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Angry Birds Revisited: Making Connections Between Mathematics and Physics Meaningful and Joyful

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ABSTRACT

Connecting mathematical reasoning with physical sensemaking remains challenging for many students, and interdisciplinary STEM instruction often struggles to integrate disciplines without one overshadowing the other. This paper presents a digital task sequence designed to support students in making meaningful and joyful connections between mathematics and physics. The design draws on research in mathematics and physics education concerning students' development of meanings for graphs as representations of relationships between quantities and for projectile motion, together with research on students' meaning-making and the affordances of game-based learning. We illustrate the task sequence through a teaching experiment involving two ninth-grade students. The students developed productive meanings for graphs and key ideas of projectile motion by coordinating these ideas throughout the task sequence, demonstrating how meaningful interdisciplinary learning can emerge through joyful engagement. These findings illustrate one approach to designing and implementing meaningful and joyful interdisciplinary STEM learning experiences.

1 | Introduction

Long-haul flights are often tedious, especially for toddlers. Yet, we once observed a three-year-old fully engaged on a flight, repeatedly playing *Angry Birds* on the seatback touchscreen. With focused attention, the child adjusted the virtual slingshot, released it, and watched to see if the bird would hit the target. The moment the bird struck the structure, his face lit up with joy and satisfaction. The child repeated it again and again, transforming an otherwise dull flight into an absorbing experience.

Albeit anecdotal, this experience illustrates the powerful role of games in fostering joyful engagement, even in less favorable contexts. Inspired by such affordances, we designed a task sequence aimed at enriching students' learning through a game resembling *Angry Birds*: launching a projectile to hit a specified target. However, the game differs from the original game in that it is intentionally structured to support students in connecting mathematics and physics. Using a joyful game-based environment to foster these connections is particularly appropriate because integrating the two disciplines is inherently complex. It requires students to coordinate mathematical reasoning with physical sensemaking, a process that many students find challenging (Levi et al. 2026; Redish and Kuo 2015).

Furthermore, the task sequence responds to challenges identified in the literature on interdisciplinary STEM education. When two or more STEM disciplines are integrated, one discipline often overshadows the others (English 2016; Martín-Páez et al. 2019). For example, when science learning is prioritized, mathematics may be reduced to “voodoo maths”, that is, the rote application of mathematical procedures without conceptual understanding, to support scientific inquiry (Wong and Dillon 2019). Conversely, when mathematics is the primary focus, science is often treated merely as a contextual entry point to develop mathematical ideas (Yang et al. 2025). To avoid either discipline overshadowing the other, our task sequence positions both mathematics and physics as the goal. In mathematics, the goal is to support students in developing productive meanings for graphs, whereas in physics, the goal is to develop productive meanings for the fundamental ideas underlying projectile motion.

We argue that our proposed task sequence provides learning experiences that are both meaningful and joyful in connecting ideas of graphs and projectile motion. In the next section, therefore, we briefly discuss what we mean by meaningful and joyful learning. Following that, we describe the study context and present the task sequence, explaining how it supports students in making connections between the mathematics of graphs and the physics of projectile motion.

2 | Meaningful and Joyful Learning in Connecting Mathematics and Physics

We conceptualize "meaningful" in this paper from two complementary points of view: first, our view of the productive meanings students are intended to develop for graphs and projectile motion; and second, students' perspectives in their meaning-making. We drew on a growing body of research in mathematics education on graphs, which suggests that when interpreting and constructing them, students need to conceive of graphs as representations of relationships between quantities (e.g., Moore and Thompson 2015; Paoletti et al. 2023). Such graph reasoning is considered productive because it not only supports the development of more advanced mathematical ideas but is also compatible with ideas outside of mathematics that involve relationships between quantities, such as projectile motion.

Regarding projectile motion, studies in physics education (e.g., Dilber et al. 2009; Rizcallah 2018; Xiong-Skiba et al. 2024) indicate that a key idea is to decompose the motion into two independent yet simultaneous components: horizontal and vertical motions, both coordinated by the variable of time. Conversely, the position of the projectile at any given moment can be understood as the superposition of these horizontal and vertical motions. By representing these motions with graphs and connecting them to the projectile's trajectory, students are provided with opportunities not only to learn the key ideas of projectile motion but also to construct and use their meanings for graphs.

