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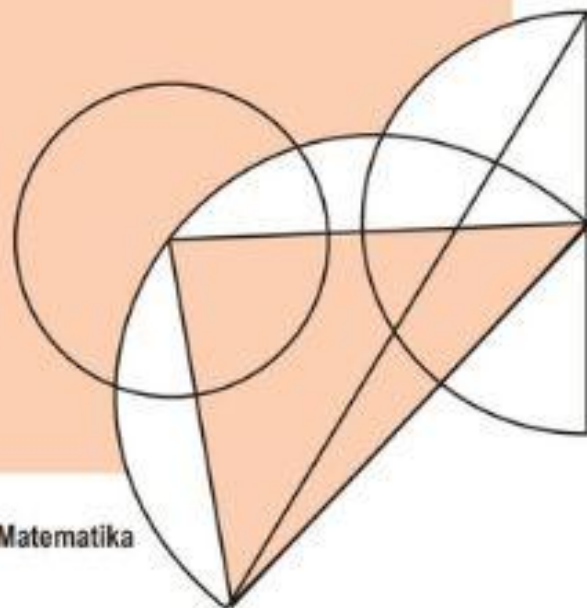
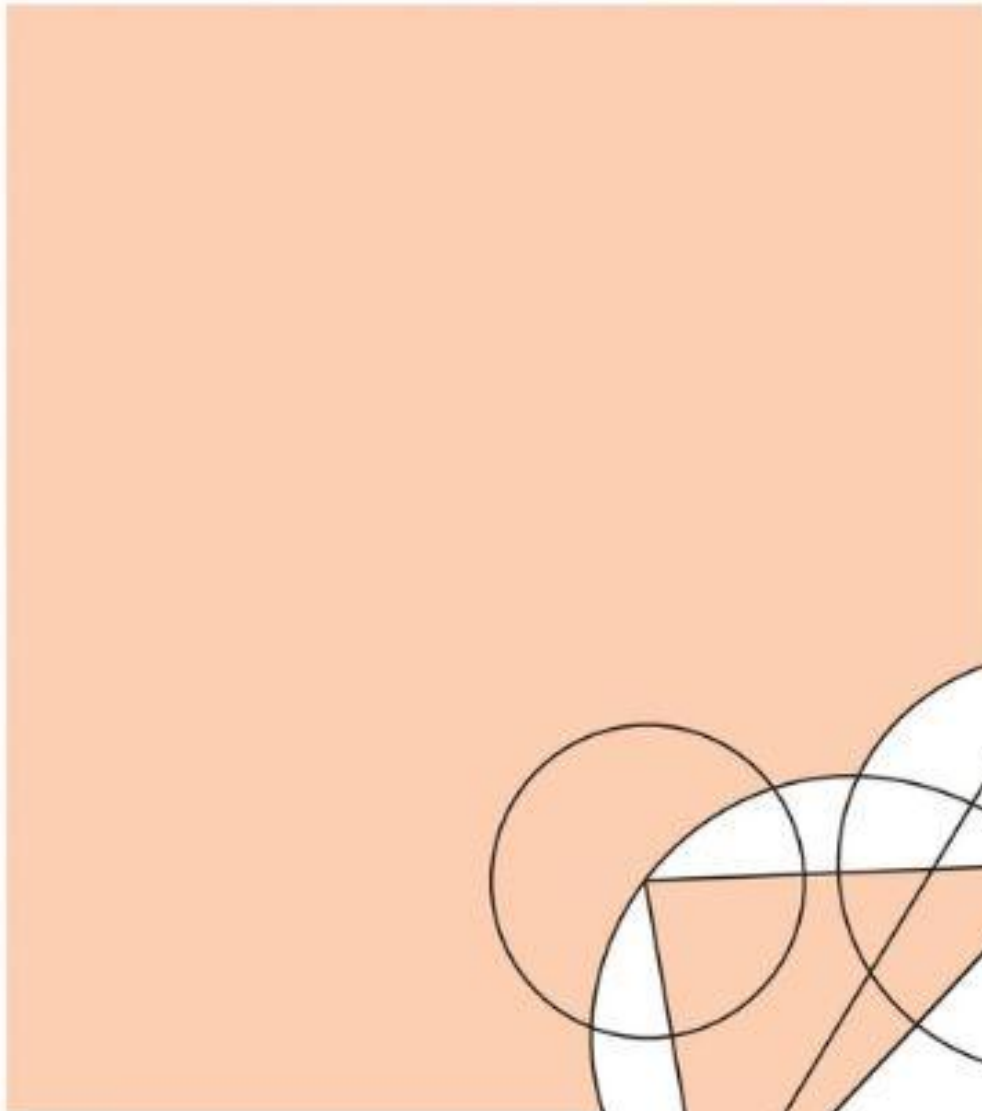
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Guided Discovery Learning to Improve Students' Mathematics Representation and Collaboration at Senior High School 6 Yogyakarta

