting students in class discussions to connect new material with the concepts they have learned and drawn conclusions.

- The contexts of ball-taking experiments and OSIS selection are used by researchers to help students construct their knowledge about the addition and multiplication rules.
- The RME approach can be used to assist students in constructing knowledge about the rules of addition and multiplication.

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