From students' perspective, the knowledge they have is closely related to their ways of knowing (Cook and Brown 1999); that is, knowledge becomes meaningful when it is grounded in situations that allow students to make sense of that knowledge. Accordingly, task design intended to support the development of ideas (graphs and projectile motion, in our case) should involve the creation of problematic situations that foster students' need for those ideas (Harel 2008). In this paper, the problematic situation takes the form of a challenge within a game (which we elaborate on later). As students construct meanings for graphs and projectile motion in response to this situation and apply them to resolve it, they regard these meanings as having "the perceived reasons for its birth in the eyes of the learner" (Harel 2008, 897). In other words, the situation serves not only as a catalyst for students to learn about graphs and projectile motion, but also as a basis for why these ideas become meaningful for them.

Beyond being meaningful, learning experiences can also be made joyful through games (Hwang et al. 2023; Wang et al. 2016). Games are particularly effective in this regard because they typically have three key playful elements: challenge, response, and feedback, all of which are shaped by the selected game design features (Plass et al. 2015). In our digital game, "Hitting the Target," the gameplay involves launching a ball toward a specific target, with the learning focus on graphs and projectile motion. Based on this focus, we designed the challenge, response, and feedback as summarized in Figure 1A. The challenge is to launch the ball so that it hits the target in as few attempts as possible. Students' primary response is to adjust the arrow, which represents the initial velocity vector of the ball (Figure 1B, left panel). While adjusting the arrow, students can anticipate the ball's height and horizontal distance over time by interpreting the corresponding graphs (Figure 1B, right panel). These graphs are presented in a non-conventional way, with the time variable represented on the vertical axis, to foster students' reasoning about relationships between quantities rather than treating the graphs (particularly the height-time graph) as a literal picture of the ball's trajectory (Kristanto et al. 2025; Moore et al. 2014). Students, then, can launch the ball; if it hits the target, they receive on-screen feedback reading "Success!", which is intended to evoke feelings of joy and satisfaction.

Taken together, these considerations informed the design of our task sequence to provide learning experiences that are both meaningful and joyful in connecting mathematics and physics. First, the task sequence is intended to support students in developing productive meanings for graphs as representations of covarying quantities and for the key ideas underlying projectile motion. Second, it presents a problematic situation that serves as both the catalyst for students to construct these meanings and the basis for why these ideas are important to them. Together, these two design considerations are intended to foster meaningful interdisciplinary learning by supporting students in making coherent connections between mathematics and physics. Finally, the game-based environment is intended to promote joyful learning as students engage with ideas of graphs and projectile motion.

3 | Context of the Study

Informed by the aforementioned design considerations, the task sequence was designed to consist of three main tasks (see the Supporting Information for a detailed description).[1] The first task is to explore the vertical and horizontal components of projectile motion by interpreting or constructing their respective graphs. The second task is to sketch the ball's trajectory. Finally, the third task is to play the game.

We conducted a teaching experiment (Steffe and Thompson 2000) with two ninth-grade middle school students, referred to by the pseudonyms Fania and Bianca. They were selected based on their willingness to participate and their ability to articulate