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Abstract. Classroom Action Research aimed to assess the mathematical representation ability and collaboration in understanding circle material by implementing the guided discovery learning model. The data collection took place in April 2023 at SHS 6 Yogyakarta, involving 35 students from XI science. The research comprised two cycles, with a diagnostic test conducted in the first cycle focusing on the equation of a circle, particularly the position of points and lines related to the circle. In the second cycle, the diagnostic test concentrated on tangents to the circle. Data were gathered through tests, interviews, and questionnaires. The findings indicated that guided discovery learning effectively enhances both mathematical representation ability and collaboration. The average scores for the pre-cycle test were 78.87, for cycle I were 79.17, and for cycle II were 85. Moreover, the percentage of students achieving a score of 75 or higher increased from 60% in the pre-cycle to 71% in cycle I and further rose to 77% in cycle II. Regarding collaboration skills, in cycle I, 23 students were categorized as having medium-level skills, and 12 were classified as having high-level skills. In cycle II, the distribution changed with 16 students falling into the very high category, 12 in the high category, and seven in the medium category.

Keywords: Guided Discovery Learning, Mathematical Representation, Collaboration Ability

INTRODUCTION

Mathematics is a field of knowledge that can be used to improve the ability to think logically, critically, and rationally. It can be achieved but requires understanding and competence in mathematics (Sinaga, 2016). One of the goals of mathematics learning at all levels of education is to improve students' mathematical abilities. Students can better understand and apply the concepts they have learned in various contexts if this ability is developed. They can use it in various situations.

Representation is a model or form that replaces a problem to help find a solution to the problem, usually shown using images, words, or mathematical symbols (Jones & Knuth in Sabirin, 2014). According to NCTM (2000), the first

standard of representational ability is organizing, recording, and communicating mathematical ideas by creating and using representations. The second standard is selecting, using, and translating between representations to solve problems. Meanwhile, the third standard uses representations to model and interpret mathematical, social, and physical phenomena. According to the NCTM process standards (2000), students must have five mathematical abilities that they must master when participating in mathematics learning, namely: 1) problem-solving, 2) reasoning and proof, 3) mathematical communication, 4) mathematical connection, and 5) mathematical representation. One of the five most critical mathematical abilities mastered by students is mathematical representation abilities (Ramziah, 2016; Yenni & Sukmawati, 2020). Considering that these five abilities are interconnected and intertwined during the learning period, these five abilities cannot be separated from mathematics learning. In addition, mathematical representation emphasizes connecting and expressing mathematical concepts through symbols, tables, charts, and graphs. Students must understand this as a means of expressing ideas.

Villegas, Castro, and Gutierrez (2009) stated that indicators of verbal, pictorial, and symbolic mathematical representation abilities can all be categorized. Villegas provides the following explanation for the three types of representation in his research: 1) Verbal representation, which can be in the form of an oral or written statement that describes the problem; 2) pictorial representations, such as diagrams, tables, graphs, or other types of representation; 3) Symbolic representation, which is in the form of a numerical model or conditions created by numerical images. Conditions created by these numerical images. Villegas also stated that verbal, pictorial, and symbolic representations were closely related.

The inadequacy in mathematical representation proficiency is evident in the prevalent practice where numerous educators predominantly emphasize textbook instruction. The instructional methodologies employed by teachers still exhibit a degree of conventionalism, characterized by the routine presentation of material, followed by assigning students practice questions devoid of mathematical representations. This deficiency is substantiated by the findings of a study

conducted by Mulyaningsih, Marlina, and Effendi (2020), revealing a tendency among students to approach questions with a lack of attentiveness and precision in execution. Additionally, students exhibited limited proficiency in problem-solving through graphical representations, hindering their comprehension of presented information. Consequently, an imperative arises for implementing instructional practices incorporating elevated cognitive challenges, encompassing sophisticated questioning techniques and increased integration of mathematical representation indicators within the learning process.