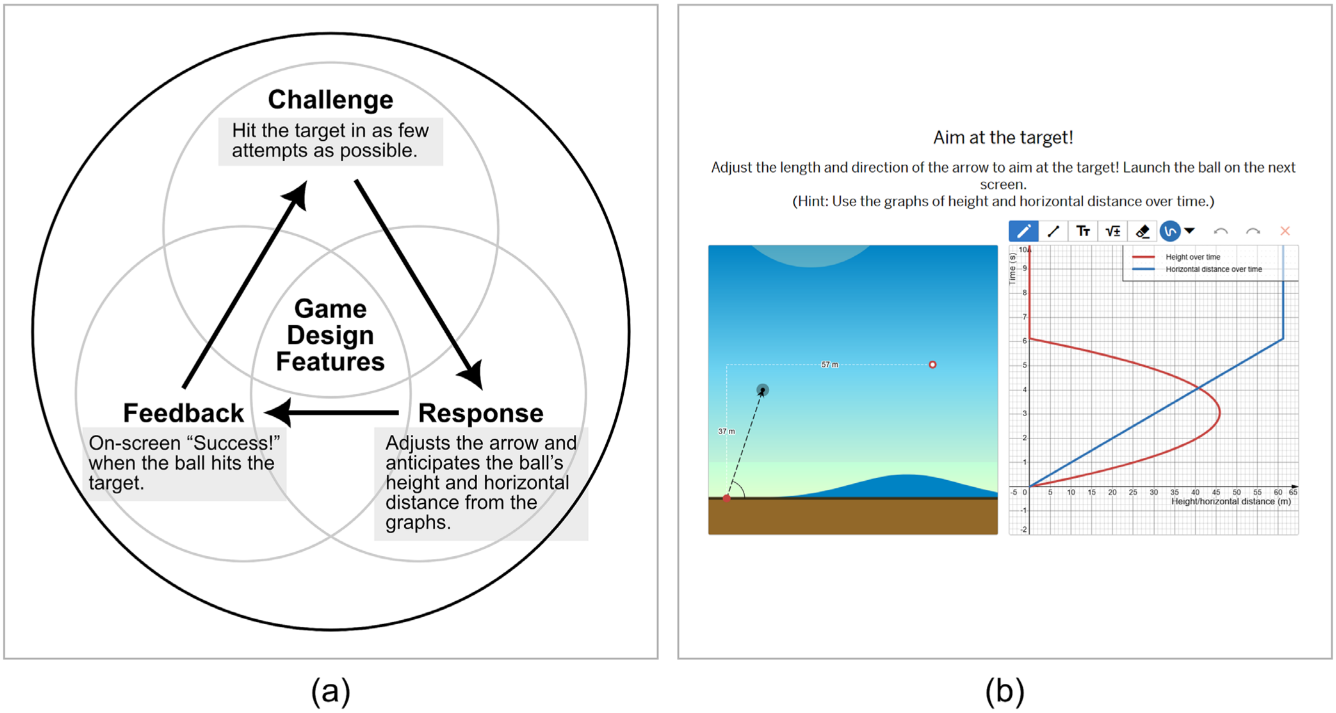

FIGURE 1 | Overview of the “Hitting the Target” game. The game structure was adapted from Plass et al. (2015).

their reasoning during discussion. We chose the teaching experiment methodology, a specialized form of the clinical interview, because our goal was to examine students' learning processes from both mathematical and physical perspectives as they engaged with the task sequence. During the teaching experiment, the first author served as the teacher-researcher (TR). As is characteristic of teaching experiments, the students and the TR learned from one another throughout the instructional interactions (Borba and Confrey 1996). The TR learned from the students by attending to and interpreting their activities in order to provide appropriate responses and follow-up questions when needed. In turn, the students learned through the TR's questions, prompts, and instructional support as they developed their meanings for graphs and projectile motion.

4 | Design and Implementation of the Task Sequence

Here, we describe each task in detail, including its objectives and the learning opportunities it affords. We also present evidence of students' learning as they engaged with the tasks.

4.1 | Task 1: Graphing for Vertical and Horizontal Motions

The objective of Task 1 is for students to develop meanings for graphs of the ball's height and horizontal distance over time while simultaneously learning the key characteristics of the vertical and horizontal components of projectile motion. To support this objective, the task provides learning opportunities for students to interpret graphs in the context of vertical motion and to construct graphs in the context of horizontal motion.

To support students in interpreting the vertical motion, they were provided with linked visualizations consisting of two panels (Figure 2): the left panel displays the situation as the ball moves, while the right panel synchronously represents the relationship between the ball's height and time in that situation. In representing this relationship, we adapted the graph reasoning framework proposed by Paoletti et al. (2023). Guided by this framework, variations in the magnitudes of the quantities of the ball's height and time as the ball moves are respectively represented by the lengths of line segments along the x- and y-axes, and the pairing of these magnitudes is represented as a moving point on the coordinate system. By attending to how this point moves within the coordinate system, students can reason about how the ball's height changes over time.

In the visualization shown in Figure 2, students can also record the trace of the moving point. This feature supports students in developing meanings for the graph of the ball's height over time, and for graphs more generally. Meanings for graphs that may emerge from such an activity include conceiving a graph as a representation of relationships between quantities, as well as understanding that its direction and shape depend on the rate of change of one quantity with respect to another. Such meanings are productive because they can serve as a foundation for students to engage in subsequent activities related to the horizontal component of projectile motion.

For both Fania and Bianca, the linked visualization in Figure 2 helped them interpret the movement of the point on the coordinate plane as representing the ball's height over time. They described the point's movement in three cases: moving upward to the right, upward to the left, and vertically upward. Bianca first interpreted the point's movement upward to the right, and Fania elaborated on and extended Bianca's interpretation.

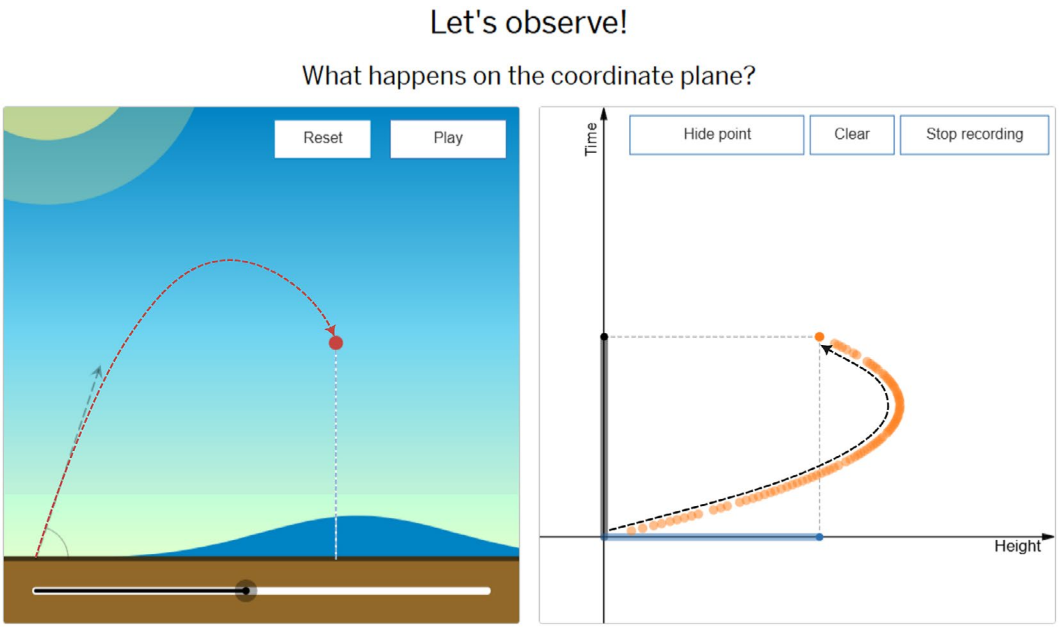

FIGURE 2 | Linked visualizations for vertical motion. The left panel shows the ball's motion, and the right panel displays the graphical representation of its height over time. The red and black dashed arrows in the left and right panels, respectively, are added to illustrate the movement of the ball and the movement of its corresponding point on the coordinate plane.

> When [the point] moves upward to the right, it means that the ball gets higher after it is thrown. Then, after it reaches the highest point, [the ball] goes down.
>
> (Bianca)

> When [the point] moves upward to the right, it means that the ball, which was initially lower, becomes higher. When [the point] moves upward to the left, it means that the ball, which was initially higher, becomes lower. Then, when [the point moves] straight upward, it means that there is no height at all because [the ball] is already on the ground.
>
> (Fania)

These excerpts suggest that both students interpreted the direction of the point's movement on the coordinate plane as representing changes in the quantities of time and the ball's height. Specifically, they recognized that the point's vertical movement represented the passage of time, whereas its horizontal movement represented changes in the ball's height. Beyond this, they also interpreted the curvature of the height-time graph as indicating that the relationship between the two quantities is nonlinear. A more detailed analysis of Fania's and Bianca's interpretations of the graph's curvature is provided in our previous work (Kristanto et al. 2025).

The learning activity for interpreting the horizontal motion focuses on constructing graphs. This activity provides a visualization of the ball's launch, allowing students to observe how the ball's horizontal distance changes over time (Figure 3, left panel). Through such observations, students may notice, for example, that the horizontal distance increases as time increases and that the rate of change is constant. Building on these observations and the meanings for graphs developed in the previous activity, students are expected to construct a linear graph to represent the ball's horizontal distance over time. In this way, the activity provides opportunities for students to apply their meanings for graphs while also identifying a key characteristic of the horizontal component of projectile motion.

As expected, both Fania and Bianca constructed an upward-sloping linear graph to represent the relationship between the ball's horizontal distance and time from the moment it was launched until just before it reached the ground. Fania justified their graph as follows:

> As [the graph] goes to the right, the [horizontal] distance gets farther. As [the graph] goes upward, time also passes. The speed is constant.
>
> (Fania)

Together with their interpretations of the height-time graph, these responses suggest that Fania and Bianca developed meanings for graphs while simultaneously learning about the vertical and horizontal components of projectile motion. They interpreted the height-time graph as representing the ball's height over time and constructed the horizontal distance-time graph to represent the ball's horizontal distance over time. In doing so, they also developed meanings for key ideas of projectile motion by distinguishing the different characteristics of its vertical and horizontal components: the ball's height changes nonlinearly over time, whereas its horizontal distance changes linearly over time.

4.2 | Task 2: Constructing the Ball's Trajectory

The objective of Task 2 is for students to construct the ball's trajectory by superposing its vertical and horizontal motion components. In this task, students are provided with graphs of the ball's height over time and horizontal distance over time on the same coordinate plane. Using this information, they are asked to construct the ball's trajectory on a separate coordinate plane.