Based on the results of students' daily tests and observations made in class Apart from that, when writing mathematical symbols, sometimes students write them incompletely, so it can be concluded that students' symbol representation abilities still need to be improved. Apart from representational abilities, based on interviews with mathematics teachers and observations of mathematics learning, information was obtained that students only group with close friends and tend not to want to change groups. It shows that students' collaboration skills are still lacking. Therefore, there needs to be a learning breakthrough that can improve students' representation and collaboration abilities.

One alternative to improve students' representation and collaboration abilities is to use the guided discovery learning model. Guided discovery learning is scientific learning which aims for students to solve problems in groups using the steps of simulation, problem statement/identification, data collection, data processing, verification, and conclusion (Simamora, Saragih, Hasratuddin, 2019; Shieh & Yu, 2016; Yang et al. al., 2010; Yerizon et al., 2018; Alfieri et al., 2011). Therefore, researchers are interested in conducting classroom action research using the guided discovery learning model to improve students' mathematical representation and collaboration abilities.

RESEARCH METHODS

This research used the type of Classroom Action Research. In classroom action research based on the model of Stephen Kemmis and Robyn McTaggart, there were four stages: planning, action, observation, and reflection. At the planning

stage, researchers designed learning by applying guided discovery learning to circular material.

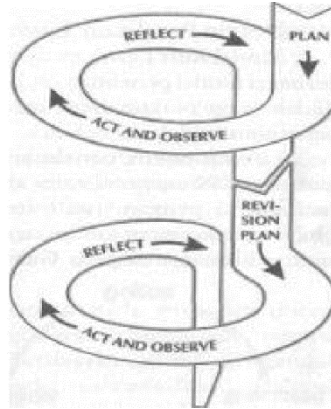


Figure 1. PTK cycle, according to Kemmis and Taggart

At this stage, the researcher created a research instrument in the form of a mathematical representation ability test and also created a questionnaire to test students' collaborative abilities. In the action stage, the researcher directed students in learning by applying guided discovery learning to circle material. At the observation stage, researchers observed and recorded students' activities during learning, and at the end of the material, students were given a questionnaire to determine students' collaboration abilities. Then, in the final stage, namely reflection, the researcher carried out data analysis and evaluated the success of implementing guided discovery learning on circular material. This research was conducted in April 2023 at SHS 6 Yogyakarta. The subjects used were students in class XI science, with 35 students.

Data collection techniques were as follows:

1. Pre-cycle I: diagnostic test topic equations in circles
2. Cycle I: mathematics representation test and questionnaires of student collaboration
3. Cycle II: mathematics representation test and questionnaires of student collaboration.

Data analysis techniques were as follows:

1. Questionnaire of Student Collaboration

The collaboration questionnaire created by researchers was scaled from 1 to 5 with scale information: 1 = very inappropriate; 2 = not appropriate; 3 = not suitable; 4 = appropriate; 5 = very appropriate. The criteria for students' collaboration abilities were divided into five categories: deficient, low, medium, high, and very high.

Criteria	Information
$0\% < k \leq 20\%$	Deficient
$20\% < k \leq 40\%$	Low
$40\% < k \leq 60\%$	Medium
$60\% < k \leq 80\%$	High
$80\% < k \leq 100\%$	Very high

Table 1 . Criteria of student collaboration

2. Mathematics representation test

The results of the mathematical representation test were divided into three assessment aspects: symbol representation, visual representation, and verbal representation. The Minimum Completion Criteria (MCC) at school in mathematics was 75.

RESULTS AND DISCUSSION

The researcher performed a diagnostic test in the pre-cycle stage using circle equation material and interviews. The results of the mathematical representation diagnostic test were divided into three assessment aspects: symbol representation, visual representation, and verbal representation. The results of the mathematical representation ability test in the pre-cycle or initial diagnostic test are presented below.