To solve the problem in Task 2, Fania and Bianca first plotted two key points of the trajectory: the ball's highest point and the point at which it reached the ground (Figure 4). To determine the coordinates of the ball's highest point, they first identified the vertex of the height-time graph to determine the ball's maximum height.

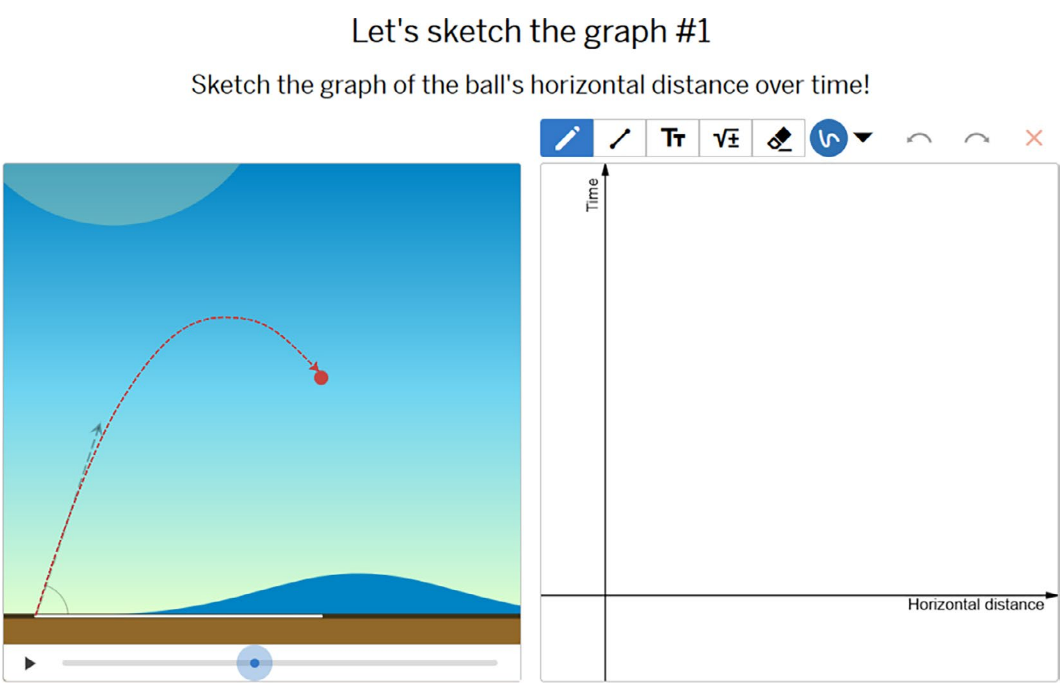

FIGURE 3 | Visualization and sketching panel for horizontal motion. The left panel shows the ball's motion, and the right panel provides the sketching area for students to construct the graph of the ball's horizontal distance over time. The red dashed arrow in the left is added to illustrate the movement of the ball.

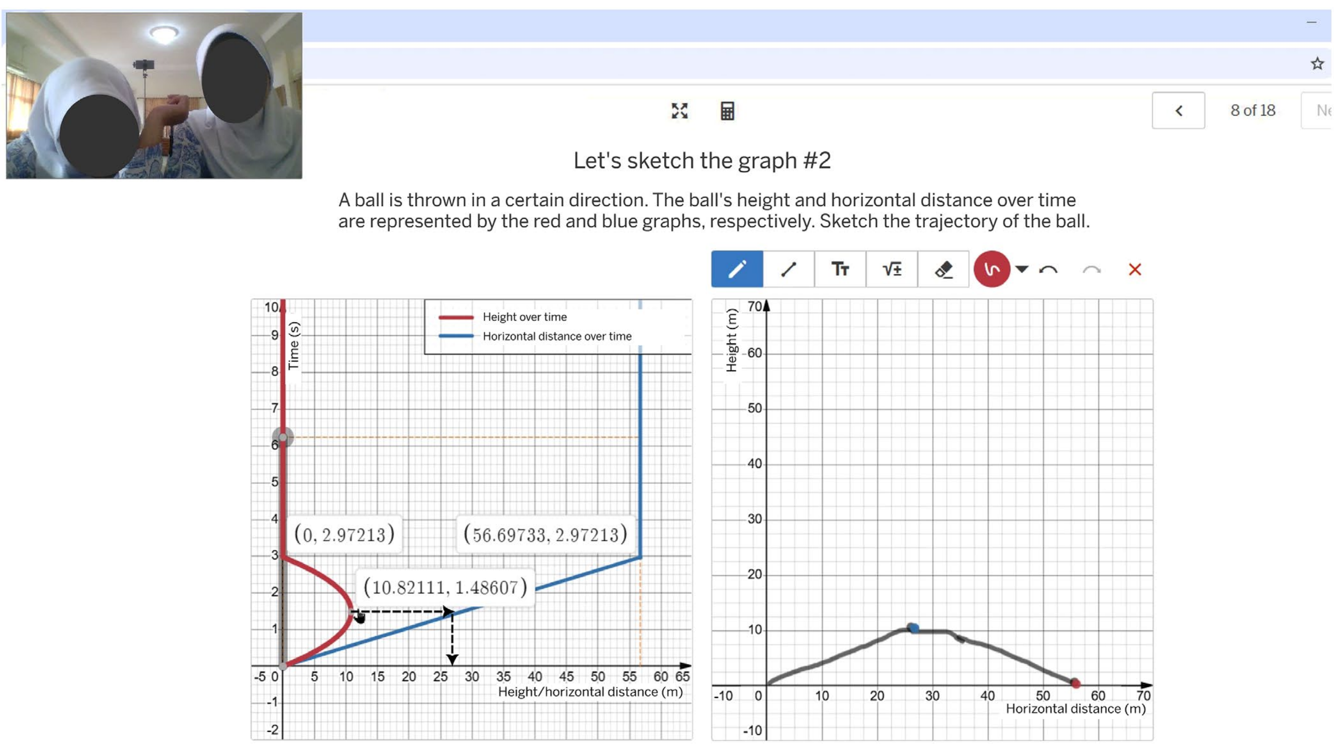

FIGURE 4 | Constructing the ball's trajectory from the height-time and horizontal distance-time graphs. The labels originally displayed in Indonesian have been translated into English for clarity. The black dashed arrows in the left panel are added to illustrate the process of determining the coordinates of the ball's highest point by coordinating information from the two graphs.

They then used the same moment in time to locate the corresponding horizontal distance on the horizontal distance-time graph. This process of determining the coordinates of the ball's highest point is illustrated by the arrows in Figure 4. Fania explained her reasoning for determining this point as follows:

> If it's here [*pointing to the vertex of the red graph, then moving the pointer horizontally to the right*], it's at ... [*clicking the blue graph to display the coordinates of a point*] twenty-seven. So, the highest point is here [*plotting the ball's highest point at approximately (27, 10)*].
>
> (Fania)

Fania and Bianca used the same approach to determine the point at which the ball reached the ground. They first identified the moment when the ball touched the ground using the height-time graph and then used this information to determine the corresponding horizontal distance from the horizontal distance-time graph. After identifying these two key points, they connected them with the origin, that is, (0, 0), by sketching a parabola to construct the ball's trajectory.

The way Fania and Bianca determined these two key points suggests that they drew on their meanings in at least two ways. First, they used their meanings for graphs as representations of covarying quantities to interpret both the height-time and horizontal distance-time graphs. Second, they demonstrated an understanding of the simultaneous nature of the vertical and horizontal components of projectile motion. Specifically, they coordinated the ball's height and horizontal distance at each moment in time to determine its position. More broadly, their construction of the ball's trajectory could be interpreted as a collection of all points representing the ball's position at each moment in time.

4.3 | Task 3: Playing the Game

The objective of Task 3 is for students to apply what they learned from the previous tasks by playing the "Hitting the Target" game. Since the game structure has been described earlier, we focus here on illustrating what Fania and Bianca did while engaging with the game. In this game, they were given a target with a horizontal distance of 48 m from the ball's starting position and a height of 5 m above the ground (Figure 5, left panel). To discourage trial-and-error strategies, they were required to explain their reasoning to the TR before launching the ball. If the TR considered their reasoning sufficiently justified, they were allowed to launch the ball.

After some discussion on how to solve the challenge, Fania came up with the idea that, "When [the ball] is at a [horizontal] distance of 48 meters, its height should be 5." She articulated this idea by adjusting the launch arrow so that a point on the red graph corresponded to 5 and was horizontally aligned with a point on the blue graph corresponding to 48 (Figure 5, right panel). We infer that Fania's idea, and the way she represented it on the graphs, was grounded in her coordination of meanings for graphs as well as her understanding of the simultaneous nature of the vertical and horizontal components of projectile motion.