Aspect	The highest score	Lowest Value	Average
Symbol Representation	75	25	64.11
Visual Representation	100	25	73.21
Verbal Representation	100	0	12.86

Table 2. Pre-Cycle Learning Results in Every Aspect of Mathematical Representation

Based on Table 1, the highest and lowest scores from the diagnostic test results in the symbol representation aspect were 75 and 25, averaging 64.11. In the visual representation aspect, the highest and lowest scores were 100 and 25, with an average of 73.21. Meanwhile, the highest and lowest values for the verbal representation aspect were 100 and 0, with an average of 12.86. If we looked at the students' grades and completion based on the Minimum Completion Criteria (MCC) at school, namely 75, several students had not yet completed the minimum criteria. The following data on student completion is presented in Table 2.

	Score	Frequency	Percentage
MCC not completed	< 75	14	40%
Complete MCC	≥ 75	21	60%
The highest score	94.6		
Lowest Value	29.7		
Average	78.87		

Table 3. Distribution of Final Diagnostic Test Scores

Based on Table 2, more students were in the complete category than students in the incomplete category. Of the 35 students, 14 (40%) were in the incomplete category, and 21 (60%) were in the complete category. The highest and lowest scores from the final cycle I test were 94.6 and 29.7, averaging 78.87. From this table, many students still had not completed the MCC. Therefore, researchers carry out learning using the *guided discovery learning model*.

Based on the results of interviews with students, students' collaboration abilities could be divided into groups, independently, and both. The interview results found that the characteristics of group colleagues that students liked were working together well, having flexible time, accepting other people's opinions, and being diligent. In resolving differences of opinion in groups, students conducted discussions and resolved problems satisfactorily by considering the opinions expressed. To build cooperation and good relationships, students applied an attitude of mutual respect, mutual assistance, and responsibility.

A. Cycle I

Researchers and teachers discussed preparation. The Lesson Plan used the

discovery learning model, learning materials, student worksheets, learning assessments, and mathematical representation ability test questions, which were tested in cycle I. In cycle I, the teaching material was the position of points and lines relative to the circle.

In cycle I, researchers divided students into nine groups, each with four students. The researcher will divide these nine groups into Category One and two categories. The group with category one will identify the position of the point on the circle based on the picture. Meanwhile, the group in category two will identify the position of the point on the circle based on their similarities. Below is a presentation of the student worksheet researchers used in cycle 1 in Figures 1 and 2. The students' worksheet was designed using the Desmos application.

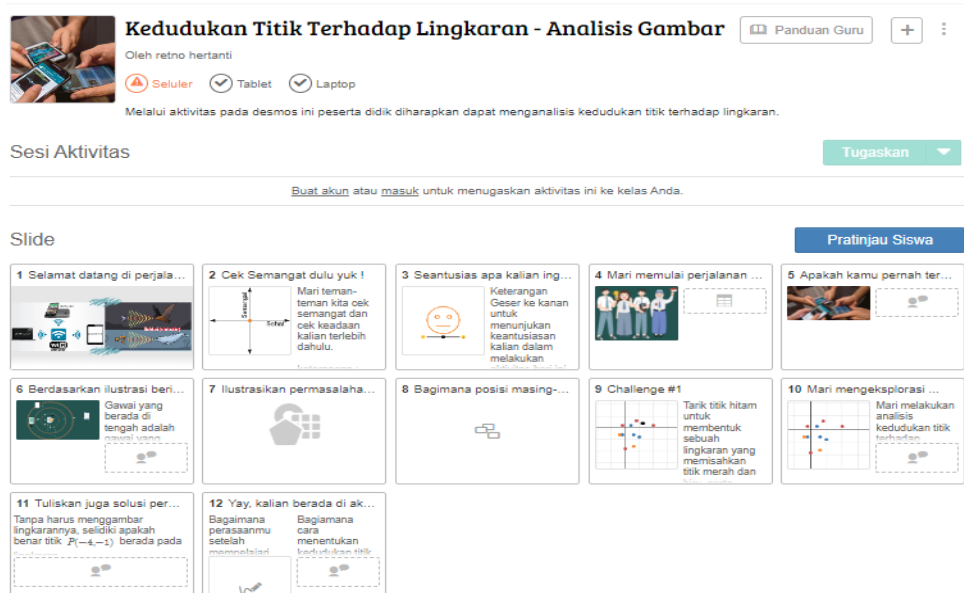


Figure 2. Students' worksheet cycle 1

At the end of cycle I, the teacher gave a representation ability test which was carried out individually. This test was conducted to determine the extent of students' mathematical representation abilities. Apart from that, researchers also provided a questionnaire. This questionnaire was used to assess students' collaboration abilities during group learning.