Bianca agreed with Fania's strategy, and they decided to launch the ball using this setting. The ball indeed hit the target, and on-screen feedback reading "Success!" was displayed. They spontaneously expressed joy and satisfaction, with Bianca clapping her hands and Fania raising both arms. After a moment, Fania reflected spontaneously and said:

> It turns out that [to solve the challenge] the way to read them [the graphs] really has to be, like, understood.
>
> (Fania)

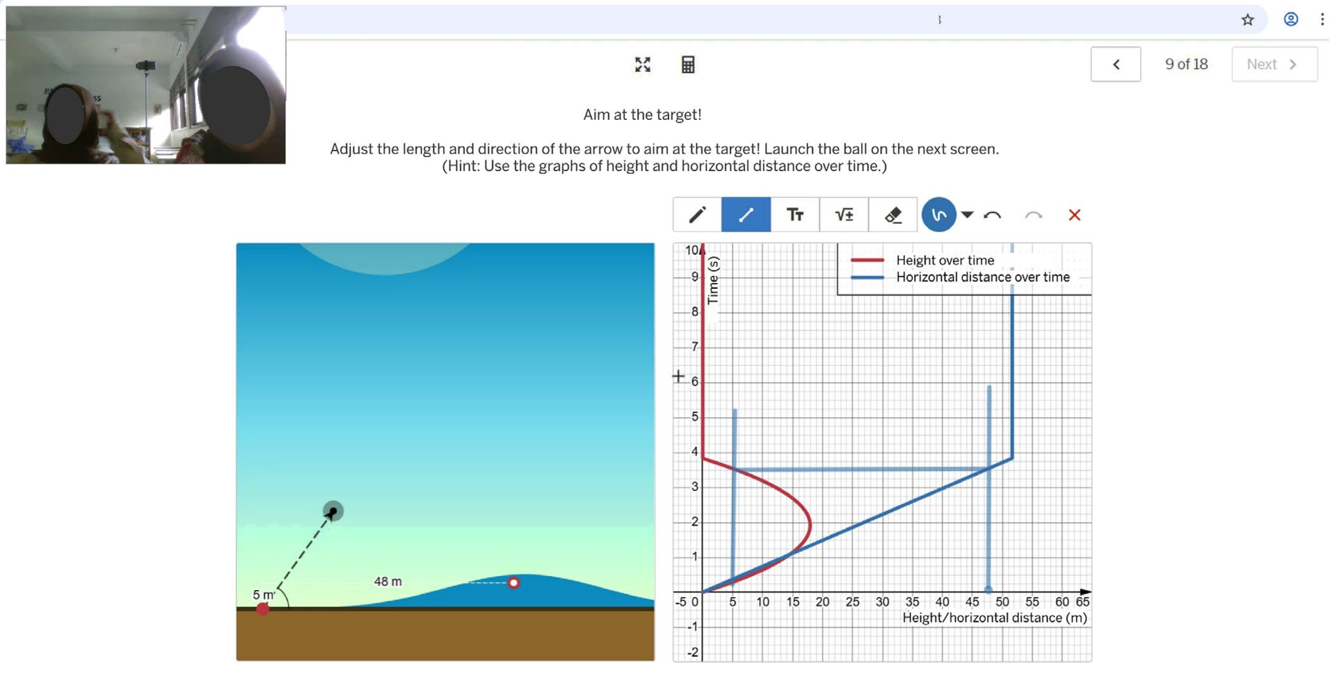

FIGURE 5 | Fania and Bianca's screen during the "Hitting the Target" game. The labels originally displayed in Indonesian have been translated into English for clarity.

Fania's statement demonstrates how she perceived the value of her meanings for graphs and projectile motion in resolving the challenge. This indicates that the game not only produced joyful experiences but also made the knowledge she gained meaningful.

5 | Discussion and Conclusion

We have exemplified the design of tasks that make connections between mathematics and physics both meaningful and joyful, and illustrated how students learned through engaging with the tasks. We next discuss how our task design contributes to the literature on connecting mathematics and physics, as well as to broader efforts to foster interdisciplinary STEM learning. We conclude by considering the pedagogical implications of our work, its limitations, and opportunities for future research.

5.1 | Students Connecting Mathematics and Physics

The task sequence in this study supported Fania and Bianca in developing productive meanings for graphs and key ideas of projectile motion, with the development of these mathematical and physical ideas being closely intertwined. In Task 1, for example, observing the dynamic phenomenon of projectile motion helped them make sense of graphs as representations of covarying quantities. Conversely, their meanings for graphs enabled them to clearly articulate the relationships among the quantities involved in the vertical and horizontal components of projectile motion. Thus, they not only achieved the intended learning goals in both mathematics and physics, but did so by developing these ideas in connection with one another. This meaningful connection between mathematics and physics in students' learning is consistent with recommendations in the literature on connecting mathematics and science (Pang and Good 2000; Panorkou and Germia 2021; Tekerek et al. 2023), as well as the broader STEM education literature (Bybee 2013).

The game in our task sequence created a joyful learning experience for Fania and Bianca. This observation aligns with previous research suggesting that well-designed games can promote joyful and engaging learning experiences (e.g., Brizuela and Strachota 2024; Plass et al. 2015). The challenge it presented allowed them to apply the meanings they had developed for graphs and the key ideas of projectile motion. Beyond providing an engaging context for applying these ideas, the game also gave students a justification for why developing meanings for graphs and projectile motion was important, as evidenced by Fania's final remark. Obtaining such a justification for the importance of an idea is essential for creating meaningful learning experiences (Harel 2024).

5.2 | Pedagogical Implications, Limitations, and Opportunities for Future Research

Drawing on both the design of the task sequence and the ways in which students engaged with it, we discuss several implications for task designers and teachers. First, we argue that designing interdisciplinary tasks should begin with clearly articulated intended meanings or ways of reasoning that cut across multiple disciplines. In our case, we began with the value of graph reasoning grounded in relationships between quantities. Such reasoning about graphs is not only crucial for learning mathematics, but also for learning and working in other STEM fields (Paoletti et al. 2022). This starting point enabled us to connect this form of reasoning to key ideas in STEM disciplines, namely, projectile motion in this paper.

Second, we contend that providing students with opportunities to articulate their reasoning is essential. Such opportunities make students' connections between mathematical and physical ideas visible through their reasoning, rather than merely through their final answers (Johnson et al. 2020; Slavit et al. 2021). For instance, if we had not known Fania's reasoning behind her success in solving the challenge presented in Task 3, we could not determine whether her success was based on trial and error. In contrast, by attending to her reasoning, we witnessed a brilliant moment in which she coordinated the graphs of the vertical and horizontal motions. For teachers who wish to implement this task sequence in their classrooms, we therefore recommend incorporating scaffolds that invite students to explain and justify their reasoning (see the Supporting Information for a more detailed teacher's guide). For example, asking, "What do you notice? What do you wonder?" (see Rumack and Huinker 2019) can initiate students' exploration of the underlying ideas. Teachers should also intentionally facilitate productive classroom discussions, for instance by drawing on the discussion practices proposed by Smith and colleagues (2018; 2008). Productive classroom discussions enable students to construct and evaluate both their own and one another's reasoning, thereby fostering collective sensemaking (Stein et al. 2008).

Before concluding, we acknowledge several limitations of this study. First, we presented the activities of only two students. Other students may engage with the task sequence in different ways and develop different meanings. Future studies could therefore investigate how a broader range of students engage with the task sequence. Second, facilitating learning with two students is considerably different from facilitating learning in a whole-classroom setting, where interactions are inherently more complex. Further research is therefore needed to examine how students collectively develop meanings for graphs and key ideas of projectile motion in classroom settings.

5.3 | Concluding Remarks

In conclusion, we have elaborated a task sequence designed to support students in making meaningful and joyful connections between mathematics and physics. By coordinating graph reasoning with key ideas of projectile motion, the task sequence addresses the challenge of helping students connect mathematical reasoning with physical sensemaking while positioning both disciplines as explicit learning goals. Rather than using mathematics merely to support science, or science merely as a context for mathematics, the task sequence illustrates one way of fostering interdisciplinary learning. Future work could investigate how this design approach can be adapted to other STEM contexts in which students must coordinate mathematical and scientific ideas, such as rates of change in calculus, chemical reaction rates, or other dynamic phenomena represented through graphs. Just as *Angry Birds* inspired the joyful experience described at the beginning of this paper, we hope this work encourages teachers, task designers, and researchers to look beyond the game itself and consider how its engaging qualities can be thoughtfully combined with meaningful disciplinary learning. In doing so, joyful and meaningful learning experiences can become a foundation for designing interdisciplinary STEM learning in which disciplinary ideas are developed through one another rather than in isolation.

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Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

Endnotes

[1] The complete digital task sequence is accessible online and can be copied or adapted: https://classroom.amplify.com/activity/6a5b501520d7c296792eca8e. A standalone version of the game is available at https://people.usd.ac.id/~ydkristanto/index.php/learning-resources/problem-solving-projectile-motion/.

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Supporting Information

Additional supporting information can be found online in the Supporting Information section. **Data S1:** Supporting Information.