Researchers conducted observations to assess the learning process and student learning outcomes. This observation of the learning process was based on cycle I

test scores and a collaboration questionnaire created by the researcher. Based on the results of observations made while working in groups, many students needed time to adapt. Students worked by dividing each slide on Desmos. However, because this work was done in groups, the teacher finally reminded them to discuss completing the student's worksheet.

Mathematical representation ability

The results of the first cycle of mathematical representation tests based on assessment aspects will be divided into three assessment aspects: mathematical representation abilities based on symbols, visuals, and verbal.

Aspect	The highest score	Lowest Value	Average
Symbol Representation	100	25	78.29
Visual Representation	100	11	79
Verbal Representation	100	50	76.79

Table 4 . Cycle I Learning Results on Every Aspect of Mathematical Representation

Table 3 shows an increase in results compared to Table 1, especially in symbolic and verbal representation. In symbolic representation, initially, the student's highest score was 75, but at the end of cycle 1, some got a score of 100. In the verbal representation aspect, initially, the lowest score was 0, then increased to 50, meaning that all students had begun to develop in the verbal representation aspect. If we look at the average obtained, all aspects have increased. The following also presents the completeness of students' grades.

	Score	Frequency	Percentage
MCC not completed	< 75	10	29%
Complete MCC	≥ 75	25	71%
The highest score	100		
Lowest Value	40		
Average	79.17		

Table 5. Distribution of Final Scores for Cycle I Tests

Based on Table 4, it can be seen that students' completeness scores in cycle one increased when compared to diagnostic tests. Initially, students' completeness

was 60%, which increased to 71%. Apart from that, the average student score also increased from 78.87 to 79.17. In this way, discovery learning can help students improve their representation abilities.

Collaboration capabilities

After carrying out the first cycle test, students were given a questionnaire to measure their collaboration abilities during the first cycle of learning. The indicators in the student collaboration questionnaire were active collaboration, working productively, showing flexibility, showing an attitude of responsibility, and showing respect. Based on the collaboration questionnaire that students filled out, the results of students' collaboration abilities are presented below in Table 5.

Criteria	Information	students
$0\% < k \leq 20\%$	Very low	0
$20\% < k \leq 40\%$	Low	0
$40\% < k \leq 60\%$	Medium	23
$60\% < k \leq 80\%$	High	12
$80\% < k \leq 100\%$	Very high	0

Table 6. Results of the Student Collaboration Ability Questionnaire

Upon examination of Table 5, it is evident that most students exhibit moderate levels of collaboration abilities, while a subset demonstrates high levels of proficiency in collaboration. Notably, there is an absence of students categorized as having very low, low, or very high collaboration abilities. The ensuing data presents an overview of the accomplishment of indicators about students' collaboration abilities.

No	Questionnaire Indicator	Percentage
1	Active collaboration	84.31%
2	Work productively	84.17%
3	Demonstrate flexibility	83.33%
4	Demonstrate an attitude of responsibility	62.78%
5	Show respect	86.11%

Table 7. Results of the Student Collaboration Ability Questionnaire

Table 6 presents a comprehensive overview of collaboration indicators,

delineating percentages for each criterion. Active collaboration was notably high, registering at 84.32%, followed closely by working productively with a percentage of 84.17%. Demonstrating flexibility exhibited a percentage of 83.33%, while displaying a responsible attitude recorded a percentage of 62.78%. Conversely, the indicator for exhibiting an attitude of respect attained a percentage of 86.11%. Evaluating these findings against predetermined conclusion criteria, it is evident that active collaboration, working productively, demonstrating flexibility, and exhibiting respect all meet the criteria for proficiency. It implies that students can effectively engage in active group collaboration, work productively, display flexibility in group dynamics, and manifest appreciation for their peers during collaborative endeavours. Contrarily, the indicator about the manifestation of a responsible attitude exhibits the lowest percentage among the assessed indicators. It suggests that students' demonstration of responsibility requires further enhancement and improvement.

In cycle I, mathematical abilities increased compared to pre-cycle, but mathematical abilities in the verbal aspect were lower than in the other two aspects. Meanwhile, for students' collaboration abilities, the indicator showing an attitude of responsibility is also lower than other collaboration ability indicators. So, based on this reflection, the researcher retook action by conducting research in cycle II.

B. Cycle II

Based on the reflection results in cycle I, researchers and teachers discussed again to design various things needed and preparations for learning cycle II. Things that are needed and must be prepared include a lesson plan different from Cycle I with continuing material, students' worksheets, learning assessments, and test or post-test questions that will be tested in Cycle II. The material used in cycle II is the tangent line to a circle whose tangent point is known and the tangent line to the circle whose gradient is known.

At the action stage, researchers carry out learning using the lesson plan that has been designed. Cycle II was carried out with three learning meetings, namely two learning meetings and one cycle 2 test meeting. The cycle 2 test meeting was

divided into ten groups, with students themselves choosing groups, each group consisting of 2 or 4 people.



Figure 3. Students' worksheet Cycle II

At the end of cycle II, the teacher gives the final test of cycle II, which is carried out individually to measure students' mathematical representation abilities, which have improved from before during cycle I. After giving the final test of cycle II, the teacher also gives a questionnaire that students fill out to assess the participants' collaboration abilities during group learning.

The observation stage is carried out to find out whether the learning process that has been carried out is by the previous one or not, and observations are carried out to assess the teaching and learning process and student learning outcomes. Learning begins with the teacher providing an overview of the learning that will be carried out using a Desmos-based student worksheet in groups. The students choose the group members, each consisting of 2 or 4 students. At the beginning of the lesson, the teacher displays several pictures of the position of the line to the circle and then asks the students to show which one is the tangent line to the circle, along with the reasons. The teacher also reviews class 8 material so that later, the students can find the formula for a tangent line to a circle that passes through a point, namely material about how to find the gradient of a straight line followed by students in groups working on Desmos-based students worksheet. At the third meeting of cycle II, the teacher carried out the cycle II test, and the results showed an increase in the indicators for each mathematical representation (symbols, verbal, and images).

Mathematical Representation Ability

The results of the cycle II test are a continuation of the actions that started from the cycle I reflection. The following presents students' representation abilities based on the three aspects assessed.

Aspect	The highest score	Lowest Value	Average
Symbol Representation	100	25	88
Visual Representation	100	0	82
Verbal Representation	100	0	79

Table 8. Cycle II Learning Results for Each Mathematical Representation

Based on Table 7, It can be seen that students' abilities in the aspects of symbolic, visual, and verbal representation have increased if seen based on the average value. Symbol representation ability increased from 78.29 to 88. Visual representation ability increased from 79 to 82. Verbal representation ability increased from 76.79 to 79. If seen based on completeness scores, the following is presented as a percentage of students' completeness scores in Table 8.

No	Score	Frequency	Percentage
KKM not completed	< 75	8	23%
Complete KKM	≥ 75	27	77%
The highest score	100		
Lowest Value	23		
Average	85		

Table 9 . Distribution of Final Scores for Cycle II Tests

Table 8 shows an increase in student learning completeness. In cycle one, student learning completeness was 71%, increasing in cycle 2 to 77%. The average student score also increased from 79.17 to 85. It shows that students' representation abilities are improving from pre-cycle, cycle 1, and cycle 2.

Collaboration Capabilities

Researchers distributed questionnaires that were used to measure students' collaboration abilities. Questionnaires were distributed after carrying out the second cycle test. The following are the results of the student collaboration questionnaire.

Criteria	Information	Many students
$0\% < k \leq 20\%$	Very low	0
$20\% < k \leq 40\%$	Low	0
$40\% < k \leq 60\%$	Currently	7
$60\% < k \leq 80\%$	Tall	12
$80\% < k \leq 100\%$	Very high	16

Table 10. Results of the Collaboration Ability Questionnaire for Cycle II Students

From Table 9, it can be seen that there is an increase in students' collaboration abilities from cycle I to cycle II. In cycle I, no students reached the very high category; in cycle II, 16 students reached the very high category. In addition, the collaboration criteria are being reduced from 23 to 7 students. It shows that students' collaboration abilities increase after learning using the *discovery learning model*. Data on students' collaboration abilities based on each indicator is presented below.

No	Questionnaire Indicator	Percentage
1	Active collaboration	83.71%
2	Work productively	74.00%
3	Demonstrate flexibility	71.43%
4	Demonstrate an attitude of responsibility	60.00%
5	Show respect	75.43%

Table 11. Results of the Collaboration Ability Questionnaire for Cycle II Students

Table 10 delineates that there has been no discernible enhancement in students' collaborative proficiency across various facets when juxtaposed with the outcomes from cycle I. Additionally, scrutinizing the five collaboration ability indicators reveals that the fourth criterion, specifically indicative of a responsible attitude, persists as the least developed compared to the remaining indicators.

The results of cycle II show a significant increase in each indicator of mathematical representation. In cycles I and II, verbal representation had the lowest average compared to symbolic and visual representation. Symbolic, verbal, and visual mathematical representation indicators have a high average related to collaboration ability. The indicator of active collaboration ability has the highest average, while the lowest is the ability to demonstrate an attitude of responsibility. So, based on this reflection, the researcher only needed to take action until cycle II.

In the pre-cycle, there were 21 students (60%) who completed it; in the first cycle, there were 25 students (71%) who completed it, while in the second cycle, there were 27 students (77%) who completed it. It means an increase in mathematical representation ability of around 17% from the initial condition. It is

in line with what was stated by Kurniasih and Sani (2014) that *guided discovery learning* can help improve students' mathematical representation abilities and strengthen memory because the knowledge obtained through independent discovery is student-centred. Students better understand basic concepts and students' ideas well. Thus, *guided Discovery learning* influences students' mathematical representation abilities. Research by Annajmi and Afri (2019) also shows that guided discovery learning improves students' mathematical representation abilities. Research by Simangunsong (2022) shows that the Discovery learning learning model significantly influences students' mathematical representation abilities. According to Firyal and Tina (2022), Guided discovery learning and CTL can improve students' mathematical representation abilities.

Students' collaboration abilities from cycle I to cycle II can be seen based on collaboration ability indicators that have not experienced a significant increase. After the teacher identified why these ability results tend to remain constant, this is due to, among other things: 1) students are bored of working in groups because, in every mathematics subject, both interest and mandatory, many tasks are done in groups; 2) group members who change frequently; 3) students feel bored using Desmos. Apart from that, the use of cellphones, laptops, and tablets to complete students' worksheets makes the distraction of using these devices to open other applications outside of learning also high; 4) mathematical material is increasingly abstract so that the formulas to be found become more difficult so that group discussions are difficult to build; 5) the teacher determines the selection of group members, this causes students not to be in the same group as their close friends. It was discovered through diagnostic tests that most liked tasks done in groups but not too often. It is in line with research conducted by Masruroh and Arif (2021) that ineffective group work results in a lack of cooperation, not being actively involved in discussions, and boredom using the same learning media. Based on research by Saeful (2022), students' collaboration abilities are still lacking based on the five aspects observed. Students do not have collaboration abilities, especially in group contribution, responsibility in carrying out tasks, and participation in problem-solving.

If we look at the scores obtained by students on the collaboration skills questionnaire, the scores have increased significantly. In cycle I, there were 23 students in the medium category and 12 in the high category. However, after cycle II, students' collaboration abilities increased very significantly. There are 16 students in the very high category, 12 in the high category, and 7 in the medium category. It aligns with research conducted by Pardede (2015), who found that this learning model can improve students' collaboration abilities using media. In the learning that has been carried out, researchers use Desmos media as students' worksheets, which students complete in groups. From this discussion, *guided discovery learning* can improve students' representational and collaborative abilities in circular material.

CONCLUSION

Guided Discovery Learning can improve the mathematical representation abilities of class XI students at SHS 6 Yogyakarta on circle material. The results of the mathematical representation test showed that students' mathematical representation abilities increased; namely, the average pre-cycle score was 78.87; in cycle I, it was 79.17; and in cycle II, it was 85. In addition, the percentage of students' score completion also increased from pre-cycle where 60% of students completed, in cycle I there was 71%, and in cycle II increased to 77%.

Guided Discovery learning can also improve the collaboration skills of class XI students at SHS 6 Yogyakarta on circle material. In cycle I, there were 23 students in the medium category and 12 in the high category. However, after cycle II, students' collaboration abilities increased very significantly. There were 16 students in the very high category, 12 in the high category, and 7 in the medium category.

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